# Essentials of ALTERNATING CURRENTS

Second Edition

W. H. TIMBIE · H. H. HIGBIE

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Essentials of
Alternating Currents

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# ESSENTIALS OF ALTERNATING CURRENTS

BY

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## PREFACE TO SECOND EDITION

The past decade has seen a wide extension in the application of alternating current power, both to the home and to industrial plants. This extension has been particularly noticeable in improved methods of starting and protecting alternating-current motors, the wider use of static condensers for power factor correction, the introduction of more efficient rectifying devices, and the development of the small capacitor motor, especially for household appliances.

This second edition has been prepared in order to include the operating principles of these latest devices, of which the practical electrician must have a thorough understanding.

The work of revision has been done by Professor Samuel H. Caldwell, of the Department of Electrical Engineering at Massachusetts Institute of Technology, who brought to the task a wide experience in the field of modern alternating-current practice. To him the authors express their sincere thanks and deep appreciation.

W. H. Timbie
H. H. Higbie

Cambridge, Mass.
Ann Arbor, Mich.
*March*, 1939.

## PREFACE TO FIRST EDITION

In writing this text the authors have tried to include in it only that material which really represents the essentials that a worker on alternating-current appliances should know and know well. Since the education of many of these men is limited to that given in the ordinary grammar school, a method of presentation has been adopted which, it is believed, will enable them to grasp and retain the fundamental information.

We believe this book has these four desirable qualities:

(1) It deals with the information and problems of alternating-current practice which an electrical worker is most likely to meet in his trade.

(2) It is written in simple language.

(3) It avoids the use of algebra and trigonometry.

(4) It is the result of several years of experience in teaching alternating-current electricity in short intensive trade courses for electric wiremen at Wentworth Institute.

It is hoped that it will be found equally well adapted to similar practical courses given in Trade, Industrial and Technical High Schools, where it is desired to impart in the minimum time the maximum amount of information concerning the laws and practice of alternating currents — information which will be in an immediately usable form and which can still serve as a substantial foundation for more advanced work.

The authors desire to express their appreciation to Mr. Arthur L. Williston, Principal of Wentworth Institute, to Mr. Joseph M. Jameson, Vice President of Girard College,

and to our colleagues Mr. Wallace J. Mayo and Mr. George M. Willmarth for their assistance in developing the course as here outlined. Grateful acknowledgment is also extended to Mr. Ernest S. Schuman of Wentworth Institute for solution of the problems and criticism of the text.

W. H. T.
H. H. H.

Boston, Mass.
Ann Arbor, Mich.
*June*, 1918.

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# ESSENTIALS OF ALTERNATING CURRENTS

## CHAPTER I

### MODERN SYSTEMS OF POWER TRANSMISSION

In beginning the study of alternating-current electricity it is desirable to get in mind the general composition of a typical alternating-current system. It is the purpose of this chapter to describe such a system and to explain in a general way what each part is and how it is used. Later chapters take up the various machines and devices in greater detail and explain the underlying principles upon which they operate.

**1. Central Power Station.** In our present-day civilization, some form of power is needed in almost every building, whether it be located in a large city or in a small village, or even on an isolated farm. This power may be used in many ways and frequently the amount of power required at any one place is small. To illustrate: In a manufacturing plant having several buildings scattered over a large tract of land, we find that power is used for various manufacturing purposes throughout the plant; for machine tools in the repair shop; and for crane and elevator service. Lights are supplied in every part of the buildings, and flood lights outside illuminate the yard. Clocks are electric-driven, and numerous bells and annunciators are in constant use. In the boiler room, motor-driven oil-burners are used to supply steam for manufacturing processes, and motor-driven auxiliary pumps stand ready to meet any demand caused by the outbreak of fire. Fans and air-conditioning equipment help maintain

efficient working conditions, and during rest periods a public-address system broadcasts entertainment to the employees.

Most of these devices require very little power. An incandescent lamp or a desk fan requires about $\frac{1}{10}$ horsepower, and a clock requires less than $\frac{1}{100}$ horsepower. A flood light requires $\frac{1}{2}$ horsepower to 1 horsepower, and the machine tools in the repair shop require from $\frac{1}{4}$ horsepower to 3 horsepower. To be sure, large quantities of power are required for manufacturing purposes, but the machines to be driven are located on the several floors of the different buildings.

Under such circumstances it would be neither practicable nor economical to install an engine or a water wheel at each place where power is needed. The only practical plan is to generate in one "**power station**" all the power that is needed for the entire plant, and then to distribute this power in some way to the places where it is to be used. In modern practice, we frequently go one step further and purchase the power directly from the public utility company, thus centralizing the production of power for many users in one or more large power stations. Besides the lower cost of power, the additional advantage of improved reliability is generally obtained by this method.

Fig. 1 shows a power station with generators operated by Diesel engines. These machines deliver 2300 volts and supply a comparatively small local demand. In Fig. 2 is shown part of a modern, high-power central station, operated by steam turbines. This station is part of the power system for the city of Chicago.

**2. Selecting a Method of Transmitting Power.** If a power station is installed in the plant, the next problem is to determine the best method of distributing the power that is generated to each separate machine or device where it is to be used. There are four common ways of transmitting power to considerable distances: First, mechanically by means of belts or rope-drives and shafting; second, by steam under pressure

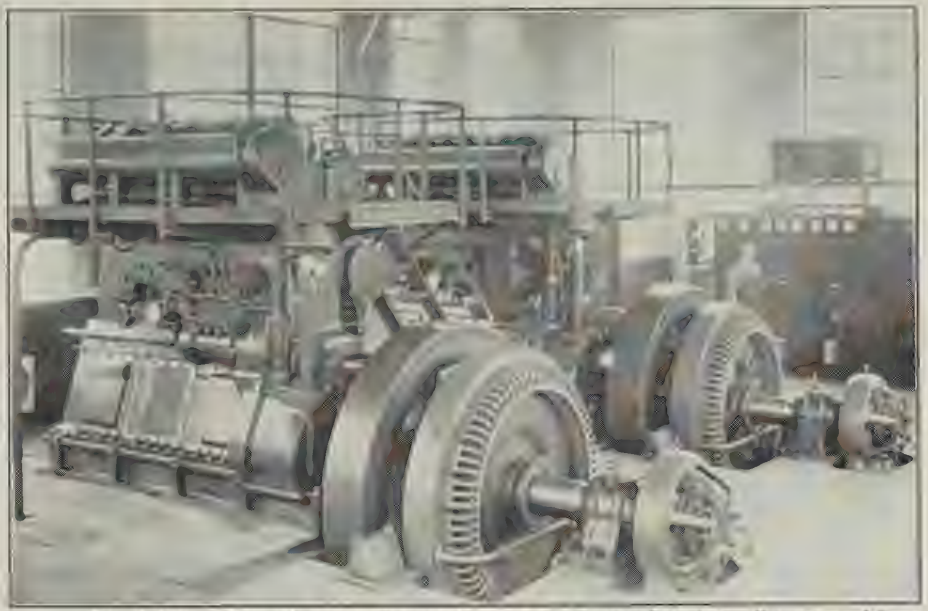


FIG. 1. Diesel-driven alternating-current generators in a small central station. Generators deliver 250 kilowatts at 2300 volts. *General Electric Co.*

FIG. 2 Modern high-power central station using turbine-driven generators. These units will produce 208,000 kilowatts. *General Electric Co.*

flowing in mains; third, by compressed air; fourth, by electricity. Which of these four methods of power transmission is best suited to the requirements of a particular plant depends upon the surrounding conditions, and is a question which requires careful study. Each of these methods has its advantages. There are instances in which a single one of these four modes of power transmission should be used in preference to any of the others. Where distances are very short, mechanical transmission of power by belts and shafting is frequently the cheapest and most efficient method. Where installations are temporary, where the distances to which power must be transmitted are not too great, and where the quantities of power required are small, steam under pressure may often be used to advantage. An illustration that might be cited of this type of transmission is the scheme for rock drills, hoisting engines, elevators and pumps needed for a subway or a large-building excavation. Compressed air is used in preference to steam where more flexibility is required, where the distances are greater, and where freezing temperatures will be encountered. The development of efficient, portable compressors has led to the wide use of compressed air for temporary power requirements.

**3. Advantages of Electrical Transmission of Power.** Electricity has many advantages over other types of power transmission, especially where the distances to which power is to be sent are great. Electricity travels on wires that do not move. These wires may be bent in any direction. They easily pass obstructions and may be supported in a great variety of ways. The cost of such a transmission line is relatively small and, when installed, the line is subject to but small depreciation and wear. Fig. 3 shows a section of a modern electric transmission line used to transmit 270,000 horsepower from water-power stations located in the Sierra Nevada range, across the Mojave Desert, to the vicinity of Los Angeles, a distance of 241 miles.

FIG. 3. Section of a modern, high-voltage transmission line, operating at 230,000 volts. *Southern California Edison Co. Ltd.*

Electric power may be started, stopped and controlled by devices that are more precise and rapid in their operation and more compact and durable in their construction than those which must be used when power is transmitted by other means; and it is suited to a greater variety of uses. Also, for long-distance transmission, electricity is more economical than any other kind of power. Central power stations have consequently become, in almost every instance, **electric generating stations.**

**4. Advantages of Alternating-current Electricity for Central Stations.** Experience has shown that large central plants and those having great diversity of service can be operated more efficiently than smaller plants or than plants having little diversity of service; hence there has been constant growth in the economical size for central generating stations. There are now examples of single stations capable of generating 300,000 kw; and plans are being made for still larger stations. To use the machinery in such plants to the best advantage and to operate at as steady a load as possible, longer and longer transmission lines are being planned. Interconnection of complete distribution systems by transmission lines known as "tie-lines" is another method employed to secure more uniform generator loads and more reliable service.

The desire to transmit electrical power as far as possible with a minimum loss has resulted in the present very general use of alternating-current electricity.

The reason is simple. Electric power depends upon two factors, current and voltage. We may transmit a given amount of power in either of two ways. First, by means of a large current at a low voltage; or, second, by means of a smaller current at a correspondingly higher voltage. The smaller the current used, however, the smaller will be the loss of power in transmission. Hence, for long-distance transmission, in order to secure a small loss, we must use a

small current and consequently as high a voltage as is practicable.

Electrical engineers have long recognized the fact that at high voltages it is more efficient and cheaper to transmit direct-current power than alternating-current power. However, no completely satisfactory system for generating and using direct current at high voltages has yet been developed. The great advances made in the development of high-power vacuum tubes and gaseous conduction devices in recent years have again aroused widespread interest in the possibility of achieving high-voltage, d-c transmission and distribution systems using these new tools.

On the other hand, alternating current may be simply and inexpensively stepped up from low voltages, at which it may be generated, to high voltages at which it may be transmitted over wires; and then it may be stepped down again to whatever voltages are desired for use. The instrument used for doing this is called a **transformer.** Unfortunately, no device of this nature is available for use with direct current.

It is now customary to generate alternating-current electricity in large central stations at voltages as high as 22,000 volts, although most installations operate at about 14,000 volts, and alternating-current motors can be built to operate on about the same voltages. Beyond these values, it becomes difficult to insulate the windings of the generator or motor.

Where high-voltage transmission is required it is customary to place in central stations **step-up** transformers which increase the voltage from that at which the current is generated to whatever voltage may be desired upon the transmission line. This voltage may be only a few thousand volts or it may be as high as 220,000 or even 275,000 volts, depending upon the quantity of power that is to be transmitted and the distance it must be sent. Such transformers

are exceedingly compact and their efficiency may be as high as 99 per cent. They are comparatively inexpensive and have no moving parts to require attendance.

It is not advisable to carry the full voltage of long transmission lines into a town on account of the danger of contact with buildings or trees or with other electrical conductors. Often municipal ordinances forbid it. Therefore, a **transformer substation,** situated on the outskirts of a town, is used to **step down** the voltage from that used on the main transmission line to a voltage that is suitable for the distribution of current to the different consumers. Voltages used on such distributing systems differ very greatly according to circumstances. In large systems, distribution voltages of from 13,000 to 22,000 volts may be used. Smaller systems frequently use a distribution voltage of 2300 volts.

Fig. 4 shows a group of step-down transformers in a substation at the Los Angeles end of the Boulder Dam lines. These units drop the voltage from 275,000 volts to 13,200 volts, and each transformer can carry 65,000 kilowatts.

Near the points where power is consumed smaller transformers are used for stepping down the voltage of the current in the distributing system to 115 volts, 230 volts, 550 volts, or to whatever other voltage may be required by lamps or other apparatus.

**5. Converter and Rectifier Substations.** While a very large percentage of the electrical power now generated in central stations in the United States is generated in the form of alternating current, many applications of electrical power require direct current. Therefore, direct current must usually be available from the distributing system, even though alternating current is required for transmission. Fortunately several devices have been developed for " converting " or " rectifying " alternating current into direct current.

One of the most widespread types of rectifiers is that found

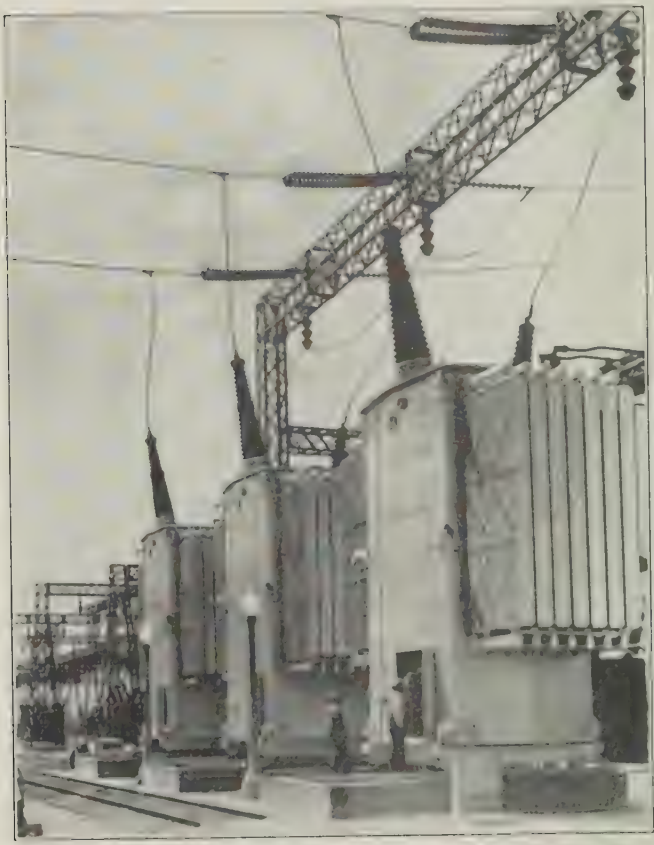


FIG. 4. Transformer substation receiving power from Boulder Dam at 275,000 volts and supplying distribution system in Los Angeles at 13,200 volts. *Westinghouse Elec. and Mfg. Co.*

in every radio set which operates from the alternating-current supply. In some cases this is a vacuum tube, but most of the newer sets have tubes containing mercury-vapor. Similar tubes are used in the amplifiers of the sound systems in motion-picture theatres. These tubes have been developed in recent years to the point where they can handle very large amounts of power. A good example of this is shown in Fig. 5; the rectifier contains only three small tubes, but it is capable of supplying a total direct-current load of over 6 horsepower. Units of this type are used extensively to supply direct current for magnetic clutches, chucks, and separators, and for operating special types of office machinery.

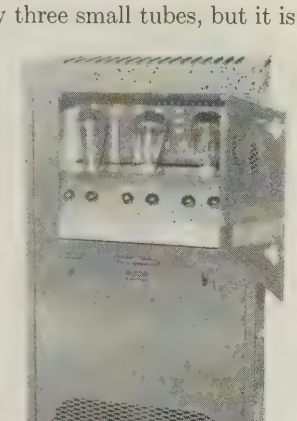


FIG. 5. A modern industrial rectifier. *Raytheon Manufacturing Co.*

Where large quantities of direct current are needed to supply communities, large manufacturing plants or electric railways, **converter substations** are erected. These substations contain apparatus for converting alternating current at the high voltage used on the transmission line into direct-current electricity at the lower voltage used by the apparatus of the consumer.

This conversion of high-voltage alternating current to low-voltage direct current is done in two stages; the high-tension alternating-current power is transformed first into alternating-current power of lower pressure by means of the step-down transformers already referred to; and then the lower-pressure alternating-current power is converted

into direct-current power at a suitable voltage by means of rotating machines, or by means of large mercury-arc rectifiers.

The rotating machine converter may be one of two types: First, the **motor-generator converter,** which consists of an alternating-current motor operated by the transmission line and mechanically coupled to a direct-current generator. This is the most flexible type of converter and is adaptable to the greatest variety of conditions. It is, however, more expensive than the second type of rotary converter.

Second, the **synchronous converter** (commonly called the **rotary converter**). This type of converter, shown in Fig. 6, does practically all the work of a motor-generator converter. It has, however, only one armature and only one field structure. It is correspondingly less flexible. It is, however, also less expensive, and usually more efficient.

There is now an increasing tendency to equip new converter substations with large mercury-arc rectifiers, and even to replace older rotary types with the mercury-arc type. Recent developments have eliminated many of the operating difficulties formerly encountered with this type of rectifier. The mercury-arc is preferred because of its higher efficiency and lower operating cost.

**6. Alternating-current System for Short Transmissions Requiring no Step-up Transformers.** An old empirical rule which gives satisfactory results within reasonable limits, states that the proper transmission-line pressure should be about 1000 volts for each mile in length of the line; for instance, 2300 volts may well be used to transmit current within a radius of about two miles from the central station.

With modern generators built to deliver 13,000 volts, it is possible to supply transmission systems covering a radius of ten to fifteen miles without using step-up transformers. One or more generators feed the station bus-bars to which the transmission line can be connected directly through the switchboard. Small distribution transformers are then con-

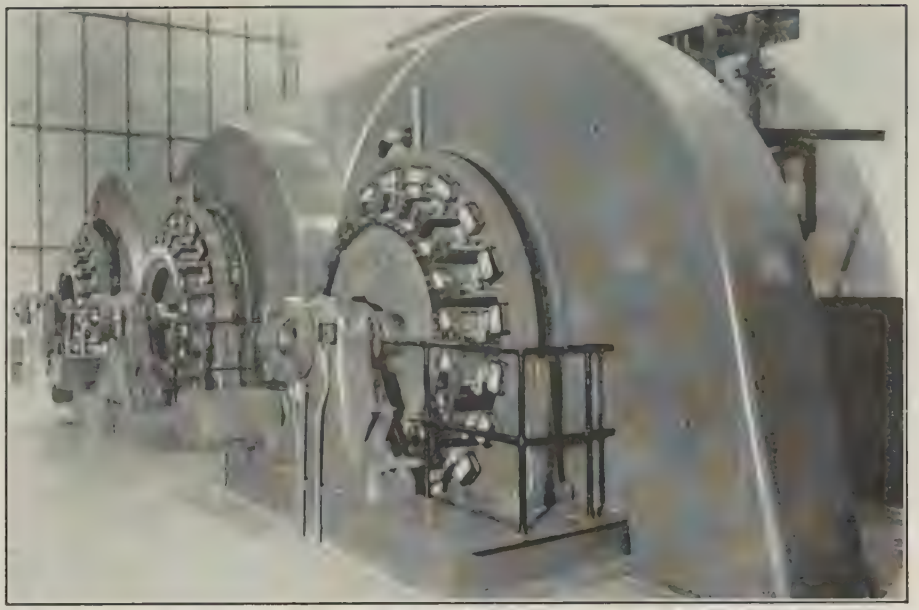


FIG. 6. Synchronous-converter substation supplying power to part of the New York subway system. *General Electric Co.*

nected to the line at various appropriate points, and from these lower voltages are obtained for use by individual consumers.

Fig. 7 is a diagram illustrating an alternating-current system of this type. It is drawn to represent the usual three-phase installation having three wires for each circuit.

"A" The main generator A is connected directly to the three-phase transmission line. The alternating-current generator must have its field magnets excited from a separate source of direct current, usually from a small direct-current generator either on the same shaft with the alternator (as shown) or driven by an independent source of power.

"B" A three-wire three-phase transmission line is represented in the figure by the three lines B, B, B. To the transmission line is attached the following service equipment.

"C" A single-phase transformer steps down the voltage of the line to supply lights and other equipment in the generating station.

"D" A special group of transformers changes the supply to six-phase instead of three-phase.

"E" The bank of transformers D supplies six-phase power to the mercury-arc rectifier E. The rectifier provides direct current for street-railway use. A more uniform direct current is obtained from a six-phase supply than from a three-phase supply. Also, the rectifier reliability is improved by the use of a six-phase power supply.

"F" A single-phase transformer steps down the voltage to supply light and power in a residential district, using three-wire, single-phase distribution.

"G" A group of single-phase transformers are connected to supply three-phase power at reduced voltage to three-phase induction motors, H, in a factory. An in-

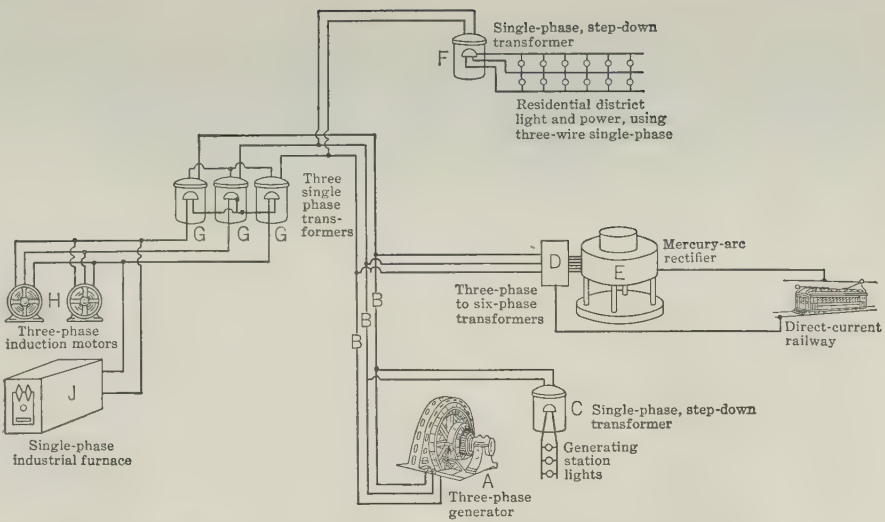



FIG. 7. A short transmission system. The generator $A$ is connected directly to the three-phase line $B$. Single-phase lighting loads are supplied by transformers $C$ and $F$. Transformers at $D$ feed a mercury-arc rectifier $E$ which converts power from the line into direct current. Three-phase motors $H$ are supplied through transformers $G$. An industrial furnace $J$ is also supplied by the same transformers.

dustrial furnace, J, is operated single-phase on the low-voltage side of the transformers.

Notice that although three-phase distribution is used, many single-phase loads are put on the system. The single-phase loads should be so connected to the line wires that the three phases are as nearly equally loaded, or **balanced,** as possible.

**7. Alternating-current Systems for Long-distance Transmission where Step-up Transformers are Required.** Where the transmission of electric power must be made to distances greater than about five or six miles or at voltages higher than from 6600 to 13,000 volts, the system is usually increased by the addition of step-up or central-station transformers and also transformer substations. A typical long-distance system of this type is illustrated diagrammatically in Fig. 8. This includes the following principal items of equipment.

"A" The main generator located at the central power station. In large systems, this generator is frequently driven by a steam turbine. The generator delivers power at about 13,000 volts to

"B" A set of three step-up transformers in which the voltage is raised to a value suitable for transmission, in this case 230,000 volts.

"C" A three-wire, three-phase transmission line distributes power in large quantities to the points where it is needed. These places are often called "load-centers" because a large amount of power is supplied for various purposes, with only one connection to the transmission line.

"D" At each load center, step-down transformers must be connected so that power can be obtained at usable voltages. In this case, the transformers reduce the line voltage to 2300 volts for transmission to various parts of the load center. Some apparatus is built to

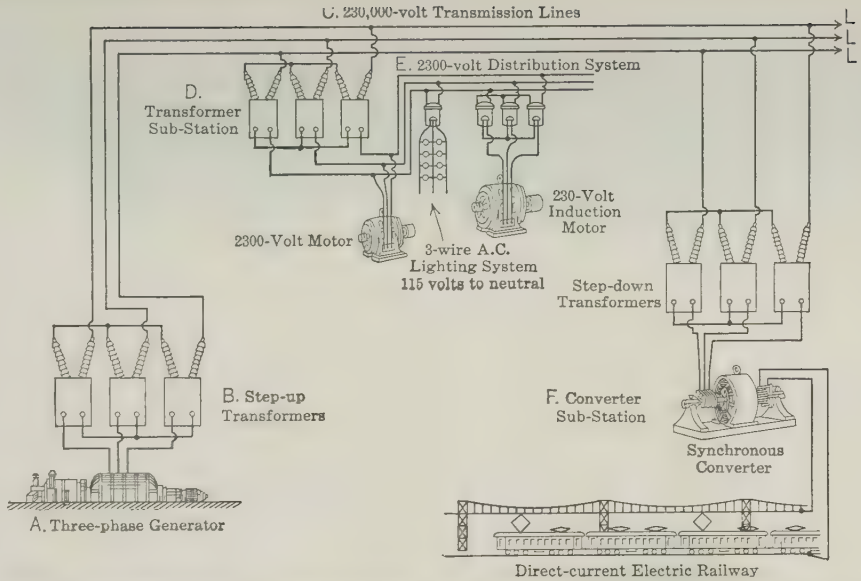



FIG. 8. A long transmission system. The generator $A$ supplies power to the step-up transformers $B$, in which the voltage is raised for transmission by the line $C$. Transformers at $D$ reduce the voltage for distribution over the 2300-volt system $E$. Some apparatus can operate directly on $E$, but most power and light is supplied through distribution transformers. At $F$, a rotary converter receives power through another set of step-down transformers, and delivers direct current to an electric railway.

work directly from this distribution voltage (as for example the 2300-volt motor shown), but generally further reduction is required.

"E" The 2300-volt distribution system is shown with additional transformers arranged to reduce the voltage to conventional light and power values.

"F" Electric railway operation usually requires that power be supplied to the trolley wire (or third rail) at many points along the line. It is more efficient to transmit the power to distant points at high voltage and then reduce the voltage and convert, if necessary, to direct current for train operation. Here a synchronous converter is shown, but many modern installations are of the mercury-arc type. Many of the converter substations are entirely automatic in their operation.

"L" The section of the transmission line L, L, L may be continued to supply other load centers. Also, a number of systems may be connected together in order to obtain higher reliability, and operating economies.

## SUMMARY OF CHAPTER I

**POWER** is obtained from coal, oil and water, by the use of prime movers in the form of steam engines, gas engines and water wheels.

**CENTRAL POWER STATIONS** are established because it is inefficient and too expensive to place a prime mover near each place where a small amount of power is required.

**CENTRAL STATIONS ARE ELECTRICAL** because electrical power can be transmitted more cheaply and more conveniently and turned to a greater number of uses than any other form.

**THE LOCATION** of these power stations is as near the center of the region to be served as possible. Water wheels, however, must be located near the water supply.

**ALTERNATING CURRENT** is generated by these central stations because remarkably efficient machinery has been de-

vised for "stepping up" the voltage and getting the great advantage of transmitting at high voltage. The same machine, a transformer, "steps down" the voltage, allowing it to be used at a low pressure. Transformers will not operate on direct current.

CONVERTER SUBSTATIONS are placed at points along the transmission line where a large amount of direct current is needed, and synchronous converters, motor-generators, or mercury-arc rectifiers are installed, which change the alternating current to direct current. For converting small amounts of alternating-current power to direct-current, thermionic tubes of various types may be used.

TRANSFORMER SUBSTATIONS are erected wherever it is desirable to step down from the transmission voltage, which may be as much as 275,000 volts, to a city circuit usually of about 2300 volts, for the sake of greater safety to human life. At the immediate points where the power is to be used, small individual transformers change this 2300 volts to the 500, 230 or 115 volts desired.

SHORT TRANSMISSION SYSTEMS for transmitting power ten to fifteen miles or less consist of an alternating-current generator of from 2000 to 13,000 volts, connected directly to the line. At the receiving end of the line, synchronous motors, induction motors or converters may also be attached directly to the line. By attaching transformers to the line, small motors, incandescent lamps and other apparatus may be run at their proper low voltage.

LONG TRANSMISSION SYSTEMS are those which transmit power more than five or six miles. The generator delivers about 13,000 volts, but this is "stepped up" by station transformers, sometimes as high as 275,000 volts, before it is delivered to the line. Wherever power is to be used, a transformer substation is erected. The transformers "step down" the transmission voltage to a distribution voltage which, depending upon the size of the system, may be from 2300 to 22,000 volts. At the load points within the area served by a distribution system, additional transformers further decrease the voltage to the values usually required for lighting and power. When direct-current power is needed, converters are usually located either at the transformer substation or at a special substation with transformers connected directly to the conversion equipment.

## CHAPTER II

## TRANSFORMERS. FUNDAMENTAL IDEAS

Most of the world's electric power is generated and used in the form of alternating current. This is due principally to the fact that the voltage of an alternating-current circuit can be increased or decreased to satisfy most efficiently the requirements of transmission, distribution, and use. The device which makes this possible is the transformer. To understand how it operates, it first will be necessary to understand clearly the nature of an alternating current.

**8. How an Alternating Current Differs from a Direct Current.** In Fig. 9 we have a wheel $W$ mounted on a shaft $S$. A series of blades $B$, $B$, $B$ . . . , extend from the outside of the wheel and dip into a stream of water moving steadily in the direction shown. We know that the moving water will push the blades and thus cause the wheel to rotate as shown.

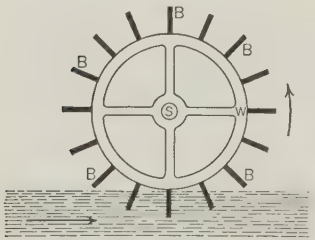



Fig. 9. The steady flow of water pushes the blades $B$–$B$ and turns the wheel in the direction shown.

Now suppose we have another wheel arranged with a piston and crank as shown in Fig. 10. By admitting steam to the cylinder alternately through the openings $A$ and $B$ we can force the piston $P$ back and forth in the cylinder, and this motion, when transmitted to the wheel by means of a crank, will cause the wheel to turn in the direction indicated. If we wish, we can mount the wheels shown in Fig. 9 and 10 on the same shaft so that the water wheel can be used when

water is plentiful, and the steam drive can be used when the water is low.

Although power can be obtained from the shaft $S$ by either of the methods shown, the forces acting in the two types of drive are quite different. In Fig. 9 the wheel is turned by a steady, continuous force, due to the continuous motion of the

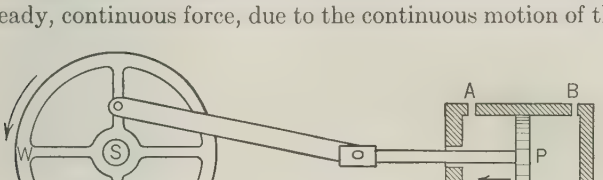



FIG. 10. Power to drive the wheel $W$ is secured from an alternating motion of the piston $P$.

water in the stream. In Fig. 10, however, the force acting to turn the wheel comes from the motion of the piston, and its

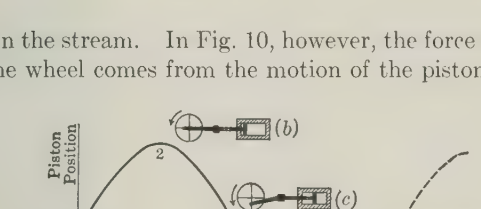



FIG. 11. The position of the piston changes with time as shown in the curve.

motion is not continuous. Instead, the piston motion changes direction twice for each revolution of the wheel, and during each revolution the speed of the piston changes.

If we plot the position of the piston at each instant of time, we obtain the curve shown in Fig. 11. Point 1 on the curve

represents the piston in the middle of the cylinder as indicated in sketch (*a*). The rising curve represents the piston's motion to the left of the mid-point, finally reaching point 2 on the curve which corresponds to the piston at the extreme left-hand displacement as in sketch (*b*). Now the piston starts back to the right, and at point 3 it again reaches the mid-position, as in sketch (*c*). At point 4, the piston reaches its extreme right-hand position, sketch (*d*), and from there it returns to point 5. Of course, point 5 is the same as point 1 in regard to the piston's position and it is hence ready to repeat the motion during the next revolution of the wheel.

In Fig. 9 the action of the flowing water in causing the wheel to turn may be compared to the flow of direct current. Similarly, the action of the piston in Fig. 10 may be compared to the flow of alternating current.

**9. What Is Meant by the Terms Cycle and Frequency.** In Fig. 11, we saw that the piston, starting at the mid-position (point 1), had to travel to the extreme left (point 2), then back to the mid-position (point 3), then to the extreme right (point 4), and, finally, returned to the mid-position (point 5), in order to produce one revolution of the wheel. The same series of events must take place for each revolution of the wheel, and as long as the piston keeps working we can think of the curve extended indefinitely, repeating itself each time the wheel makes a complete revolution.

The portion of the curve which represents one complete turn of the wheel is called a **cycle**; the number of times the cycle is repeated in one second is called its **frequency.** Thus if the wheel is turning 1800 revolutions per minute, it turns $\frac{1800}{60}$, or 30 revolutions per second, and since the piston completes one cycle for each revolution of the wheel, the piston frequency is 30 cycles per second.*

* Usually the word "cycle" is used to mean "cycles per second." Thus, a 60-cycle current is one with a frequency of 60 cycles per second.

The curve of Fig. 11 is the same as the curve which represents an alternating current, and Fig. 12 shows a typical alternating-current wave. At the instant marked 1 the current is zero, at the instant marked 2, it has the value in one direction denoted by the line $a$, in this case 2 amperes; at 3 it has again become zero, while at 4 it has the value $b$, or 2 amperes in the opposite direction but equal to $a$; at 5 it has once more become zero and has completed the cycle. The loops above the zero line represent the values of the current at

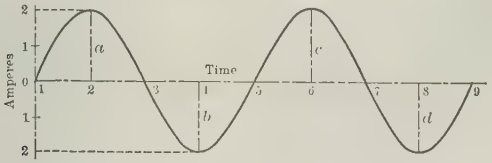



FIG. 12. Alternating-current wave, showing how the current varies in strength and in direction with time.

different instants when it is flowing in one direction. The loops below the line represent the values at the instants when it is flowing in the opposite direction. The values at 5, 6, 7, 8 and 9, merely show that the current goes through the same set of values during the next cycle. Note that in each cycle the current has two instants when it is zero and two other instants when it is a maximum value, though these values are in opposite directions.

The frequency of an alternating-current power system is a matter of great importance. Thousands of electric clocks are now in use, operated from alternating-current supply lines. These are driven by small motors which operate in synchronism with the cycles, or alternations, received over the power line, and their accuracy therefore depends on maintaining accurate frequency at the central station. Fig. 13 shows a clock motor of this type.

**Prob. 1–2.** The drive wheels of a locomotive are 5 feet in diameter. What is the frequency of the pistons when the locomotive is travelling 60 miles per hour? Assume no slipping between the wheels and the rail.

**Prob. 2–2.** How fast would the locomotive of Prob. 1–2 travel if its pistons operated at the same frequency as a 60-cycle alternating current?

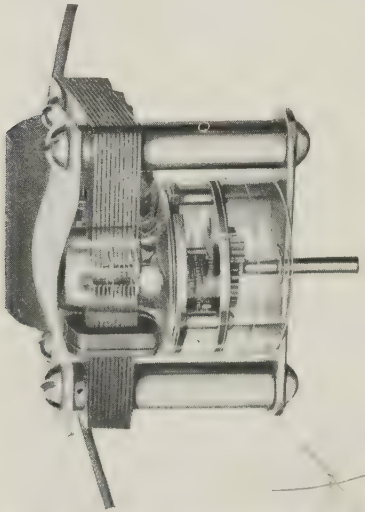


Fig. 13. Clocks driven by small synchronous motors depend upon accurate control of frequency. *Warren Telechron Co.*

**Prob. 3–2.** In order to drive a certain automobile at 40 miles per hour, its engine speed is 2400 rpm. What is its speed when the piston frequency is 25 cycles per second?

**Prob. 4–2.** In a Telechron clock motor (Fig. 13), the motor armature makes one complete revolution per cycle of current. What must be the gear ratio between the armature and the second hand of a clock if it is to operate on a 25-cycle circuit?

**Prob. 5–2.** An alternating current increases in value from zero to a maximum value of 3 amperes in 0.004 second. What is its frequency?

**Prob. 6–2.** In a 60-cycle circuit, what is the time interval between successive zero values of current?

**10. Induction. Induced Voltage.** The following experiment may easily be tried. Attach a long wire across the terminals of a low-reading voltmeter (millivoltmeter) and move part of the wire rapidly across the end of a strong bar magnet, as shown in Fig. 14. The voltmeter will indicate that a voltage is **induced** in the wire which causes a current

to flow along the wire. If we now move the wire in the opposite direction across the face of the magnet, the voltmeter will show a deflection in the opposite direction, indicating that a voltage has been **induced** in the opposite direction. In order to understand more clearly what the conditions are by which a magnet may induce an electric pressure in a circuit, it is necessary to make a brief study of magnetic fields.

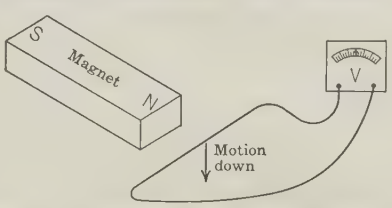



FIG. 14. When the wire is moved across the face of the magnet, a voltage is set up which is indicated on the voltmeter *V*.

If we place a glass plate over a bar magnet, scatter iron filings on the plate and tap it gently, the filings will arrange

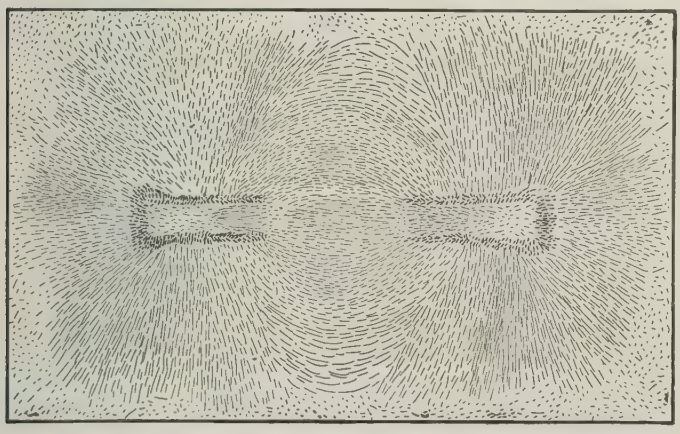


FIG. 15. Lines of magnetic flux shown by iron filings.

themselves in lines, called magnetic lines of force, as shown in Fig. 15. Note how these lines all seem to come out of one

end of the magnet and go into the other end. Since both ends look exactly alike, we cannot tell from which end the lines are coming out and into which end they are going, except by the use of a compass. If we place a compass near one end of a magnet and its North points away from that end, we say that end of the magnet is the **North** pole, and that the lines coming out of this north pole push the compass around so that it points away from the north pole. Similarly, the lines going into the **South** pole pull the compass around so that it points toward the south pole. Thus we say that:

**The north pole of any magnet is the place where the magnetic lines come out, and the south pole is the place where the lines enter the magnet.**

The lines, then, run through the magnet from the south pole to the north pole, out of the north pole, through the air,

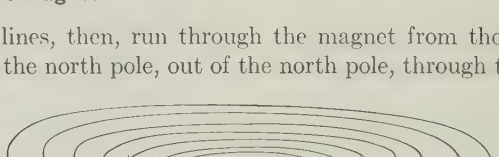



FIG. 16. Magnetic lines of force in and about a bar magnet.

and enter the south pole again, making a complete loop. The space occupied by these lines, around the magnet is called the **magnetic field.** Fig. 16 is a diagram of the magnetic lines of a bar magnet. Strictly speaking, a magnetic field is simply a space where force is exerted upon any magnet which may be put there, and it includes all the space around the magnet, not merely the place where the lines are drawn. The lines are drawn to represent the field; we make the direction of the lines indicate the direction of the force, and the closeness and number of the lines indicate the amount

of the force that would be exerted upon a standard magnet.

The wire in Fig. 14, when moved, cuts these magnetic lines of force. When the lines are cut in one direction, a voltage is induced in the wire which tends to cause an electric current to flow in one direction. If the magnetic lines are cut in the opposite direction, the voltage set up is in the opposite direction. It is immaterial whether we move the wire across the magnetic lines, or move the magnetic lines across the wire. As long as the lines are being cut by the wire, a voltage is induced in the wire. It will be remembered that this principle is applied in electric generators. A voltage is set up in the wires on the armature, by causing the armature or the field to revolve in such a way that the wires on the armature cut the magnetic lines of the field. When the motion or the cutting ceases, the voltage in the armature coils dies out. Similarly, when a wire is held motionless at the end of a magnet, no voltage is set up. It is only when either the wire or the magnet moves so that the magnetic lines cut the wire, that a voltage is set up in the wire.

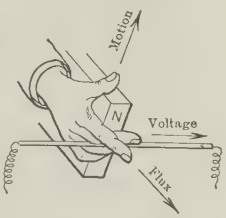



FIG. 17. Right-hand rule for induced voltage.

Then what is really induced is voltage, not current, because when the circuit is open and no current can flow, the voltage is still there between the terminals of the wire which is cutting magnetic lines (or magnetic "flux").

One rule for finding the direction of the induced voltage is as follows:

**Extend the THUMB, FOREFINGER and MIDDLE FINGER of the RIGHT hand at right angles to one another. Let the THUMB point in the direction of the motion, the FOREFINGER in the direction of the magnetic lines, then the MIDDLE FINGER will be pointing in the direction of the induced voltage.**

The hand in Fig. 17 illustrates the application of the right-

hand rule to the case of a wire being moved across the face of a magnet.

**Prob. 7–2.** If the wire $AB$ in Fig. 18 is moving down across the magnetic lines, in which direction will a voltage be set up, from $A$ to $B$ or from $B$ to $A$?

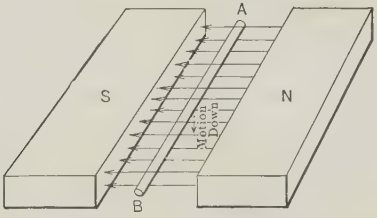



FIG. 18. Wire cutting magnetic lines of force.

**11. The Magnetic Field Due to an Electric Current in a Wire.** Most magnetic fields, however, are not those of bar magnets, but are made by sending an electric current through a coil of wire. When the wire carrying the electric current is straight, a magnetic field is produced which is circular, with the wire at the center of the circles representing lines of force. A compass needle (which is merely a small bar

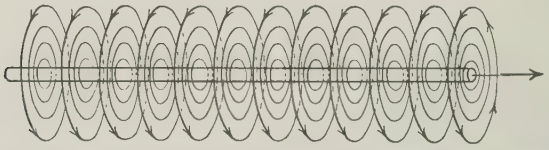


FIG. 19. Magnetic field about a straight wire carrying a current of electricity. Field is circular, not spiral.

magnet) placed in the magnetic field of a straight wire would be deflected from its normal position, tending to take up a position parallel or tangent to the circular lines of magnetic force. If the current in the wire is reversed, the compass needle swings around and points in the opposite direction. Fig. 19 shows this circular field about a straight wire.

If we now look along the wire in the direction in which the current is flowing, we see that the magnetic lines of force consist of a series of circles concentric with the wire. As we increase the current in the wire, these circles widen out, just as the ripples on the surface of water widen out around the spot where a stone has been dropped in. If we now decrease the current, these circles will contract until, when we shut the current off, they disappear entirely. Fig. 20 shows a cross-section of the wire and magnetic field, and represents the way

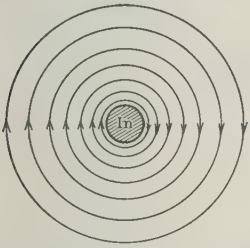



FIG. 20. End view of magnetic field about a straight wire.

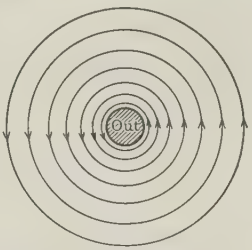



FIG. 21. Field about a wire; current reversed from Fig. 20.

the field would appear if we looked at the end of the wire with the current going away from us. In Fig. 21 the current is reversed. Notice that the field is also reversed in direction.

A simple way to find the direction of the magnetic field about a wire carrying an electric current is by the thumb rule.

### THUMB RULE

**Grasp the wire with the right hand, so that the THUMB points in the direction of the current; the FINGERS will point in the direction of the magnetic field.**

Similarly, if we know the direction of the magnetic field, we can find the direction of the current. For if we wrap the fingers in the direction of the lines of force, the thumb will then point in the direction of the current.

Fig. 22 shows by means of iron filings the appearance of the circular field about a wire carrying current.

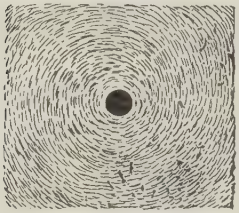


Fig. 22. Iron-filing picture of magnetic field about a wire carrying current.

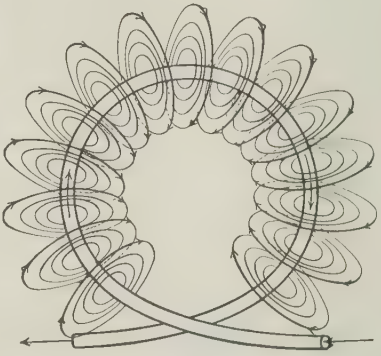


Fig. 23. Field about a single loop of wire carrying current.

**12. Field About a Coil Carrying a Current. Ampere-turns.** If now the wire is made into a loop as in Fig. 23, we find, by the THUMB rule, that the lines of force, which everywhere encircle the wire, all enter the same face of the loop and all come out of the opposite face.

If we place several loops together into a loose coil as in Fig. 24, most of the lines will thread the whole coil. If we make a close coil, practically **all** the lines will thread the whole coil, and return outside the coil to the other end.

The reason that practically no lines of force encircle the separate loops of a closely wound coil, but all thread the entire coil, is explained by referring to Fig. 25. This drawing represents an enlarged longitudinal section of the coil in Fig. 24. The current entering the ends of half-loop at $A$, $B$ and $C$, comes out again at $D$, $E$ and $F$. If the turns $AD$ and $BE$ were pushed nearer one another, the field on the right side of $A$ (being in the opposite direction) would neutralize the field on the left side of $B$. The space between the wires $A$ and $B$ would thus be neutral, or free of lines of force. The

lines now would be compelled to continue on through the whole length of the coil, and would not slip into the spaces between the loops and encircle each wire with a separate field.

We thus have the same shaped field as in and about a bar magnet; one end of the coil being a North pole, since all the lines come out of it, and the other end a South pole, since all

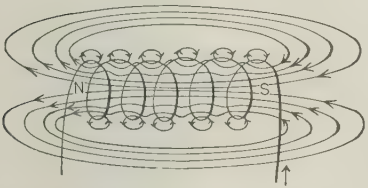



FIG. 24. Field due to a loose coil carrying current.

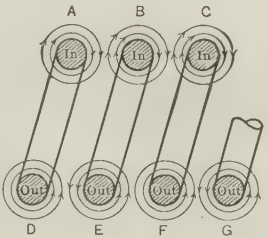



FIG. 25. Longitudinal cross-section of coil in Fig. 24.

the lines enter it. It must be kept in mind, however, that the whole field starts from small circles formed around each wire through which current flows. These circles combine to form the field of Fig. 24 when the turns are placed close together.

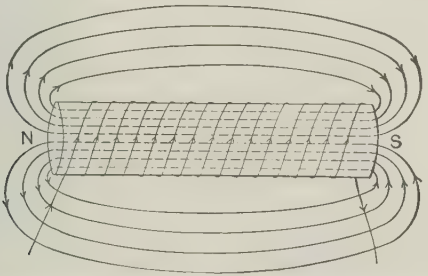



FIG. 26. Magnetic field of a "solenoid" coil carrying an electric current.

**13. Electromagnets.** Thus it is not necessary for the magnet in Fig. 14 to be a bar magnet. It may be, and generally is, an electromagnet. Consider Fig. 26 and 28.

When an electric current is sent through these coils, a magnetic field is created, the direction of which depends upon the direction of the electric current in the coils. The rule for this direction is as follows:

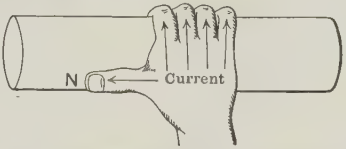



FIG. 27. Fingers of right hand point in the direction of the electric current; thumb points in the direction of the north pole of the coil.

**Grasp the coil with the right hand as in Fig. 27 and 29, so that the fingers point in the direction of the electric current; the thumb then points in the direction of the North pole.**

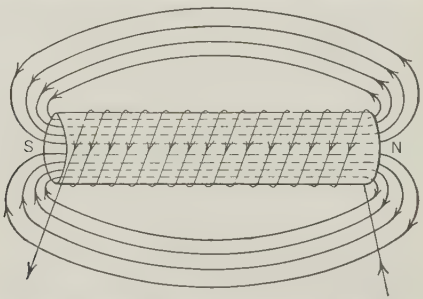



FIG. 28. Reversed current flowing in coil of Fig. 26. Note that magnetic lines and poles also are reversed.

Note that the field of such an electromagnet is exactly like the field of a bar magnet. If we want a stronger magnetic field, we can either send a larger current through the turns of wire, or keep the same current flowing, but wind on more turns. The product of the **amperes** times the **turns** is called the **ampere-turns,** and determines the magnetizing force of the coil. If a weak magnet is required, only a few ampere-turns are used per inch length of coil, and no iron

core is inserted. For a strong magnet, a large number of ampere-turns to the inch is wound on an annealed steel or iron core.

To form a permanent magnet, it is necessary to use a very hard steel core and apply a strong magnetizing force.

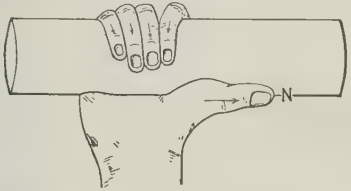



FIG. 29. Right-hand rule of Fig. 27 applied to the reversed current of Fig. 28.

When the current is turned off, the core will be found to retain a part of the magnetism set up. With modern materials the magnet will keep its strength for a long time. These magnet materials are generally alloys, such as cobalt steel or "Alnico," which is a steel alloyed with aluminum, nickel and cobalt.

**Prob. 8–2.** Draw the internal and external magnetic field for an iron core with electric current flowing around it as indicated in Fig. 30.

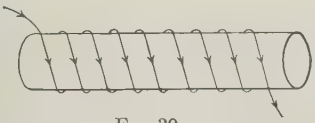


FIG. 30.

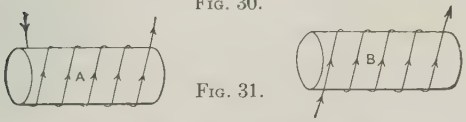



FIG. 31.

**Prob. 9–2.** Draw the field between coils *A* and *B*, in Fig. 31.

**Prob. 10–2.** Draw the field between coils *A* and *B*, Fig. 31, when the current in coil *B* is reversed.

**14. How a Transformer Operates.** Transformers operate by taking advantage of the two principles:

(*a*) A coil of wire carrying an electric current constitutes a magnet.

(*b*) When a wire is cut by magnetic lines of force a voltage is set up in the wire

Suppose we consider two coils arranged as in Fig. 32. If a current is passed through the outside coil *B*, lines of magnetic flux are set up around the coil as shown in Fig. 24 and

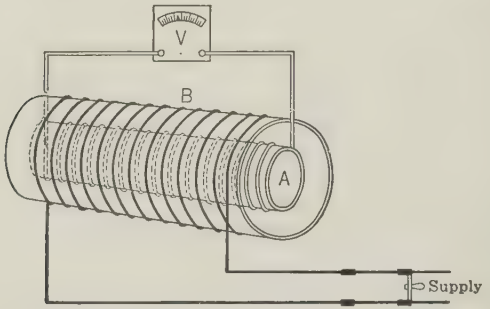



FIG. 32. Any change of current in coil *B* induces a voltage in coil *A*.

25. Many of these flux lines cut across the wires of the coil *A*, and if we continually increase the current in *B*, more and more lines will cut the wires of *A*.

This is indicated by the voltmeter *V*, Fig. 32, which shows a deflection as long as the current in *B* is increasing, although there is absolutely no electrical connection between the coil *A*, in which a voltage is produced, and coil *B*, in which a current is increasing. The voltage in coil *A* is set up merely by the magnetic lines around the wires in coil *B* **cutting** the wires of coil *A*, just as a current was set up in the wire passed across the face of a bar magnet (Fig. 14). As soon as the current in coil *B* reaches its full value

and flows steadily, then the voltage in coil $A$ dies out, although coil $B$ remains an electromagnet.

If now we break the current in coil $B$, a voltage in coil $A$ is set up in the opposite direction, as shown by the deflection of the voltmeter needle in the opposite direction. This voltage in coil $A$ is due to the collapse of the magnetic field around the wires of coil $B$. As the current in $B$ decreases, the strength of the magnetic field decreases and the lines in contracting again cut across the wires of coil $A$, only, of course, **in the opposite direction.** As soon as the current in coil $B$ ceases to flow, the voltage in coil $A$ disappears.

Note that the voltage in coil $A$ is only momentary. It appears only while there is **a change** in the current of coil $B$.

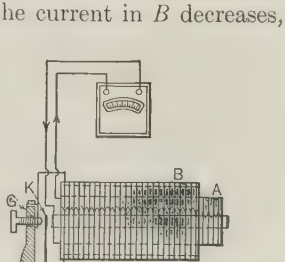



FIG. 33. Induction coil with vibrating-contact interrupter. An intermittent direct current in the primary (outer) coil is used to induce a high alternating voltage in the secondary (inner) coil.

The production of voltage in the inner coil $A$ can be understood better by considering Fig. 33. As a current is sent into coil $B$ the magnetic field starts around each turn of coil $B$. These magnetic fields all join together and sweep inward across the wires of coil $A$. As this field sweeps across the wires of coil $A$, a voltage is induced in each turn of the coil. If 1 volt is induced in each turn and there are 1000 turns, then the voltage across the terminals of coil $A$ is 1000 volts. It is on this principle that induction coils are made. Coil $B$ is generally made of comparatively heavy wire and of enough turns to produce a strong magnetic field when a current is sent through it. Coil $A$ consists of a large number of turns of finer wire. The center of the coil is generally

filled with soft iron wires to produce a stronger magnetic field.

When a current is sent into coil $B$ the growing magnetic field sweeps across the turns of coil $A$ and produces a voltage in each turn. As there is usually a large number of turns in the coil, a high total voltage is induced. But the magnetism of the core is also used to pull over a vibrator $K$ as in a bell; this motion of the vibrator breaks the current in coil $B$ and the dying magnetic field again sweeps across the wires of coil $A$ (this time in the opposite direction), and sets up a voltage in the opposite direction in coil $A$. When the magnetic field has died out of the coils, the spring brings the vibrator back so that a contact is made at $G$ and it allows the current to rush again into coil $B$ and thus to set up the magnetic field which again generates a voltage in coil $A$. This action takes place many times a second, depending upon the rapidity of the motion of the vibrator. The current in coil $B$ is a direct current but is not continuous because we **make** and **break** it continually by the vibrator in order to keep the magnetic field sweeping back and forth across the wires of coil $A$.

But if an alternating current is sent into coil $B$, it does not have to be interrupted in order to produce a rising and falling magnetic field. We have seen (from Fig. 12) that an alternating current not only grows to its greatest value and dies out to zero but it also reverses its direction, many times a second. Thus, if an alternating current flows in coil $B$, the field (1) grows in one direction, (2) dies out, and (3) grows in the opposite direction, and finally (4) dies out again only to repeat the cycle, over and over, many times a second. A strong magnetic field is therefore continually sweeping across the wires of coil $A$ inducing a voltage in this coil every time it cuts the wires. Such an arrangement is called a **transformer.**

**15. Transformer Construction. Primary and Secondary Coils.** The actual construction of a transformer depends upon the type of service for which it is designed. We do not

make power transformers as shown in Fig. 32 because it is necessary to obtain much stronger magnetic fields than we can set up in air. Fig. 33 shows a somewhat better arrangement, because the coils are wound over a soft iron core and a much stronger field can be produced in iron than in air. To get the best results, however, it is necessary to provide an iron path for practically all the magnetic flux; Fig. 34 shows a very simple type of transformer in which an **iron core** is provided to carry the flux.

Note that when a complete iron core is used, it is no longer necessary to put one coil inside the other. Even though the coils are separated as in Fig. 34, the core conducts the flux set up by a current in coil *B* through coil *A*, and we know that if the flux in *A* is **changing**, there is a voltage induced in *A*.

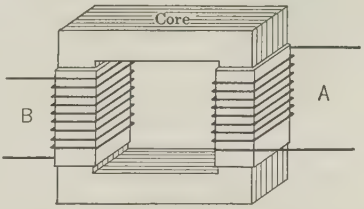



FIG. 34. In power transformers, the magnetic flux is produced in an iron core, instead of in air.

The coil to which the power is supplied is called the **primary** coil of the transformer. The coil from which power is taken is called the **secondary** coil of the transformer. Thus if a transformer is used to **step down** the pressure from 110 volts to 6 volts, the 110-volt coil which receives power is called the **primary** and the 6-volt coil which delivers power is called the **secondary**. On the other hand, if a transformer is used to step voltage up from 6 volts to 110 volts, the 6-volt coil is the primary and the 110-volt coil the secondary. To avoid error, it is usually better to speak of the high-voltage or high-tension coil and of the low-voltage or low-tension coil.

We have seen that in order to have a higher voltage across the secondary coil than across the primary coil, it is necessary

merely to wind the secondary with more turns than the primary. This is because, in a well-designed and well constructed transformer, the voltage induced in each turn of the secondary is practically equal to the voltage in each turn of the primary.

Suppose we have a transformer with a single turn in the primary, and only one turn in the secondary. If one volt is impressed on the one turn of the primary coil, enough current will flow and enough magnetic flux will be produced to generate very nearly one volt in the single turn of the secondary. If we wind the secondary with two turns and keep the same pressure (one volt per turn) on the primary, then two volts pressure will be set up between secondary terminals, or one volt per turn, the same as in the primary. If we wind the primary with 200 turns and the secondary with 10 turns, and impress 100 volts upon the primary terminals, then we shall have enough magnetic flux produced to give one-half volt in each turn of both primary and secondary, and five volts will be produced between secondary terminals. This will be explained more fully in Chapter III.

**16. Power Transformers.** It was the development of an economical power transformer that led American electrical engineers to use alternating instead of direct current where large quantities of power were to be transmitted over considerable distances.

Electric power depends on two factors — voltage and current — the power in watts being the product of the voltage times the current. For a given amount of power, the higher the voltage is, the less the current must be. Thus 1100 watts may be produced by 110 volts and 10 amperes or 1100 volts and only 1 ampere, or any combination of voltage and current the product of which equals 1100.

Since the amount of current determines the size of the wire required, a wire intended to transmit 1100 watts at 110 volts and 10 amperes would need to be about 10 times as heavy as

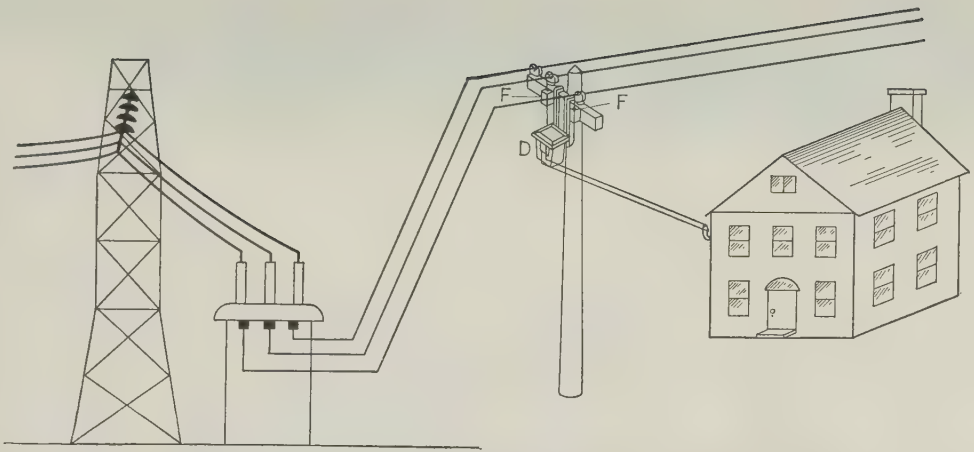



FIG. 35. House service at low tension is usually obtained from a high-tension transmission line through two successive transformations: a transformer substation lowers the line pressure to about 2300 volts, then a distributing transformer lowers it still further to 110 or 220 volts for service entrance to the house.

another wire to transmit the same power at 1100 volts and 1 ampere. That is, to transmit power over considerable distances, it is necessary to use high voltages and small currents. Transmission lines designed to be operated at pressures as high as 275,000 volts are in use at the present time. When electric power from a line using any very high voltage is to be supplied to a town, it is customary to erect a transformer substation just outside the town and to step the voltage down from the very high values to some lower value, usually about 2300 volts, for distribution about the town. This voltage is less likely to injure life and property in the district. Fig. 35 shows such a substation receiving high-voltage wires and sending out medium-voltage wires to distributing transformers, *D*.

**17. Distributing Transformers.** But 2300 volts is much too high for most commercial uses; 230 and 115 volts is the pressure required for the great mass of electrical appliances, such as motors, lamps, and radio sets. Accordingly, the 2300-volt wires are run through the streets, and wherever power is desired a small distributing transformer is set up on one of the poles, as shown in Fig. 35. The primary coils of this transformer are attached to the 2300-volt wires, and leads from the 115- or 230-volt secondary coils are brought into the house or factory. The high-voltage terminals are usually connected to the high-voltage lines through fused cut-outs, similar to those shown in Fig. 36*a* and 36*b*.

Fig 36(*a*). Fused cut-out for distribution transformer. *General Electric Co.*

Fig. 37 shows the external appearance of a typical distribution transformer. The leads on the left-hand side are the primary leads, and are usually connected to the high-voltage lines through fuses *F*, *F*, as shown in Fig. 35. The trans-

former in Fig. 37, however, has a circuit breaker inside to protect the windings in case of overload or short-circuits. This circuit breaker is operated by means of the lever shown near the top of the case. In Fig. 38 the outer case of the same transformer is cut away to show the internal construction. Fig. 39 shows the arrangement of the coils and the iron in a transformer of this type. The magnetic circuit consists of an iron shell made up of a central leg, around which the

Fig. 36(*b*). Cut-out of Fig. 36(*a*) in open position.

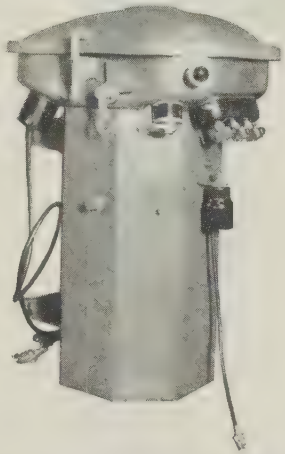


Fig. 37. A modern distribution transformer. *Westinghouse Elec. and Mfg. Co.*

coils are wound, and four outside legs. This gives a magnetic path through the center of the coils, and four paths by which the magnetic lines return to the other end of the central leg. This magnetic circuit is composed of thin sheets of annealed steel. The primary coils are insulated from the

secondary coils by mica shields, as shown in the diagram of Fig. 39. These shields prevent the high voltage of the primary coils from puncturing the insulation and sending into the secondary circuit a high voltage which would be dangerous to life. The coils and core are placed in a tank filled with transil oil. The oil carries away the heat generated in the coils and core and prevents the whole apparatus from becoming too hot. It also aids in the insulation. This construction is called the **shell** type.

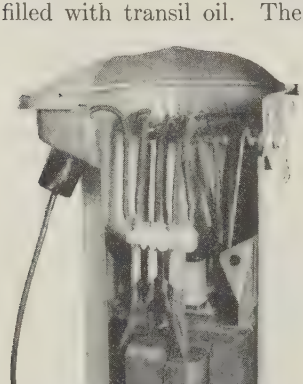


Fig. 38. View of transformer of Fig. 37 with oil tank cut away. *Westinghouse Elec. and Mfg. Co.*

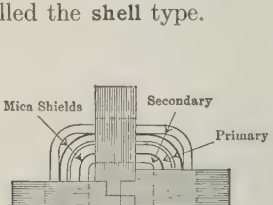



Fig. 39. Relative positions of coils and iron in a *shell-type* distribution transformer.

Fig. 40 shows the other common form of distributing transformers, — the **core** type. Note that the coils are wound on two legs which are joined top and bottom. Fig. 41 shows the relative position of the core and the coils.

**18. Arrangement of Coils in Transformers.** In most distributing transformers the primary circuit is divided into two coils of an equal number of turns and the secondary coil likewise is divided into two coils of an equal number of turns as in Fig. 42. All four terminals of the secondary or low-voltage coils are brought out as $x$, $y$, $xx$, $yy$ of Fig. 42. Any

connections between the two low-voltage coils are then made

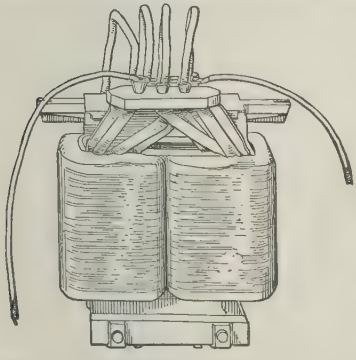


FIG. 40. *Core-type* transformer in which the iron is mostly surrounded by coils, instead of having coils mostly surrounded by core as in Fig. 39.

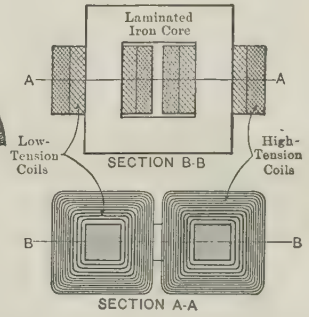



FIG. 41. Core type of transformer.

on the outside of the case by means of these leads. Only two terminals of the high-voltage coils are generally brought outside of the case, as *A* and *B*, Fig. 42. The connections between the two high-voltage coils are made inside the case, by means of the links and studs, *R*, *S*, *T*, *U* and *V* of Fig. 42.

The primary leads *A* and *B* coming to the outside are connected to studs *R*

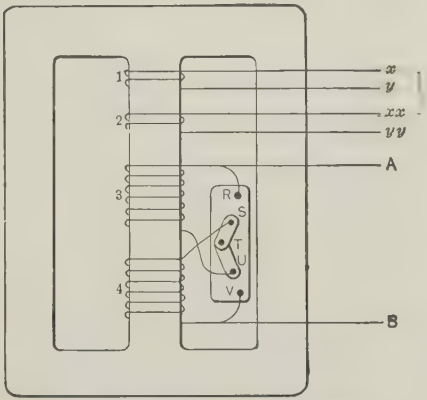



FIG. 42. High-tension coils are marked *A*, *B*, and low-tension coils *x*, *y*, and *xx*, *yy*.

and $V$. Coil 3 is connected to studs $R$ and $U$; coil 4 to studs $S$ and $V$. By connecting the links as shown in Fig. 42, the primary coils 3 and 4 are put in series. By swinging one link between $S$ and $R$ and the other between $U$ and $V$, the primary coils 3 and 4 are put in parallel. To connect the low-voltage coils in series, terminal $y$ is joined to $xx$. To put the low-voltage coils in parallel, terminal $x$ is joined to $xx$, and $y$ to $yy$.

In the transformer of Fig. 38, the links for changing primary connections can be seen mounted on a porcelain block. However, it will be noticed that only three secondary leads are brought out of this transformer (see Fig. 37). This is because in modern practice, it is customary to use these transformers with the secondaries connected in series to supply 115 volts on either side of the center point (see Fig. 43). Hence, unless there is some special reason for wanting all the coil connections brought out, the transformer is supplied with the series connection already made.

**Prob. 11–2.** In the transformer of Fig. 42, each primary coil is built to operate on 2300 volts. At what primary voltage should the transformer be operated with the connections as shown?

**Prob. 12–2.** If each secondary coil of the transformer of Prob. 11–2 is built to deliver 115 volts, show how the coils should be connected so as to deliver the maximum current at 115 volts with the primary connected to a 2300-volt line.

**Prob. 13 2.** If each primary coil of the transformer of Prob. 12 2 is wound with 2600 turns, how many turns must there be in each secondary coil?

**Prob. 14 2.** A transformer as shown in Fig. 42 is built with 1250 turns in each primary coil and is operated at 2300 volts with the primary coils in series. Secondary coil 1 is wound with 250 turns but coil 2 has 255 turns. If lead $x$ is connected to lead $xx$, what voltage will there be between lead $y$ and lead $yy$?

**19. Grounded Secondaries.** Since there is always some danger that the low-voltage secondary wires may get into contact with the high-voltage primaries, either outside or

inside of the transformer, the neutral lead of the secondary coil is always grounded. Fig. 43 shows the neutral point of the secondary tapped at point $P$ and brought to the ground. This is usually done inside the building which is supplied with power by the transformer. A tap from the neutral wire connects to the water pipe in the cellar. When it is neces-

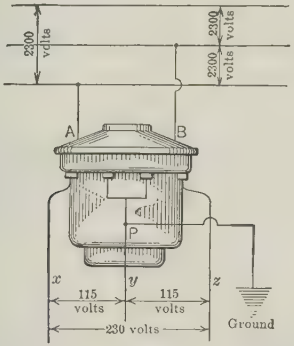



FIG. 43. Secondaries connected in series and tapped to a three-wire (single-phase) low-tension system. Neutral wire should be thoroughly grounded as protection to life.

sary to ground the wire outside the building, a galvanized iron pipe is usually driven about 8 ft into the ground and the neutral wire is connected to it. A good electrical connection must always be used to ground, either a soldered connection or a special ground-clamp. A corroded or broken ground connection is worse than none. The grounding of the neutral point of the secondary makes it impossible, except under extraordinary conditions, for more than normal voltage to exist between the ground and any part of the low-voltage system. For instance, in a system installed as in Fig. 43, no part of the low-voltage wires can attain a higher pressure than 115 volts. This greatly decreases the danger to life and property.

**Prob. 15–2.** In a system installed as in Fig. 43 three wires are brought into the house in a conduit which is grounded. If the insulation on wire $x$ becomes broken so that the copper wire comes into contact with the conduit, what will be the result?

**Prob. 16–2.** If the copper wire $y$, Prob. 15–2, comes into contact with the conduit, what will happen?

**Prob. 17–2.** If the copper wire $z$, Prob. 15–2, comes into contact with the conduit, what will happen?

**Prob. 18–2.** If the insulation between primary lead $B$ of Fig. 43 and secondary lead $z$ breaks down, what will be the greatest voltage between the ground and the lead $z$?

**20. Conduit Rule for Alternating-current Circuits.** The circular magnetic field around a wire carrying alternating current is continually spreading out into wider circles, then

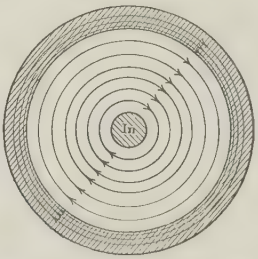


FIG. 44. Putting a single wire of any a–c circuit in an iron conduit leads to excessive voltage drop in the circuit and heating of the conduit, due to the varying magnetic field set up in the iron.

contracting into smaller and smaller circles until the magnetic field dies out. Then the field spreads out again, only with the magnetic field acting in the opposite direction, and again contracts and dies out. The field does this 60 times each second around a wire carrying a 60-cycle current. If such a wire were installed in a metallic conduit the magnetic lines would sweep across the metal of the conduit four times each cycle as shown in Fig. 44: first, as they spread out in

one direction; second, as they contracted; third, as they spread out in the opposite direction; fourth, as they again contracted. This would cause them to cut the metallic conduit $4 \times 60$ or 240 times each second, and electric currents would be set up in the conduit which would waste power and heat the conduit.

For this reason, when it is advisable to run in a metallic conduit, a wire carrying an alternating current, all the return wires for that current are also run in the same pipe. Since the return wires are at all times carrying a current of strength equal to that in the line wire and in opposite direction, the magnetic fields at all instants are exactly equal and opposite to each other and thus neutralize each other.

**21. Special-purpose Transformers.** One of the advantages of the use of alternating current is the ability to use the lighting and power circuit for the operation of radio equipment, control and signalling equipment, electric toys, oil-burner ignition, etc. All of these applications require small transformers, and many of our modern comforts and conveniences are available largely because these small transformers can be built at low cost.

FIG. 45. Small transformer for operating door bells. *Westinghouse Elec. and Mfg. Co.*

One of the most common types is the bell-ringing transformer, an example of which is shown in Fig. 45. A unit of this type is usually built to deliver about 10 volts on the secondary side, with the primary connected to the 115-volt supply.

A modern radio receiver will often contain four different types of transformers. The main power transformer is built with a single primary coil which connects to the 115-volt line. It will have a secondary coil delivering 600 volts

or more to a rectifier and "smoothing" circuit which provides high-voltage direct current for the plates of the various tubes. In addition, other secondary windings are placed on the same iron core, and from these lower voltages (2 to 8 volts) are obtained to heat filaments of tubes and to light the dials.

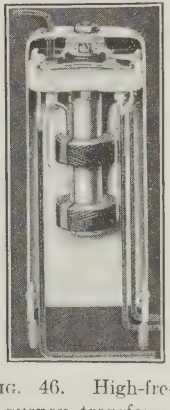


FIG. 46. High-frequency transformer for use in radio receivers. The core is made of powdered iron alloy held in shape by a chemical binder. *Hammarlund Mfg. Co.*

The radio waves which are received by the antenna produce currents of very high frequency in the receiver. Ordinary transformers cannot be used for stepping up the voltage of these high-frequency currents because of the large amount of power which would be needed to set up the magnetic field in the iron. Many of these high-frequency transformers are therefore built without iron cores and consist merely of two coils placed near each other. Recently, however, a new type of iron core has been developed for such use. The core is made of special "low-loss" iron alloys which are first powdered and then mixed with a chemical binder and molded into the desired core shape. Fig. 46 illustrates a transformer of this type for use in the "intermediate-frequency" stages of a radio receiver.

In many applications, it is desirable to be able to vary the secondary voltage of a transformer. Usually this is done by providing "taps" in the winding so that by changing connections the number of secondary turns actually used can be varied. There is also now available a transformer built with a sliding contact which can be moved over part of the secondary winding, and thus vary the number of

secondary turns used in the circuit. Fig. 47 shows a transformer of this type, which is called a Variac.*

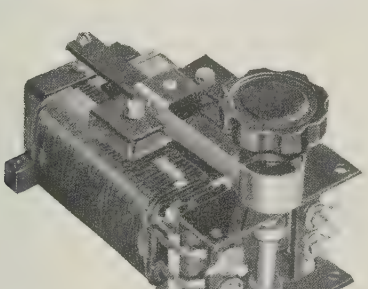


FIG. 47. A variable-voltage transformer in which a sliding contact is used to change the number of secondary turns used. *General Radio Co.*

## SUMMARY OF CHAPTER II

**An ALTERNATING CURRENT is one which rises in strength to a maximum in one direction, then subsides and reverses, rises in strength in the opposite direction and again subsides, repeating this complete set of changes over and over in equal periods of time.**

**A CYCLE is one complete set of values or changes in both directions. The number of cycles completed in one second is called the FREQUENCY. In the United States, the frequencies used for distribution of current for motors, lights and heating appliances have been standardized at 60 cycles or 25 cycles per second.**

**A required amount of power (watts) may be had as a small current (amperes) at a high pressure (volts), or as a large cur-**

* The name Variac is a copyrighted name applying only to the variable-voltage transformers manufactured by the General Radio Co.

rent at a small pressure, so long as the product (volts × amperes) is the same. Large currents require correspondingly large wires to carry them.

To transport power economically in large quantities or over long distances much higher pressures must be used for transmission than can be applied to the distributing circuits where power is consumed. This is accomplished by means of transformers.

The TRANSFORMER consists of two sets of coils wound over the same iron core; one set, the PRIMARY, receives current and power and produces flux or " lines " of magnetism in the core, which periodically change in value and direction, or alternate, like the current. The changing flux " cuts " the other set of coils, the SECONDARY, which thereby has generated in it a voltage and will deliver current and power if an external circuit be completed between the secondary terminals.

The RATIO OF VOLTAGE between primary terminals to voltage between secondary terminals is very nearly the same as the ratio of primary turns in series to secondary turns in series, in well-designed transformers of the most usual types. When this ratio is greater than one, we have a STEP-DOWN transformer; when it is less than one, we have a STEP-UP transformer. The same transformer may be used either to step down or to step up the voltage.

Any conductor carrying electric current is surrounded by a circular magnetic field, or circular " lines of force " around the conductor. The direction of the lines or flux of magnetism at any place is assumed to be the direction in which the north-seeking pole of a compass needle would point at that place. If the wire be grasped in the right hand so that the fingers point in the direction of the magnetic flux or compass needle, then the thumb will point in the direction of current flow in the conductor which produces that flux.

In general, a voltage is produced or " induced " in any wire when it is located in a CHANGING magnetic field. The voltage may be produced by movement of either wire or magnetism relative to the other, or by a change in strength of magnetic field surrounding a motionless wire. The voltage is produced only while such changes are occurring. Dynamo machines illustrate the former, and alternating-current transformers the latter. There is a definite relation always existing between direction of magnetic flux or lines, of motion, and of induced voltage due to

the motion, which may be remembered by rules such as given in Art. 10 and illustrated in Fig. 17.

The MAGNETIC STRENGTH of an electromagnet, or of a coil carrying current, is proportional to the product of amperes $\times$ turns, or to the total number of AMPERE-TURNS. This enables us to determine the strength of magnets.

All wires belonging to the same circuit or system must be run side by side in the same iron conduit, otherwise there will be a large pressure-drop in the circuit and an excessive amount of power will be lost from the electric circuit to the iron conduit, which not only is bad economy but also overheats the conduit and increases the fire risk. Metal conduits enclosing electric circuits are usually grounded by making metallic connection to water pipes or to the steel frame of a building. The neutral of a three-wire system is usually grounded in similar manner.

## PROBLEMS ON CHAPTER II

**Prob. 19–2.** In a certain automobile, the engine turns 3.5 times for each revolution of the wheels. If the outside diameter of the tires is 30 inches, at what frequency do the pistons oscillate when the car is driven 40 miles per hour?

**Prob. 20–2.** In Prob. 19–2, how fast would the car have to travel to have the pistons oscillate at 60 cycles per second?

**Prob. 21–2.** A certain cyclic phenomenon is observed and it is found that it reaches its positive maximum once every 4 hours. What is its frequency?

**Prob. 22–2.** In a 60-cycle circuit, what interval of time (seconds) elapses between a zero value of voltage and the next maximum? What time elapses between a maximum value and the next maximum value in the same direction? Between maximum values in opposite directions?

**Prob. 23–2.** Answer Prob. 22–2 for a voltage at a frequency of 1,250,000 cycles per second picked up on a radio antenna.

**Prob. 24–2.** There is a time interval of 0.0025 second between two successive zero values of alternating voltage in a certain circuit. What is the frequency in cycles per second?

**Prob. 25–2.** Unless they are closely fitted to each other, there is likely to be a very noticeable chatter between an alternating-current electromagnet and its keeper or armature. How many

blows or vibrations per second are there in this noise, if the magnet is energized from a 60-cycle circuit? If from a 25-cycle circuit?

**Prob. 26 2.** Explain how you can determine the direction of current in a circuit, or the electrical polarity of the circuit, by aid of a compass.

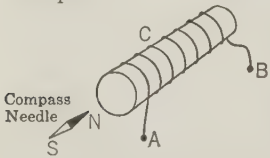



FIG. 48. Is the current in the coil flowing from $A$ to $B$, or from $B$ to $A$?

**Prob. 27–2.** In Fig. 48, the north pole of the compass needle points to the end of the coil shown when a direct current flows in the coil $C$. What is the direction of the current in the coil?

**Prob. 28–2.** If the current flowing in the coil of Prob. 27–2 were alternating at 60 cycles per second, how would you expect the compass needle to behave?

**Prob. 29–2.** How will a compass point:

($a$) When laid on top of a bus-bar carrying direct current?

($b$) When the bus-bar carries alternating current?

**Prob. 30-2.** How would a compass needle behave when held between the blades of a two-pole knife switch carrying ($a$) direct current, ($b$) alternating current?

**Prob. 31–2.** The electromagnet shown in Fig. 49 is wound with two separate coils. Each coil has the same resistance and is designed to operate on a 115-volt d-c circuit.

($a$) How should the coils be connected for operating the magnet, at maximum strength, on a 115-volt circuit?

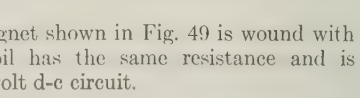



FIG. 49. An electromagnet with two coils.

($b$) How should the coils be connected for operation on a 230-volt circuit?

($c$) Compare these two connections with respect to the strength of the magnet produced.

**Prob. 32 2.** In Fig. 49, the terminals $B$ and $C$ are connected to the positive terminal of a d-c circuit, and $A$ and $D$ are connected to the negative side of the line. Sketch the magnetic field thus produced, showing the directions of the lines of flux.

**Prob. 33–2.** With a magnet connected as in Prob. 32–2, in what direction will the north pole of a compass point when brought near the end of the magnet adjacent to the terminal *D*? How will the compass needle behave when brought opposite the terminals *B* and *C*?

**Prob. 34–2.** Show that the rule given in Art. 13 for the polarity of an electromagnet is really the same as (or follows from) the " Thumb Rule " given in Art. 11 for the magnetic field around a straight wire.

**Prob. 35–2.** Explain why the voltage induced in a coil by a change in the number of lines or amount of magnetic flux passing through it, is increased in proportion to the increase in the number of turns in the coil. How would you wind a coil so that no voltage would be set up in it, regardless of how the flux might change?

**Prob. 36–2.** It was stated in Art. 15 that " the voltage in each turn of the secondary is practically equal to the voltage in each turn of the primary." Actually, the secondary voltage per turn (with no load on the secondary winding) is slightly less than the primary voltage per turn. How do you explain this?

**Prob. 37–2.** In what direction would the induced electrical pressure in the wire of Fig. 17 be, if it were moved along the side of the magnet from the north pole to the south pole, while being kept parallel to the position shown?

**Prob. 38–2.** Describe the pressure that would be induced in the wire of Fig. 18, if it were: (*a*) Moved parallel to itself from *S* toward *N*; (*b*) rotated about an axis passed through the letters *S*, *N*.

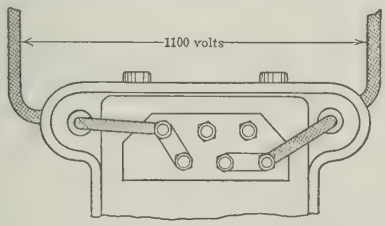



Fig. 50. Terminal block for high-tension coils.

**Prob. 39–2.** The Wagner Electric Mfg. Co. use in some of their transformers a high-voltage terminal board arranged as in Fig. 50.

The connections from the primary coils to the board are shown in Fig. 51. Each primary coil consists of 1760 turns and each is tapped at $A$ so that there are 1520 turns in each coil on one side of $A$ and 240 turns on the other side. Each of two secondary coils has 184 turns. When the links are arranged as in Fig. 50, what is the voltage between the secondary terminals?

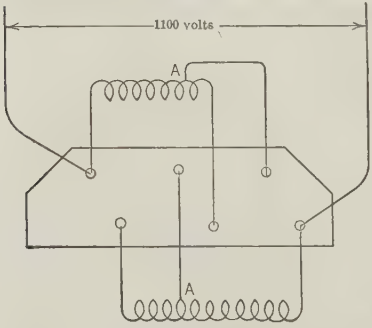



FIG. 51. Showing how terminal block of Fig. 50 is tapped to the high-tension coils.

(*a*) When the secondary coils are in parallel?

(*b*) When the secondary coils are in series?

**Prob. 40–2.** How would you arrange the links on the primary connection board of the transformer of Prob. 39–2 (Fig. 50), if the transformer was to be used on a line of 950 volts?

**Prob. 41–2.** What would be the voltage between the low-voltage terminals of a transformer connected as in Prob. 40–2 if the secondary coils were

(*a*) In series?

(*b*) In parallel?

**Prob. 42–2.** Show the link arrangement of the transformer of Prob. 39–2 if it is to be used on 2200-volt line.

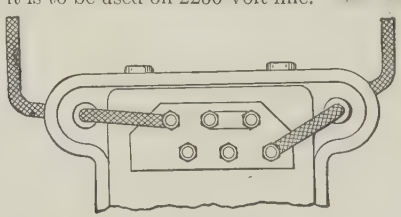


FIG. 52.

**Prob. 43–2.** (*a*) On what voltage can the transformer of Prob. 39–2 be used if connected as in Fig. 52?

(*b*) What would be the voltage across each secondary coil?

**Prob. 44–2.** Show the link arrangement for the transformer of Prob. 39–2 if it is to be used on a 2050-volt line.

**Prob. 45–2.** Using the transformer of Prob. 39–2, how should the primary be connected across a 1100-volt line in order to induce the least possible secondary voltage? What is the value of the voltage thus produced?

**Prob. 46–2.** What is the greatest voltage that can exist between the ground and any part of the low-voltage system in case of a breakdown at any one point between the primary and secondary coils of the transformer in Fig. 43?

**Prob. 47–2.** If the conduit of Prob. 15–2 were not grounded and the copper wire $x$ came into contact with the conduit, what voltage would a person be subjected to who stood on the ground and touched the conduit?

**Prob. 48–2.** The following tests were made on the secondary leads labeled 1, 2, 3, 4 of a transformer. Voltage between 1 and 2, 2 and 4, 1 and 3, 3 and 4 = 0. Voltage between 2 and 3, 4 and 1 = 115.

When 1 and 2 were joined, the voltage between 3 and 4 was 230.

When 3 and 4 were joined, the voltage between 1 and 2 was 230.

When 2 and 4 were joined, the voltage between 1 and 3 was zero.

When 1 and 3 were joined, the voltage between 2 and 4 was zero.

(*a*) Make a sketch showing how the secondary coils were connected to the terminals and indicate the directions of the voltages in the coils at any particular instant.

(*b*) How should the coils be connected for 115-volt operation?

**Prob. 49–2.** A three-wire supply is desired for a factory using 230-volt motors and 115-volt lamps. This power is to be obtained from a 1150-volt distribution line using a transformer as shown in Fig. 43. The primary winding consists of two coils connected in series, and each coil being wound with 1200 turns. How many turns are required in each secondary coil?

**Prob. 50–2.** Suppose that the supply in Prob. 49–2 is to be obtained from a 4600-volt line, and the only transformers available are like the one described in Prob. 49–2. Show how these transformers should be connected to feed the three-wire low-tension circuit.

## CHAPTER III

## IMPEDANCE

When the primary of a certain bell-ringing transformer was connected for a few moments across a 115-volt direct-current line, it was found that a current of 8.5 amperes flowed in the coil. If allowed to continue, this current would have quickly burned out the winding.

From these figures the resistance of the primary coil may be found, since

$$\text{Resistance} = \frac{\text{Direct voltage}}{\text{Direct current}}$$
$$= \frac{115}{8.5} = 13.5 \text{ ohms.}$$

When the primary coil was placed on an alternating-current line of 115 volts (60 cycles), only 0.06 ampere flowed. Since this small current value would not overheat the coil, the transformer could be left attached to the alternating-current circuit for an indefinitely long time.

Now, if the resistance of the primary coil were the only thing which had restricted the flow of alternating current, we could be sure that the value of the current would have been as follows:

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}}$$
$$= \frac{115}{13.5} = 8.5 \text{ amperes.}$$

This is the value of the direct current which flowed when the voltage was direct but it is many times greater than the current which the a-c voltage forced through the coil.

**22. Impedance.** Thus it is clear that when a transformer coil is connected to an alternating-current circuit there is something other than the voltage of the circuit and the resistance of the coil that determines the flow of alternating current.

This something which restricts the flow of an alternating current, we call the **impedance** of the circuit, and it is equal to the quotient of the alternating voltage divided by the alternating current. The impedance is measured in **ohms** just as is the resistance; but, unlike true resistance, the value of impedance depends upon the arrangement of the electrical circuit and its surroundings, and upon the frequency.

$$\text{Thus,} \qquad \text{Impedance} = \frac{\text{Alternating voltage}}{\text{Alternating current}}.$$

In the case of the primary coil of this bell-ringing transformer,

$$\begin{aligned} \text{Impedance} &= \frac{\text{Alternating voltage}}{\text{Alternating current}} \\ &= \frac{115}{0.06} \\ &= 1917 \text{ ohms.} \end{aligned}$$

The impedance of this coil is therefore about 140 times greater than its resistance. Since it is the impedance which limits an alternating current, while a direct current is limited by the resistance only, it is easy to see why such a coil would soon burn up if connected to a direct-current line of the same voltage as the a-c voltage of the coil.

**Prob. 1–3.** If the above transformer coil were connected to a 90-volt a-c circuit, what current would it take? The frequency of the 90-volt line is the same as that of the 115-volt line above.

**Prob. 2–3.** A certain radio filament-heating transformer has a resistance of 4.8 ohms in the primary coil. What current will flow in the primary when it is connected across a 120-volt d-c circuit?

**Prob. 3–3.** The impedance of the coil in Prob. 2–3 is 644 ohms at 60 cycles. What current will this coil draw from a 120-volt, 60-cycle circuit?

**Prob. 4–3.** In order to measure the resistance of the coil in Prob. 2–3, it is to be connected in series with a protective resistance and an ammeter, and the series combination is to be connected across a 120-volt d-c line. If the maximum allowable current in the coil is 2.6 amperes, what is the smallest protective resistor which can be used? The resistance of the ammeter is negligible compared to the total resistance.

**Prob. 5–3.** The impedance of a certain appliance is 52 ohms at 60 cycles. If this device can take 10.6 amperes safely, at what 60-cycle a-c voltage can it be operated?

**23. Why the Impedance Is Usually Greater than the Resistance.** In the example given in the preceding article, the impedance of the circuit was much greater than its resistance. As we will see in Chapter V, the impedance is always at least as great as the resistance and in most cases is greater. We can learn more about the nature of these relations by some simple tests.

For our test circuit, let us take 345 ft. of No. 12 copper wire with a cotton cover and string it up as "line and return." This wire has a resistance of 0.55 ohm, so that we would not be safe in connecting it across a 110-volt, d-c generator. If we did this, it would allow about 200 amperes to flow, which would melt the wire. Therefore, let us put only 11 direct volts across it. An ammeter now indicates 20 amperes direct current, a safe current for this wire when it is strung up as a line in free air. The resistance is $\frac{1}{2}\frac{1}{0}$, or 0.55 ohm. Similarly, we would not be safe in placing this wire across the terminals of a 110-volt, 60-cycle, a-c generator, as approximately 200 amperes would flow. Accordingly, we will try 11 volts alternating current, at the same frequency of 60 cycles. An a-c ammeter reads now about 20 amperes, showing that the impedance of the wire so arranged is approximately the same as the resistance, $\frac{1}{2}\frac{1}{0}$, or 0.55 ohm.

Now let us wind this same wire on a round wooden core 20 inches long and $1\frac{1}{4}$ inches diameter. There would be about 1050 turns on this core, which would constitute a weak electromagnet, as in Fig. 53. If we put the coil across the 11 volts direct current, as we did the straight wire, an ammeter again indicates 20 amperes, showing that, in shaping the wire into a weak electromagnet, we have not changed, in the slightest degree, the resistance it offers to the flow of a direct current. The resistance is still $\frac{1}{20}$, or 0.55 ohm.

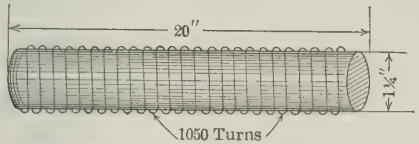



FIG. 53. A weak electromagnet made by winding wire on a wooden core.

But if we put it across the 11 volts, alternating at 60 cycles, we find that an a-c ammeter indicates a little less, about 14.7 amperes. The impedance has become $\frac{11}{14.7}$, or 0.75 ohm. Without changing the wire in any way except to wind it into the form of a weak electromagnet, we have increased the impedance about 35 per cent, while the resistance has not been changed in the slightest degree.

Examining this effect more closely, we find that in winding this piece of wire into the form of a coil, we do change its behavior in one important respect. As a straight wire carrying current, it sets up a magnetic field in its own vicinity, but the strength of the field is very small. Wound on a wooden core, as in Fig. 53, each part of the wire is now located in the combined magnetic field of itself and its neighboring portions of wire, and the total field produced is stronger.

Suspecting that a change in the magnetic properties of the

coil may have some influence on the impedance, let us make as strong a magnet as is convenient, of the same dimensions as the coil with the wooden core. We shall, accordingly, wind the 345 ft. around a soft iron ring of 5 in. inside diameter, made of round $1\frac{1}{4}$-in. stock as in Fig. 54. The length of the

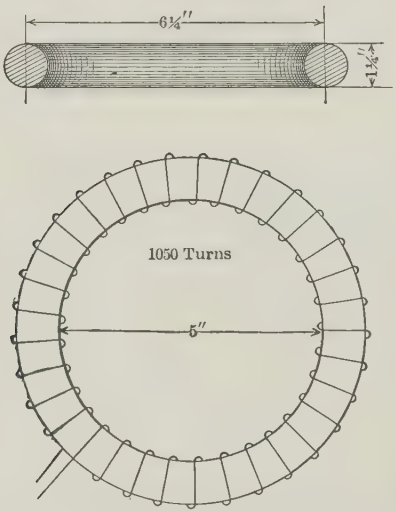



Fig. 54. A strong electromagnet made by winding the wire of Fig. 53 on an iron ring.

iron core would be nearly equal to the length of the wooden core and the 345 ft. of wire would again make 1050 turns on this ring, as on the wooden core. If we now place this coil across 11 volts direct current we will find that the ammeter again indicates 20 amperes. In winding the wire about an iron ring so as to make a strong electromagnet we have not changed, in the slightest degree, the resistance which it offers to the flow of the direct current. The resistance is still $\frac{1}{20}$, or 0.55 ohm.

Now, however, we try it across 11-volt, a-c, 60-cycle mains. An ammeter in the circuit reads about $\frac{1}{100}$ of an ampere only. Apparently the impedance has been greatly increased. In fact, we can now connect the coil across 115-volt, 60-cycle mains, and only about 0.15 ampere will flow.

In winding the wire around the iron ring, therefore, we have increased more than a thousandfold the impedance which it offers to the flow of this alternating current. The impedance has now become about $\frac{115}{0.15}$, or 767 ohms. Let us consider the causes of this great increase in the impedance.

**24. Inductive Reactance.** We started with a wire which, when stretched out approximately straight, offered only 0.55-ohm resistance to the flow of a direct current and the same amount of impedance to the flow of an alternating current. Without changing it in any way except merely to wind it about a piece of iron so as to form a strong electromagnet, we raised the impedance to over 760 ohms. This figure is so great that when we compare it with the original, 0.55 ohm, we see that practically all this impedance is due to winding the wire around the iron. In speaking of the impedance of the coil we can thus neglect the original 0.55 ohm due to resistance, and say that practically the whole impedance of the wire is the result of winding it into a coil so that it sets up a strong magnetic field when a current flows through it.

When the impedance of a circuit carrying alternating current is greater than its resistance, we say that this circuit possesses **reactance**; that is, there is some condition present in the circuit which reacts against the applied voltage and hinders it from forcing through as large an alternating current as we would expect, judging from its resistance to a direct current.

When this reactance in a circuit is largely due to the

magnetic field which a current will set up about it, we call the reactance an **inductive reactance.** Now we have seen that the inductive reactance is practically zero in the case of a short, straight line, about which a current produces a very weak magnetic field. But the inductive reactance of a coil made of the same piece of wire is quite noticeable, when the circuit is coiled into even a weak electromagnet, and very great when a strong electromagnet is produced. There seems to be something about this magnetic field, then, which produces this counter action or, as we have called it, this reactance.

In a circuit carrying direct current we know that sometimes such a strong reaction is set up that the current is cut down below the value which we would expect from Ohm's law. This always happens whenever a motor is running on the circuit. We measure the resistance of the motor with the armature at rest and find it very low, and perhaps expect the motor when running to take a current which shall have the value expressed by the fraction $\frac{\text{volts across motor}}{\text{resistance of motor}}$. But we find that the machine takes only a small fraction of this current. On investigation we decide that this great decrease in current, or large increase in apparent resistance, is due to the fact that when the armature is revolving, the conductors on it cut through the magnetic field and set up a back voltage which opposes the flow of the current. When the armature is standing still there is no back voltage and the current would be very great (as indicated by the fraction above) if the same voltage were applied. The faster the armature moves, and the stronger the motor field, the greater the back voltage that is produced, and the smaller the current through the armature.

If we investigate the reactance of a circuit carrying alternating current, we shall see that a similar process takes place. The circuit offers no reactance to an electric current unless the

conductors of the circuit are cut by the magnetic field. When we try to send an alternating current through the coils in Fig. 53 and 54, the current is continually changing in value and direction. The magnetic field is thus continually being built up in one direction, reduced to zero, and built up in the opposite direction. In this process, the lines of force must cut the wires of the coil again and again. The stronger the field the greater the voltage thus set up. From an inspection of Fig. 55, which represents a section of a coil, we can see that the voltage induced in the wire by this cutting, in each instance, is always opposed to the change of the current which produces the magnetism. Thus, if a current is trying to increase, the magnetic flux, increasing and spreading, cuts the wires in such a direction as to oppose any increase of the current. If the current is trying to decrease, the lines of the dying magnetic field cut the wires in such a direction as to oppose the decrease of current, or so as to maintain the current.

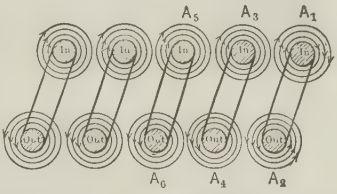



FIG. 55. The growing or dying magnetic field around each turn of the circuit, due to the increasing or decreasing current in it, cuts the other parts or turns of circuit in such manner that the voltage induced tends to oppose the change of current.

Consider Fig. 55. Assume that a current is growing in the coil. A field immediately begins to grow out from each turn of wire, as for instance from $A_1$, which spreads and sweeps the wire $A_3$, from right to left. This is equivalent to the wire $A_3$ cutting to the right across the lines as is shown in Fig. 56. Thus a voltage is set up which tends to send a current out at $A_3$ in the opposite direction to the current which is being established in it by the line voltage. This action takes place in all the wires as the current grows. The spreading field about each wire cuts the other wires in such a direction

as to set up a reacting voltage which opposes the growth of the current. A growing current is thus "choked" back in any coil where the field is strong and where the turns are numerous and close together.

By using the same figures, 55 and 56, and assuming the current to be dying out, we see that the lines in the dying field will now sweep across the wires in the opposite direction and set up a reacting voltage which tends to keep the current in these wires from dying out. Thus the necessary action of dying is also impeded, and it is easy to see that such reactions can greatly hinder an alternating current from flowing through a coil even though the resistance is almost zero. The action of the coil in Fig. 54 in a direct-current circuit is also explained. When a direct current has once reached a steady value it remains constant. Therefore, once a steady direct current is established, the field also remains constant and no cutting of the lines takes place; thus no greater hindrance is offered to the flow of the direct current than the resistance which the wire affords. But we might expect that it would be difficult to start even a direct current flowing, and such is the fact. Fig. 57 shows a curve plotted from data taken to investigate the time required to get a direct current up to its full value, the circuit containing a strong magnet coil. The coil had a resistance of 11 ohms, and was put across 110 volts, direct current. Note that 0.9 second elapsed after the circuit was closed, before the current reached its normal value of 10 amperes. The inductive property of the circuit opposed its growth by setting up a back voltage.

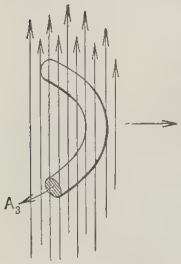



FIG. 56. In Fig. 55, it is as though turn $A_3$ moved to the right through the flux set up by turn $A_1$.

The opposition to the decrease of a current in an inductive

circuit is seen in the flash which takes place when the field switch of a large generator is opened. The sudden interruption of the field current causes the powerful magnetic field to decrease very rapidly and a very high voltage is thus set up in

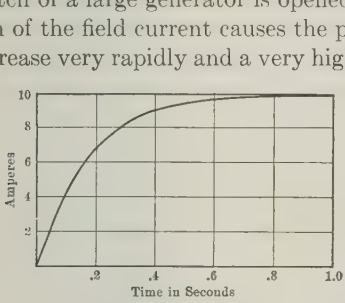



FIG. 57. Curve showing the time required to set up a direct current in a strong magnet coil.

the field coils. This voltage is high enough to break down the air path between the switch blade and clip and an arc is started.

This arc is so destructive to the copper blades of the switch that special switches are put in a field circuit which reduce the current gradually, not instantaneously. One of these field discharge switches is shown in Fig. 58. When the blades $B$ are withdrawn from the clips, it does not disconnect the field from the power which is across the upper clips, because the spring blades $SS$ are still held in the clips by their friction. When the main blades $B$ are withdrawn from their clips, the blade $X$ comes in contact with the clip $Y$. This connects a resistance across the field terminals, because a resistance $R$ is connected across $Y$ and $B_2$ as shown. As the handle is pulled farther down, the spring blades $SS$ fly out and

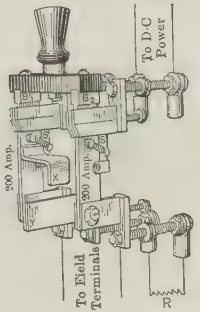



FIG. 58. A field-discharge switch. Field winding becomes shunted by a resistance $R$ just before power is disconnected; as magnetic energy of field is discharged into $R$ the field current dies slowly, and only moderate voltage is induced by the slowly dying magnetic flux.

disconnect the power from the field and the resistance $R$. But before the spring blades disengage from the upper clips, the blade $X$ makes contact with the lower clip $Y$. Thus the field is never opened. It is merely connected to a resistance, and the power is then disconnected. The voltage induced in the field coils by the dying magnetic flux produces a current

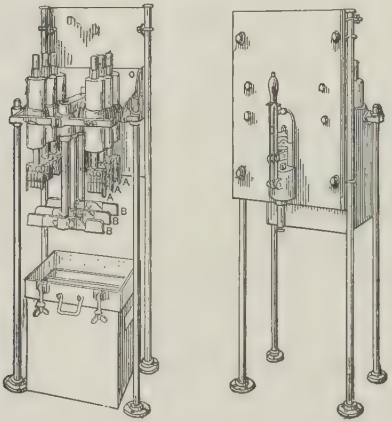



FIG. 59. A General Electric three-pole oil switch. The oil in the case (shown in position in right-hand view) smothers the arc formed between points $A$ and $B$ as the switch is opened.

through this resistance; thus the energy stored up in the magnetic field, when the current was compelled to increase against the induced back voltage, now reappears as heat in this resistance instead of an arc at the clips of the switch.

Destructive arcing is also prevented on circuits which have large inductive properties, by means of oil switches such as shown in Fig. 59. This represents a 3-pole switch, the break occurring at the points between $A$ and $B$. These points are immersed in oil which is held by the case — here shown lowered in order to make the construction clear. The oil

smothers the arc and saves the contact points from being fused or roughened.

There is greater damage likely to be done than the burning of the switch contacts when the field current is suddenly stopped. The dying lines of the magnetic field sometimes sweep in such great numbers and so rapidly across the wires in the field coils, and so high a voltage is thereby set up, that it punctures the insulation and puts the field coils out of service. The special switch shown above prevents this by closing the field circuit through a large resistance, which allows the field current to die out slowly, and so a smaller voltage is induced.

**25. Lenz' Law.** Enough has been shown concerning the effects of a strong magnetic field upon the electric circuit within it, to bring out the law which was first stated by Lenz and is, therefore, called Lenz' law. It states in part that:

**While any change is being made in the magnetic field of an electric circuit, a voltage is induced which opposes the change.**

Thus we have seen that when a current was growing, and in so doing was setting up a magnetic field, a voltage was induced in the opposite direction, so that it opposed this growth of the current and of the magnetic field. If the growing current had set up no magnetic field there would have been no opposition to its growth. The whole reaction, or opposition, is due to the creation or the destruction of a magnetic field which cuts across the wires composing the circuit and so induces the reacting voltage.

Since the magnetic field set up by an alternating current is continually changing, this induced back voltage is continually acting and thus continually limits the current throughout the system.

**26. Inductance.** When such a back voltage as we have described above is set up by these changes of current we say that the circuit is **inductive**, or contains **inductance. Induc-**

**tance may be defined, then, as that magnetic property of a circuit which causes it to oppose any change in the current flowing.**

If there is no current flowing, the inductance opposes the start and growth of one. If a current is already flowing, the inductance of a circuit opposes either any decrease or any increase of this current. **Inductance** in an electrical system is similar to inertia in a mechanical system, which opposes any change in the speed of a body. Thus, if we are standing in a rapidly moving car and the motorman applies the brakes, we feel a strong tendency to go forward in the car and we have to brace our feet in order to remain standing. It is the inertia of our bodies which is urging them to keep moving in the same direction and at the same rate while the feet are being retarded by the car. The inertia of our bodies is thus opposing the change of the speed at which they are moving just as the inductance of the electric circuit opposes any change of current flowing. When we try to stop an electric current, the inductance of the circuit tends to keep it going.

Similarly, when a car suddenly starts up, we feel a strong tendency to take a step toward the back of the car. This tendency is again due to the inertia of our bodies, which opposes the change in motion (that is, the speeding-up process), just as the inductance of an electric circuit opposes the start and growth of a current.

In fact, it is a universal law which apparently applies to all branches of science, that if we wish to make any changes we must overcome some force which tends to keep things as they are.

The force which tends to keep the current as it is in an electric circuit is the **back voltage** which the inductance of the circuit sets up whenever the current changes.

**27. Why the Transformer Voltage Depends upon the Number of Turns in the Coil.** We have seen that if we wind on an iron circuit a coil of wire having a resistance of

only 0.55 ohm, and place it across a 115-volt, 60-cycle line, it will draw only about 0.15 ampere from the line. The reason for this, we have seen, is the fact that, in the turns of the wire, a back voltage is induced which chokes back the current. If 115 volts are able to force but 0.15 ampere through only 0.55 ohm resistance, this back voltage must be very nearly equal to the impressed 115 volts. For it takes but $0.15 \times 0.55$, or 0.08 volt, to force 0.15 ampere through 0.55 ohm. Thus, at the least, the back voltage must equal $115 - 0.08$, or 114.92 volts, which value for all practical purposes equals 115 volts.*

If, therefore, there are practically 115 back volts induced in the wire and the coil of wire had 1050 turns (page 60), then in each turn there would be induced $\frac{115}{1050}$, or 0.11 back volts, because each turn surrounds the same magnetic flux.

Now if we wound another coil of wire on the core, there would be induced in each turn as many volts as are induced in each turn of the main coil, because this induced voltage is merely caused by the magnetic lines of force cutting turns of wire. It makes no difference, we have seen, whether these turns of wire are in the same coil to which the power is applied or in other coils. Suppose, therefore, we wound on this iron core another coil having 23 turns. In each turn of this coil also, there would be induced 0.11 volt. This second coil would, therefore, have a voltage of $23 \times 0.11$, or practically 2.5 volts between its terminals. We would thus have a transformer suitable for heating the filaments of 2.5-volt radio tubes.

**Prob. 6–3.** How many turns of wire would have to be wound on the core in the above example, in order to get 550 volts from the 115-volt line, using the coil of 1050 turns as the primary coil?

* For reasons which will appear in Chapter V, the back voltage is even greater than 114.92 volts.

**Prob. 7–3.** It is desired to use the 1050-turn coil of the transformer of Prob. 6–3 as the secondary at 115 volts, and to take power from an 2300-volt, 60-cycle line. How many turns must be used in the primary coil?

**Prob. 8–3.** There are 1800 turns in each of the two primary coils of a transformer, which are designed to be connected in series across a 5500-volt line.

(*a*) What is the back voltage per turn of the primary coils?

(*b*) What is the induced voltage per turn of the secondary coils?

**Prob. 9–3.** How many turns must each of two secondary coils have in the transformer of Prob. 8 if they are to deliver 115 volts when joined in parallel?

**Prob. 10–3.** What would be the total voltage of the secondary coils of the transformer in Prob. 9 when they are joined in series?

**Prob. 11–3.** The primary of a certain radio transformer is wound with 460 turns and is to be energized from a 115-volt, 60-cycle line. This transformer is to have separate windings to deliver 700 volts, 5 volts and 2.5 volts. How many turns must there be in each of the secondary coils?

**28. What Happens when the Secondary Coil Delivers Current to a Load.** In all of the previous examples, we have been considering the action of transformers with the secondary coils open and therefore carrying no current. Let us now consider what happens when a load is attached to the secondary coils and a current is drawn from them.

Fig. 60 shows a transformer having a primary of 2300 turns connected through an ammeter $A_P$ to an 1150-volt line. The secondary winding consists of 230 turns; by closing the switch $S$, the secondary will supply current to a group of lamps, and the secondary current will be indicated on the ammeter $A_S$.

As Fig. 60 is drawn, there is no load on the secondary. However, the lamp bank is known to require a certain current, say 10 amperes, and the secondary must be large enough to supply this load. Although no current is flowing in the secondary, there will be a current in the primary, the amount

depending on the primary impedance. An average value of primary impedance (at 60 cycles) for a transformer of this size is 25,000 ohms. The current in the primary therefore equals $\frac{1150}{25{,}000}$, or 0.046 ampere. This is called the **exciting current.**

If we now attach the lamp bank to the secondary coils by throwing the switch $S$, the ammeter $A_S$ will show that 10 amperes are flowing in the secondary coils. The ammeter $A_P$ will also indicate somewhat over 1.00 ampere, showing that a current of about 1 ampere is now flowing in the primary coils.

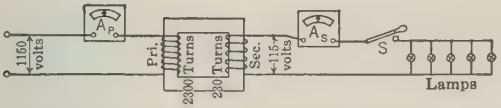



FIG. 60. When the secondary ammeter $A_S$ reads zero, the primary current read on $A_P$ is very small. When the switch $S$ is closed, the ammeter $A_S$ reads the secondary current taken by the lamps. The primary current through $A_p$ then becomes about $\frac{23.0}{2300}$, or about one-tenth, of the the secondary current.

Let us see how a current flowing in the secondary coils can cause current to flow in the primary coils, in spite of the fact that there is no electrical connection between the primary and the secondary coils. We have seen that there is an induced voltage in the secondary which is at all instants in the direction opposite to the voltage impressed on the primary. If now we allow this induced voltage to send a current through the secondary coil, this current will set up a magnetic field which will disturb the magnetic field already existing in the core, tending to neutralize it. The impedance of the primary circuit depends almost entirely upon the magnetic field set up by the exciting current of 0.046 ampere. When this magnetic field is thus disturbed and partially neutralized, the impedance of the primary coil is lowered,

and the 1100 primary volts can force more current through it. In fact, just enough additional current is forced through the primary coil to neutralize the field set up by the secondary current of 10 amperes. The magnetic field set up by this additional primary current must be exactly equal to and opposite the magnetic field set up by the secondary current. We know it must be opposite to the field of the secondary current because the voltage which produces this primary current is exactly opposite to the induced voltage which is producing the secondary current. We can see from the following that the new field set up in the primary coils by this extra current is equal to the field set up by the 10 amperes secondary current.

The strength of the field set up by the 10 amperes depends upon the product of the amperes (10) and the turns in the secondary (230) or 2300 ampere-turns. Thus in order to overcome this opposing field, enough current must flow in the primary coils to make up 2300 ampere-turns in the primary coils. Since there are 2300 turns in the primary coils, it is necessary for only 1 ampere to flow to make up 2300 ampere-turns. When this 1 ampere is added to the current in the primary coils, the opposing magnetic field set up by the secondary current is neutralized and the field becomes as it was when the original current of 0.046 was flowing and the core is magnetized as it was in the first place.

Of course, no transformer is a perfect machine with 100 per cent efficiency, so that a slightly larger current than 1 ampere would have to flow in order to make up the losses in the coils and core. But for all practical purposes this method of computing the current in the primary coils, when a given current flows in the secondary, is sufficiently correct, and is always used in practical estimates.

**Prob. 12–3.** The transformer of Fig. 60 supplies light and heating power to a small community. What current must the secondary deliver to supply forty 60-watt lamps?

**Prob. 13–3.** What current flows in the primary of the transformer in Prob. 12?

**Prob. 14–3.** Because of the installation of electric stoves, the transformer in Prob. 12 is required to supply a load of 8.8 kilowatts at 115 volts. The secondary winding can carry only 28 amperes safely. How many of these transformers are required to supply the total load?

**Prob. 15–3.** A certain 2300-volt line can carry 45 amperes. How many transformers, each rated to deliver 15 amperes at 115 volts, can be connected to the line? Assume the efficiency of each transformer is 96 per cent.

**Prob. 16–3.** At the full secondary load of 15 amperes what is the primary current in one of the transformers of Prob. 15?

**29. Rating of Transformer. Heating.** An indefinitely large amount of current cannot be taken from a given transformer, because of the excessive heating which would result. A transformer is so designed that a given voltage may be safely applied to the high-tension coils, with no danger of breaking down the insulation. But it is likewise so designed that the primary coils may carry a limited current without heating the insulation to such an extent that it is weakened. The secondary coils are designed and built to carry a current as many times greater than the primary current as the primary voltage is greater than the secondary voltage. The capacity of the transformer is then stated as the product of the secondary volts and the greatest secondary current. Thus if a transformer with two secondary coils in series were constructed for 230 volts and 21.7 amperes, it would be rated as $230 \times 21.7$, or 5000 volt-amperes. This would be called a 5 kv-a (kilovolt-ampere) transformer.*

In the design and construction of power transformers, it is necessary to provide adequate means for cooling the windings and core. If this is not done, the transformer will overheat at its rated load and only a reduced load can be supplied. In

* One kilovolt equals 1000 volts.

other words, if proper cooling is provided, a small transformer can do the work of a larger one in which no provision for cooling is made.

One of the most common methods of cooling is to immerse the transformer in oil. By using specially prepared oil, such

FIG. 61. Large power transformer showing coils provided to cool oil. *Westinghouse Elec. & Mfg. Co.*

as "transil" oil, the insulation of the transformer is improved, but the oil also serves to conduct heat from the windings and core to the outer casing where the heat is thrown off. Many transformer cases are built with fins so as to expose a

greater surface to the surrounding air. Very large transformers are frequently provided with forced circulation systems in which the oil is sent through cooling coils. Fig. 61 shows a large transformer with cooling coils on the outside of the case; this transformer contains 32,000 gallons of oil.

**Prob. 17–3.** A 50-kv-a transformer has two equal primary coils connected in series across a 2300-volt line. With its two secondary coils in parallel, it delivers 115 volts. What current can each secondary coil deliver?

**Prob. 18–3.** If the secondary coils in Prob. 17 are connected in series, what total current can the transformer deliver?

**Prob. 19–3.** What is the primary current in Prob. 17 when the transformer is delivering full load? In Prob. 18?

**Prob. 20–3.** If the primary coils of the transformer of Prob. 17 are connected in parallel across an 1150-volt line, what is the current in each primary coil when the secondaries are connected in series and delivering 185 amperes?

## SUMMARY OF CHAPTER III

**IMPEDANCE** is the volts per ampere in a circuit carrying alternating current, just as RESISTANCE is the volts per ampere in a circuit carrying direct current. In either case, both volts and amperes are to be measured between the same points in the circuit, and their ratio is the impedance of that part of the circuit, only, which is included between such points.

**IMPEDANCE** of a circuit is larger than its resistance whenever the circuit produces a magnetic field around itself. A circuit has INDUCTANCE, or SELF-INDUCTANCE, if it sets up a magnetic field around itself when current flows. The continual change in value and reversal of this field, which thereby sweeps across the circuit itself as the current alternates, induces in the circuit a voltage which always OPPOSES THE CHANGE of current. In an INDUCTIVE CIRCUIT, therefore, less current will be produced by a given alternating voltage than by the same value of direct voltage. This reduction of current is due to the INDUCTIVE REACTANCE of the circuit.

**RESISTANCE** (ohms) multiplied by current (amperes) gives volts used to produce heat in the electric circuit.

**INDUCTIVE REACTANCE** (ohms) multiplied by current (amperes) gives volts used to produce magnetic field around the circuit.

**IMPEDANCE** (ohms) multiplied by current (amperes) gives total volts used to produce current in the circuit.

If the resistance voltage is very small in comparison with the total volts in any circuit or part of the circuit, then, practically, Reactance = Impedance.

The same wire or circuit may offer widely different amounts of reactance, depending upon the way in which it is arranged (coiled, looped, or strung), the material by which it is surrounded (iron or air) and the frequency. None of these factors, however, affects the real ("ohmic") resistance, which remains the same as long as the length, cross-sectional area, temperature and material of the wire are unaltered.

Current does not rise instantaneously to its maximum steady value when a constant direct voltage is impressed on a circuit, if the circuit is inductive. The varying flux due to the rising current induces a back pressure which retards the growth of current. Similarly, the current does not fall instantaneously when the inductive circuit is broken, but tends to persist as an arc across the break.

Transformers, and the field circuits of dynamos, are highly inductive because they are designed purposely to produce quantities of magnetic flux. Special "field-break" switches may be used to interrupt such circuits, in order to avoid injury of switch points by this arc. If the current is permitted to be reduced unduly fast, the voltage induced in the circuit by the rapidly changing magnetic field may reach excessively high values, and injure the apparatus or endanger lives.

While any change is occurring in the magnetic field of an electric circuit, a voltage is induced in the circuit which tends to oppose the change. This is known as "LENZ' LAW," and there are many parallels to it in other branches of science.

The primary current of a transformer increases in proportion to the current from the secondary. The following ratios hold true approximately, unless the transformer is operating at light load:

$$\frac{\text{Primary current}}{\text{Secondary current}} = \frac{\text{secondary turns}}{\text{primary turns}} = \frac{\text{secondary volts}}{\text{primary volts}}.$$

The total current in the primary consists of two parts, the EXCITING CURRENT, and the LOAD CURRENT. The exciting

current is the zero-load current. The LOAD CURRENT, flowing through the primary turns, produces an additional magnetizing force (ampere-turns) which is at every instant equal in value and opposite in direction to the magnetizing force (secondary amperes $\times$ secondary turns) that tends to weaken the magnetic flux in the transformer.

Heating of both secondary and primary coils due to the current sets a definite limit to the amperes, and therefore to the maximum power (volts $\times$ amperes), which may be taken steadily from a transformer without injuring its insulation. This maximum power is known as the rated load, or the "size" of the transformer, and is usually expressed in volt-amperes (for small transformers) or in kilovolt-amperes $\left(\frac{\text{volts} \times \text{amperes}}{1000}\right)$ for large transformers.

## PROBLEMS ON CHAPTER III

**Prob. 21–3.** A certain coil takes a current of 3.2 amperes when connected across a 2-volt dry cell, but draws only 0.16 ampere when connected across a 6-volt, 60-cycle, secondary winding of a bell-ringing transformer. (*a*) What is the resistance of the coil? (*b*) What is the impedance of the coil at 60 cycles?

**Prob. 22–3.** What current would flow through the coil of Prob. 21 when connected across (*a*) 115 volts d-c, (*b*) 115 volts, 60 cycles?

**Prob. 23–3.** How many coils similar to that of Prob. 21 would have to be connected in series in order to make the resistance of the series circuit equal to the 60-cycle impedance of one coil?

**Prob. 24–3.** An electric circuit has an impedance of 40 ohms at 60 cycles. What voltage is required at this frequency to produce a current of 30 amperes in the circuit?

**Prob. 25–3.** A transmission line carrying alternating current has a "drop" of 88 volts over 22 miles of line when transmitting 50 amperes. What is the impedance of the line per mile for the particular frequency and spacing of line wires used?

**Prob. 26–3.** With no secondary load connected, the impedance of the primary coil of a certain transformer is found to be 3150 ohms at 60 cycles. What exciting current will this transformer draw from a 230-volt line?

**Prob. 27–3.** When the primary coil of Prob. 26 is placed in series with a 1000-ohm resistance across a 115-volt d-c line, a current of 0.11 ampere flows. What current would flow if the primary winding alone were connected across the 115-volt d-c line?

**Prob. 28–3.** What is the resistance of the primary winding in Prob. 27?

**Prob. 29–3.** A distribution transformer has two primary coils, each wound with 1800 turns. With the primary coils connected in series across a 2300-volt line, each secondary coil produces 115 volts. How many turns are there in each secondary?

**Prob. 30–3.** If the exciting current of the transformer of Prob. 29 is 0.085 amperes, what is the primary impedance at no load?

**Prob. 31–3.** If the primary coils of the transformer of Prob. 30 are connected in parallel, what exciting current will they draw from a 1150-volt line?

**Prob. 32–3.** A certain transformer has a secondary winding which delivers 640 volts. Taps are to be provided on this winding so that voltages of 340, 180, and 100 volts can be obtained from the same winding. If the full secondary winding contains 400 turns, at what points should the coil be tapped?

**Prob. 33–3.** If the voltages specified in Prob. 32 are to be obtained from a winding delivering 0.8 volt per turn, how many turns are required in the full winding and where should the taps be located?

**Prob. 34–3.** What must be the ratio of turns in a transformer intended to supply alternating current at 384 volts from its secondary to a rotary converter, while its primary takes power at 2300 volts from the transmission line?

**Prob. 35–3.** If the converter of Prob. 34 requires a current of 36.8 amperes, what current is supplied by the 2300-volt line?

**Prob. 36–3.** The transformer of Prob. 34 is rated at 25 kv-a. What is the maximum current it can deliver to the rotary converter? What is the full-load primary current?

**Prob. 37–3.** The exciting current of the transformer of Prob. 36 is 5 per cent of its full-load current. What is the no-load impedance of the primary winding?

**Prob. 38–3.** A 10-kv-a transformer supplies a group of 115-volt, 500-watt flood lights, and a 230-volt, 1000-watt immersion heater as shown in Fig. 62. What are the primary and secondary currents in the transformer?

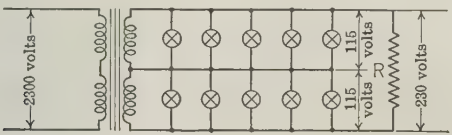



FIG. 62. A three-wire system which supplies 115-volt lamps and a 230-volt heater from a single transformer.

**Prob. 39–3.** If the transformer of Prob. 38 continues to supply the 230-volt heater as shown, but has lamps connected to only one side of the three-wire system, how many lamps can be supplied without overloading any part of the transformer? What will the primary current be under these conditions?

**Prob. 40–3.** What kv-a load is supplied by each half of the secondary in Prob. 39?

**Prob. 41–3.** How many additional lamps can be supplied in the circuit of Prob. 38 without overloading the transformer? What are the primary and secondary currents with maximum load connected?

**Prob. 42–3.** The transformer of Fig. 62 draws an exciting current of 0.23 ampere with no load on the secondary. What exciting current would it take if the primary coils were connected in parallel across an 1150-volt supply?

**Prob. 43–3.** If the transformer of Prob. 42 were used as a "step-up" transformer, with its full secondary connected across a 230-volt supply, what exciting current would it take?

**Prob. 44–3.** A two-wire transmission line 25 miles long connected to a 2300-volt, 60-cycle generator has impedance of 0.92 ohm per mile of wire. A "dead short" occurs between the wires somewhere along the line, and the station ammeter indicates 180 amperes on this line while the station voltage is pulled down to 1560 volts on account of this excessive current being drawn from the generator. How many miles out from the station, along the line, is the "short"?

**Prob. 45–3.** In some types of electric meters, fan motors, arc lamp feeding devices, and electromagnets in alternating-current circuit breakers, a " shading coil " is often wound on the magnet. This is a small coil, often only one turn, short-circuited on itself, and surrounding only one part of the pole-face. Describe and explain the effect of such a coil upon the distribution of flux over the pole face, as the current in the main winding of the electromagnet alternates.

**Prob. 46–3.** Some circuit-breakers for use on alternating-current systems are so arranged that when the main break opens, the circuit is still complete, but through a resistor which bridges or shunts the main break. After a small interval of time, an auxiliary break in this resistor opens the main circuit completely. The purpose of this is to protect the breaker against destruction, by dissipating slowly in this resistor the energy that was stored in the fields around the main circuits. Explain how this takes place.

**Prob. 47–3.** Explain how it is that a short circuit on the secondary of a transformer produces the same effect on the supply line, practically, as a short circuit across the primary.

**Prob. 48–3.** An impedance coil, wound on an iron core, is in series with a circuit which carries a constant amount of alternating current (say 6.6 amperes) at constant frequency. If another coil, insulated from the first, be wound on the same iron core and short-circuited upon itself, will the voltage drop across the series impedance coil be increased or decreased? Explain.

**Prob. 49–3.** Explain the similarity between starting a heavy flywheel into motion or stopping it, and establishing an electric current in a highly inductive circuit or stopping it.

**Prob. 50–3.** What voltages would be obtained in the secondary system of Fig. 62, instead of the values marked, if one of the secondary coils were to have its connections reversed from that shown? What would be the effect if one of the primary coils were to have its connections reversed from that shown, while the two of them remain connected in series to the same 2300-volt line?

**Prob. 51–3.** A certain distributing transformer is rated 25 kv-a, 60 cycles, 2200 : 1100/220 : 110 volts. Illustrate by sketches four ways of connecting this transformer for different voltage combinations, and for each sketch state the maximum volts and amperes input from the primary mains, and the corresponding maximum volts and amperes output to the secondary mains.

## CHAPTER IV

## POWER AND POWER FACTOR

The method of measuring the power taken by an appliance on an alternating-current circuit depends upon whether or not the appliance is inductive.

**30. Power in a Non-inductive Circuit.** An alternating current in the ordinary incandescent lamp produces but a slight magnetic field. Thus the reactance of an incandescent lamp is practically zero, and the impedance is composed almost entirely of resistance.

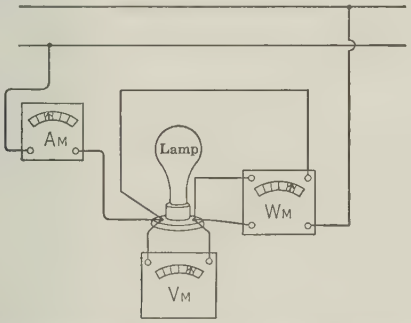



Fig. 63. The wattmeter *Wm* always measures the effective power, or true power, in watts. The product of the voltmeter *Vm* reading times the ammeter *Am* reading is the apparent power, expressed in volt-amperes. The apparent power equals the true power only when the load is a pure resistance.

In direct-current electricity the power in watts * is always equal to the product of the volts times the amperes. Suppose we connect a voltmeter *Vm* and an ammeter *Am* as in Fig. 63 to measure the alternating current and voltage of an in-

* The electric unit of power, a watt, is $\frac{1}{746}$ of a horsepower. A kilowatt (1000 watts) equals 1.34 or practically $1\frac{1}{3}$ horsepower.

candescent lamp of high candle power. If the voltmeter indicated 110 volts and the ammeter 5 amperes, a wattmeter *Wm* properly connected to the same lamp would indicate 550 watts which is also the product of the volts times the amperes, as in direct current. This shows that, in this case also, the product of the amperes times the volts equals the true watts. Thus in a non-inductive circuit we may measure the power either by a wattmeter or by a voltmeter and an ammeter.

**31. Power in an Inductive Circuit. Power Factor.** If instead of the lamp of Fig. 63, we supply power to the primary coil of a small transformer with no load on the secondary, we find an entirely different situation. The voltmeter may read 110 volts, for example, and the ammeter 0.06 ampere, but the wattmeter, instead of indicating $110 \times 0.06$, or 6.6 watts, now reads only 4 watts. The power, then, in an inductive circuit does not equal the product of the volts times the amperes, but only a certain fraction of that value. This fraction which the power is of the product of the volts times the amperes is called the **power factor** of the appliance. The product of the volts times the amperes is called the **apparent power,** and is stated in volt-amperes or in kilovolt-amperes (thousands of volt-amperes). The indication of the wattmeter is called the **effective power** and is measured, like direct-current power, in watts or in kilowatts (thousands of watts). Unless otherwise stated, the term **power** always means **effective power.**

Thus in the above example:

Apparent power $= 0.06 \times 110$, or 6.60 volt-amperes.

Effective power $= 4$ watts.

$$\text{Power factor} = \frac{4}{6.6} = 60.6 \text{ per cent.}$$

We may therefore write the equations

$$\textbf{Power factor} = \frac{\textbf{Effective power}}{\textbf{Apparent power}},$$

or $$\textbf{Effective power} = \textbf{Apparent power} \times \textbf{Power factor.}$$

Since, in a non-inductive circuit, the effective power equals the apparent power, the fraction $\frac{\text{effective power}}{\text{apparent power}}$ is equal to one or **unity.** Thus we say that the power factor of a non-inductive circuit is unity.

This property of an inductive load of consuming a real power which is only a fraction of its apparent power is used to great advantage in alternating-current systems. If, for example, we wish to protect a circuit from excessive current under short-circuit conditions,* we might place a resistance in series with the circuit sufficiently large to limit the current to a safe value. However, a coil having reactance can be used just as effectively to limit current and will consume the same number of volt-amperes as the resistance. But in the case of the resistance the number of volt-amperes is equal to the number of watts lost while with a reactance only a fraction of the volt-amperes represents actual watts lost.

**Prob. 1–4.** A certain a-c circuit which draws 5 amperes is protected by a 45-ohm series resistance. What power is lost in the resistance?

**Prob. 2–4.** If the resistance of Prob. 1 is replaced by a reactance coil of the same impedance and having a 40-per cent power factor, what power is saved by the change?

**Prob. 3–4.** An alternating-current refrigerator motor draws 3.2 amperes from a 115-volt line at a power factor of 62 per cent. What power is required to operate the refrigerator?

**Prob. 4–4.** A radio receiver draws 120 watts at a power factor of 91 per cent from a 115-volt line. What current is supplied to the receiver?

**Prob. 5–4.** What power is used by a coil of 50-ohms impedance and 85 per cent power factor, when connected to a 230-volt a-c line?

* This is frequently necessary because circuit breakers do not operate instantly and great damage may be done before a short circuit is cleared by the breakers.

**Prob. 6–4.** A reactor coil of 70-ohms impedance draws 2 kw from a 550-volt a-c line. What is the power factor of the coil?

**Prob. 7–4.** A certain transformer takes power at 65 per cent power factor when supplying its load. How much power does it take if it draws 15 amperes from a 2300-volt line?

**Prob. 8–4.** If 4.8 per cent of the power input to the transformer of Prob. 7 is lost, what power does it supply to its load?

**Prob. 9–4.** If the transformer of Prob. 8 is supplying its load at 230 volts and 70 per cent power factor, what is the secondary current?

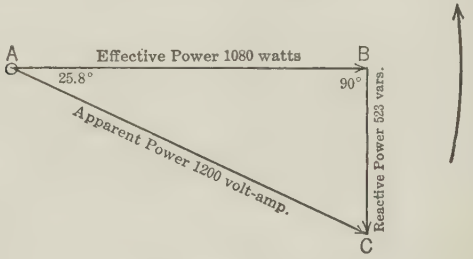



FIG. 64. Apparent power (volts times amperes) is composed of effective power (watts) and reactive power (vars — see Par. 34) at right angles to each other. The graphical representation of these relations, shown above, is called a "vector diagram."

**32. Graphical Representation.** In order to aid in solving many problems which arise in alternating-current work, it is customary to represent the various quantities by lines and angles. By working with these diagrams in front of us, we can more easily keep clearly in mind the true relations among such quantities as apparent power, effective power and power factor. Thus if we are dealing with an apparent power of 1200 volt-amperes at 90 per cent power factor, we know that the effective power is 90 per cent of 1200 or 1080 watts. And we represent it as in Fig. 64.

The 1080 watts, effective power is represented by a line

$AB$ drawn to scale. A line $BC$ is drawn down from $B$ indefinitely long at right angles to the line $AB$. Then with $A$ as a center with a pair of compasses opened to the value 1200 (to the same scale as $AB$), an arc is struck crossing the line $BC$. This crossing point is marked $C$.

We now have a right triangle, $ABC$, in which the line $AB$ is 90 per cent of $AC$, since $AB$ is drawn to scale to represent 1080 watts and $AC$ is drawn to scale to represent 1200 volt-amperes, and 1080 is 90 per cent of 1200. We can therefore say,

$$\frac{\text{Line } AB}{\text{Line } AC} = \text{power factor} = 0.90.$$

If we measure the angle between $AB$ and $AC$ we will see that it measures 25.8°. No matter how many right triangles we construct according to this method in which the effective power is 90 per cent of the apparent power, the angle between the base line and the long side (called the hypothenuse) of the right triangle will always be 25.8°.

**Prob. 10–4.** Construct the diagram for 3500 volt-amperes at 90 per cent power factor by the method shown for constructing Fig. 64 and measure the angle between the lines representing the effective power and the apparent power.

**Prob. 11–4.** Construct the diagram for 180 volt-amperes at 90 per cent power factor in the same way in which Fig. 64 was constructed and measure the angle between the lines representing the effective power and the apparent power.

**Prob. 12–4.** Repeat Prob. 11, using any value for the volt-amperes at 90 per cent power factor.

It is clearly seen from the above problems that when the power factor is 90 per cent, the effective power and the apparent power can always be represented by two lines drawn to scale at an angle of 25.8° to each other. Exactly the same method can be used for any value of volt-amperes and any value of power factor, but the value of the angle will be

**different for each different value of the power factor.** Thus if 1200 volt-amperes has a power factor of 85 per cent, the diagram will be like Fig. 65. The line $AB$ represents the effective power, 85 per cent of 1200 or 1020 watts, and the line $AC$ represents the apparent power 1200 volt-amperes. Note that this time the angle between $AB$ and $AC$ is 31.8°. When the power factor is 85 per cent, the effective power and the apparent power can always be represented by lines drawn to scale at an angle of 31.8°.

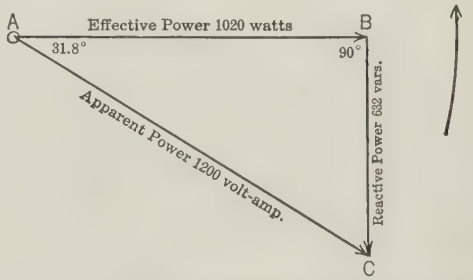



FIG. 65. The angle between effective power and apparent power in the vector diagram is different for each value of power factor (see Appendix, Table I), but is always the same for a given value of power factor regardless of the amount of power.

**Prob. 13 4.** By constructing diagrams similar to Fig. 64 and 65, find at what angles to each other we must draw lines to scale to represent a power factor of:

(*a*) 95 per cent.
(*b*) 80 per cent.
(*c*) 75 per cent.
(*d*) 70 per cent.
(*e*) 65 per cent.
(*f*) 60 per cent.
(*g*) 55 per cent.
(*h*) 50 per cent.

**33. Use of Power Factor Table.** For convenience in construction and solving diagrams, a table of angles and the corresponding power factors has been compiled. See Appendix, Table I. Accordingly, when we wish to construct a

diagram representing, say, 50 kv-a with a power factor of 92 per cent, we look in Table I under the column headed Power Factor for the number nearest to 0.92. We find it is 0.921. The angle corresponding to this power factor is 23°. Thus we construct Fig. 66, first drawing line $AB$, representing effective power equal to 92 per cent of 50, or 46.0 kw. At an angle of 23° to $AB$ we draw $AC$ making it have a value of 50 to the same scale as that to which $AB$ was drawn.

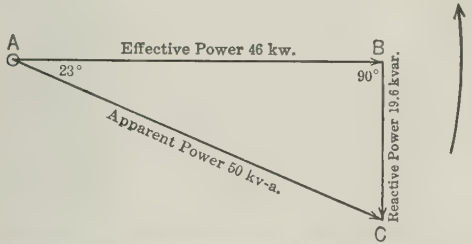



FIG. 66. Effective power is consumed in the circuit — changed into heat or into mechanical work. Reactive power is not consumed; it merely shuttles to and fro between the generator and the circuit, being the power used to establish the magnetic field around the circuit.

**Prob. 14–4.** Determine from Table I the power factor corresponding to the following angles between apparent power and effective power and construct diagrams for the same, showing how the volt-amperes necessary to deliver any required amount of power (watts) at the given power factor may be found immediately, without calculation:

(*a*) 88°.
(*b*) 45°.
(*c*) 30°.
(*d*) 59°.
(*e*) 72°.
(*f*) 12°.

**Prob. 15–4.** Determine from Table I the angles between apparent power and effective power for the following power factors:

(*a*) 0.500.
(*b*) 0.707.
(*c*) 0.866.
(*d*) 0.242.
(*e*) 0.000.
(*f*) 0.105.

**34. Reactive Power. Reactive Factor.** Thus far in referring to Fig. 64, 65 and 66, we have not considered the line $BC$, extending down perpendicularly from the end of $AB$ to the end of $AC$. This line in each case represents the **reactive power.** In other words the apparent power is made up of the effective power which does the work and the reactive power which does no work and is returned to the line.

An electric circuit carrying a large amount of reactive power may be likened to a gas engine with a large flywheel. The explosion of the gas in the cylinder delivers a large amount of power to the flywheel. That part of this power which the flywheel in turn delivers to the shaft and connected machinery corresponds to the effective power. But the flywheel then returns part of the power to the gas engine to keep it running until a new explosion takes place. This power, returned by the flywheel to the engine, corresponds to the electric reactive power; that is, to the electric power returned by the circuit to the generator.

Thus we see that the apparent power is really composed of two components, — the effective power and the reactive power. Note carefully, however, that the apparent power is not the arithmetical sum of the two. For instance, in Fig. 64, the effective power is 1080 watts and the reactive power is 523 vars,* while the apparent power is only 1200 volt-amperes. In Fig. 66, the apparent power 50 kv-a is composed of 46 kw and 19.6 kvars; the sum of the latter two being equal to much more than 50 kv-a.

However, for a given power factor, the reactive power is

* Although reactive power is still commonly referred to as volt-amperes or kilovolt-amperes, most engineers have adopted the **var** and the **kilovar,** respectively, as the names of the units of reactive power. The name var is derived by forming the word from the initial letters of " volt-ampere reactive," and the kilovar is simply 1000 vars. These names serve to identify reactive power clearly just as watts and kilowatts identify effective power; the volt-ampere and kilovolt-ampere now refer only to apparent power.

always a certain definite fraction of the apparent power, just as the effective power is a certain definite fraction of the apparent power.

Thus in Fig. 64, where the effective power is 90 per cent of the apparent power, the reactive power is $\frac{5\,2\,3}{1\,2\,0\,0}$, or 43.6 per cent of the apparent power. In all cases where the power factor is 90 per cent, the reactive power is 43.6 per cent.

The fraction which the reactive power is of the apparent power is called the **Reactive factor.** In other words,

$$\text{Reactive factor} = \frac{\text{Reactive power}}{\text{Apparent power}}.$$

**Prob. 16–4.** What is the reactive factor in Fig. 65?

**Prob. 17–4.** Determine by drawing diagrams similar to Fig. 64, the reactive power and the reactive factors for:

(*a*) 12,000 kv-a at 83 per cent power factor.
(*b*) 850 volt-amperes at 58 per cent power factor.
(*c*) 10 kv-a at 95 per cent power factor.

**Prob. 18–4.** Find, by drawing to scale, the reactive factors when the power factors are:

(*a*) 0.850. (*c*) 0.707.
(*b*) 0.940. (*d*) 0.250.

**Prob. 19–4.** A circuit draws 1500 watts effective power and 800 vars reactive. Draw the vector diagram for this condition and determine the apparent power taken by the circuit.

**Prob. 20–4.** What is the angle between the apparent power and the effective power in Prob. 19?

**Prob. 21–4.** What is the power factor in Prob. 19? What is the reactive factor?

**35. To Determine the Reactive Power.** Since the reactive factor is the fraction which the reactive power is of the apparent power, then

$$\text{Reactive power} = \text{Reactive factor} \times \text{Apparent power}.$$

By referring to Table I, we can find the reactive factors

corresponding to any power factor. Thus to find the reactive power of 1200 volt-amperes at 90 per cent power factor, we can look in the table for the reactive factor corresponding to a power factor of 90 per cent and find it to be approximately 0.436. We can then find the reactive power by using the equation:

$$\begin{aligned}\text{Reactive power} &= \text{Reactive factor} \times \text{Apparent power} \\ &= 0.436 \times 1200 \\ &= 523.2 \text{ vars.}\end{aligned}$$

This is the result obtained in Fig. 64 by drawing the values to scale.

**Prob. 22–4.** Check by means of Table I and the above equation the values obtained by scale drawings in Prob. 17 and 18.

**36. Relation between the Apparent Power, the Reactive and the Effective Power.** There is a third way of finding the reactive power, which is often used. Referring to Fig. 64, if we square the effective power, represented by line $AB$ and the reactive power, represented by the line $BC$, we shall find that the sum of these squares equals the square of the apparent power, represented by the line $AC$. Thus in Fig. 64,

Effective power squared $= AB^2 = 1080^2 = 1{,}166{,}000$;
Reactive power squared $= BC^2 = 523^2 = 274{,}000$;
Sum of squares $= AB^2 + BC^2 = 1{,}440{,}000$;
Apparent power squared $= AC^2 = 1200^2 = 1{,}440{,}000$.

Thus the apparent power squared equals the effective power squared plus the reactive power squared. This is stated in a more general way as follows:

In any right triangle,

**The square of the hypothenuse equals the sum of the squares of the other two sides.**

Or

The square of either side of a right triangle equals the

square of the hypothenuse minus the square of the other side.

In Fig. 64, 65, 66,

$$AC^2 = AB^2 + BC^2$$

or $$AB^2 = AC^2 - BC^2$$

and $$BC^2 = AC^2 - AB^2.$$

Therefore, if we know the effective power and the reactive power, we merely have to square these values and add them to get the square of the apparent power. By taking the square root of this sum we can find the value of the apparent power. Written in the form of an equation this becomes:

$$\text{Apparent power} = \sqrt{(\text{Effective power})^2 + (\text{Reactive power})^2}$$

or

$$\text{Effective power} = \sqrt{(\text{Apparent power})^2 - (\text{Reactive power})^2}$$

and

$$\text{Reactive power} = \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2}.$$

This is illustrated by Fig. 66 as follows:

$$\text{Apparent power} = \sqrt{(\text{Effective power})^2 + (\text{Reactive power})^2},$$

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{46^2 + 19.6^2}$$
$$= \sqrt{2116 + 384}$$
$$= \sqrt{2500}$$
$$= 50;$$

or $$\text{Effective power} = \sqrt{(\text{Apparent power})^2 - (\text{Reactive power})^2}.$$

$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{50^2 - 19.6^2}$$
$$= \sqrt{2500 - 384}$$
$$= \sqrt{2116}$$
$$= 46;$$

and Reactive power $= \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2}$;

$$BC = \sqrt{AC^2 - AB^2}$$
$$= \sqrt{50^2 - 46^2}$$
$$= \sqrt{2500 - 2116}$$
$$= \sqrt{384}$$
$$= 19.6.$$

Solve by the Law of Squares, the following problems:

**Prob. 23-4.** The apparent power in a certain circuit measured 10,200 volt-amperes and the effective power 8000 watts. What was the reactive power?

**Prob. 24-4.** Find the effective power in a circuit in which the apparent power is 5000 kv-a and the reactive power 450 kvar.

**37. The Three Methods of Solution.** There are thus three methods of using the relations which exist among the apparent power, effective power, reactive power, power factor and reactive factor. Any one method may be used to solve a problem, or any combination of methods.

**METHOD I.** By the use of diagrams drawn to scale. This is the simplest but least precise of the three.

**METHOD II.** By the use of tables of power factors and reactive factors, and diagram, not necessarily to scale.

**METHOD III.** By the use of the law of squares for the right triangle, and diagram, not necessarily to scale.

**Example.**

The effective power in a given circuit is 400 kw, and the apparent power is 500 kv-a. Find by all three methods:

(*a*) The power factor.
(*b*) The reactive power.
(*c*) The reactive factor.
(*d*) The angle between apparent power and effective power.

**Method I. Diagrams Drawn to Scale.**

(*a*) THE POWER FACTOR.

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \tfrac{4\,0\,0}{5\,0\,0}$$
$$= 80 \text{ per cent.}$$

(*b*) THE REACTIVE POWER.

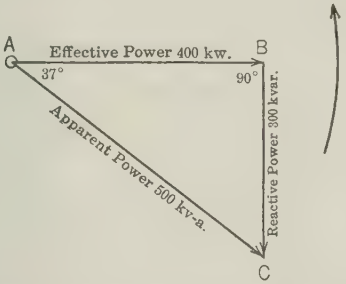



FIG. 67. In reactive circuits, lagging reactive power is represented by a vector drawn 90° clockwise from the vector representing effective power. Leading reactive power would then be represented by a vector 90° counterclockwise from the vector of effective power.

Construct Fig. 67 as follows:

Draw line $AB$ to scale to represent the effective power of 400 kw. At right angles to $AB$ draw line $BC$ down of indefinite length. With compasses opened to equal 500 on same scale as $AB$ draw an arc with $A$ as a center to cut line $BC$ at $C$.

Draw line $AC$ to represent 500 kv-a apparent power. The line $BC$ now represents the reactive power.

Scaling off $BC$, we find the reactive power equals 300 kvar.

(*c*) THE REACTIVE FACTOR.

$$\text{Reactive factor} = \frac{\text{Reactive power}}{\text{Apparent power}}$$
$$= \tfrac{3\,0\,0}{5\,0\,0}$$
$$= 60 \text{ per cent.}$$

(*d*) THE ANGLE BETWEEN LINES OF APPARENT AND EFFECTIVE POWER.

This angle by measurement is 37°.

**Method II. By Table of Power Factors and Reactive Factors.**

(*a*) THE POWER FACTOR.

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \tfrac{4}{5}\tfrac{0}{0}\tfrac{0}{0}$$
$$= 80 \text{ per cent.}$$

(*b*) THE REACTIVE POWER.

$$\text{Reactive power} = \text{Reactive factor} \times \text{Apparent power.}$$
$$= 0.60 \times 500$$
$$= 300 \text{ kvar.}$$

(*c*) THE REACTIVE FACTOR.

By referring to Table I of power factors and reactive factors, we see that for the power factor of 80 per cent (0.799) the corresponding reactive factor is 60 per cent (0.602).

(*d*) THE ANGLE.

By referring to Table I we see that the angle which corresponds to a power factor of 80 per cent and a reactive factor of 60 per cent is 37°. (Actually, 36.9°.)

**Method III. Law of Squares for Right Triangles.**

(*a*) THE POWER FACTOR.

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \tfrac{4}{5}\tfrac{0}{0}\tfrac{0}{0}$$
$$= 80 \text{ per cent.}$$

(*b*) THE REACTIVE POWER.

$$\text{Reactive power} = \sqrt{(\text{Apparent power})^2 - (\text{Effective power})^2}$$
$$= \sqrt{500^2 - 400^2}$$
$$= \sqrt{250{,}000 - 160{,}000}$$
$$= \sqrt{90{,}000}$$
$$= 300 \text{ kvar.}$$

(*c*) THE REACTIVE FACTOR.

$$\text{Reactive factor} = \frac{\text{Reactive power}}{\text{Apparent power}}$$
$$= \frac{800}{1000}$$
$$= 60 \text{ per cent.}$$

(*d*) THE ANGLE.

From Table I the angle corresponding to a power factor of 80 per cent and a reactive factor of 60 per cent = 37°.

**Prob. 25–4.** Solve Prob. 23–4 by all three methods and find the power factor and reactive factor.

**Prob. 26–4.** Find the reactive factor and power factor in Prob. 24–4, solving the problem by all three methods.

**38. Vector Diagram of Power.** In using any one of the three methods of solving the power relations in a circuit, always draw a diagram. When using the second or third method, one need not draw the diagram to scale. In fact a rough freehand drawing will serve the purpose of keeping the relations clearly before the mind.

Considering again Fig. 64, 65, 66 and 67, the lines labeled *AB*, *BC* and *AC* are generally called **vectors** * and therefore the diagrams are called **vector diagrams.**

Also, a curved arrow will be noted at the right-hand corner pointing up in a counterclockwise direction. This arrow has been put on these diagrams to indicate that we consider the whole triangle to be rotating in a counterclockwise direction about the point *A* as a center, at a speed in revolutions per second equal to the number of cycles per second. Thus a diagram representing conditions in a 60-cycle circuit would be assumed to make 60 revolutions per second about the point *A*.

* Any quantity which requires both magnitude and direction to define it is called a vector quantity. Alternating-current power is such a quantity. A line whose length represents the magnitude, and whose direction represents the direction of the quantity is called a vector.

All vector diagrams of alternating-current conditions are assumed to be rotating **counterclockwise** even if they are not so marked. For this reason the vector $AC$ is said to be lagging behind $AB$, or $AB$ may be said to be leading $AC$ (because $AB$ reaches any given point in each revolution before $AC$ reaches that same point).

In all vector diagrams of the power in an **inductive circuit,** the vector for apparent power is drawn **lagging** behind the vector of effective power. The reactive power is called the **lagging reactive power** and the power factor is called the **lagging power factor.** The term lagging, therefore, should always be closely associated with the term inductive, because it is possible, by means explained later, to produce leading reactive power and leading power factors and to have the apparent power vector lead the effective power vector.

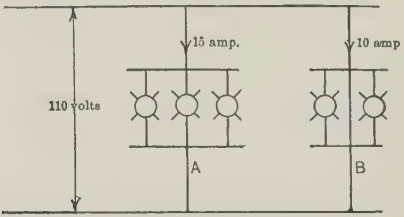



FIG. 68. If loads in parallel branches are all non-inductive, the apparent power on the line equals the arithmetic sum of the apparent powers in the branches; likewise the total power from the line is the arithmetic sum of the effective powers in the branches.

**39. Total Power Taken by Two Appliances.** It is often necessary to compute the apparent and the effective power supplied to two or more appliances. This is very simple when both of the appliances have the same power factor, because the power factor of the combination is the same as that for each appliance. Thus in Fig. 68, as each set of lamps has unity power factor, the effective power in each equals the apparent power, or:

For Bank $A$,

$$\begin{aligned}\text{Effective power} &= \text{volts} \times \text{amperes} \times 1 \\ &= 110 \times 15 \times 1 \\ &= 1650 \text{ watts.}\end{aligned}$$

For Bank $B$,

$$\begin{aligned}\text{Effective power} &= 110 \times 10 \times 1 \\ &= 1100 \text{ watts.}\end{aligned}$$

As they both have unity power factor, the power factor of the combination must also be unity. Therefore, we can add the powers of both arithmetically and obtain the total power delivered to both:

$$\text{Power to } A \text{ and } B = 1650 + 1100 = 2750 \text{ watts.}$$

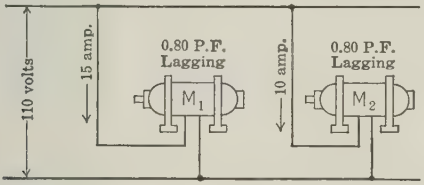



FIG. 69. If loads in parallel branches are inductive but have equal power factors, the apparent power on the line equals the arithmetical sum of the apparent powers in the branches. Total effective power from line is here also equal to the arithmetic sum of the effective powers taken by the individual branches.

Similarly we can deal with the two motors of Fig. 69 for, although the power factor is less than unity, it is the same for both motors (same numerical value, and both lagging), and therefore the power factor of the combination is the same as for each motor.

For motor $M_1$,

$$\begin{aligned}\text{Apparent power} &= 110 \times 15 \\ &= 1650 \text{ volt-amperes.}\end{aligned}$$

For motor $M_2$,

$$\begin{aligned}\text{Apparent power} &= 110 \times 10 \\ &= 1100 \text{ volt-amperes.}\end{aligned}$$

When loads have the same power factor, and both lagging or both leading, then the apparent power for both can be added arithmetically; thus,

$$\text{Total apparent power} = 1650 + 1100 = 2750 \text{ volt-amperes.}$$

For motor $M_1$,

$$\text{Effective power} = \text{volts} \times \text{amperes} \times \text{power factor} = 110 \times 15 \times 0.80 = 1320 \text{ watts.}$$

For motor $M_2$,

$$\text{Effective power} = \text{volts} \times \text{amperes} \times \text{power factor} = 110 \times 10 \times 0.80 = 880 \text{ watts.}$$

$$\text{Total effective power} = 1320 + 880 = 2200 \text{ watts.}$$

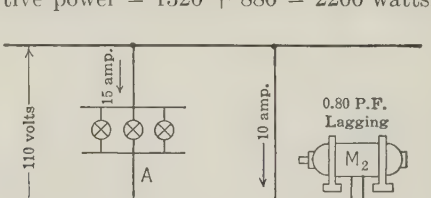



FIG. 70. If loads in parallel branches have different power factors, the total apparent power from the line is less than the arithmetical sum of the apparent powers in the branches. In every case, however, total effective power equals the arithmetical sum of the effective powers in the branches.

If, however, we combine motor $M_2$ with its 80 per cent power factor and the lamp bank $A$ with its unity power factor, as in Fig. 70, we can no longer use this arithmetical method but must resort to vector diagrams, as follows:

The single line $A_1B_1$ in Fig. 71 represents the vector diagram of the power in the lamp bank $A$, since the whole ap-

parent power is also effective power. In Fig. 72, the vector $AC$ represents the apparent power, 110 volts $\times$ 10 amperes, or 1100 volt-amperes, of motor $M_2$. The vector $AB$ represents 0.80 $\times$ 1100, or 880 watts, the effective power at 80 per cent power factor. From Table I we see that the angle at $A$

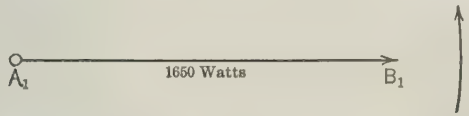



FIG. 71. Vector $A_1B_1$ represents the effective power consumed by lamp load $A$ in Fig. 70. There is no reactive power in $A$.

is 37°, and that the reactive factor is 60 per cent. The reactive power as represented by the vector $BC$ must, therefore, equal 0.60 $\times$ 1100, or 660 vars. Having thus separated the apparent motor power into the reactive power and the effective power, we can add the effective power of the motor to the

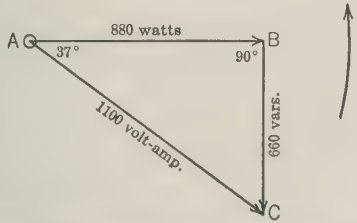



FIG. 72. Vector $AB$ represents effective power, and vector $BC$ reactive power, taken by motor $M_2$ in Fig. 70.

effective power of the lamps, and obtain the total effective power of the combination. We may also add the reactive power of one to the reactive power of the other, and obtain the total reactive power of the combination. From this combined effective power and combined reactive power we can find the apparent power and the power factor of the combination. Thus Fig. 73 is constructed by adding the vector

$A_1B_1$, of 1650 watts representing the effective power of the lamps as shown in Fig. 71, to the vector $AB$ of 880 watts which represents the effective power in the motor $M_2$ as shown in Fig. 72. This produces the vector $A_1B$ of Fig. 73. Since there is no reactive power in the lamps, the reactive power of the motor alone represented by the vector $BC$ in Fig. 72, and by $BC$ in Fig. 73, is combined with the total effective power represented by the vector $A_1B$. The appar-

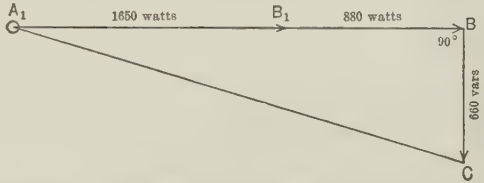



FIG. 73. Vector $A_1B$ represents total effective power in $A$ and $M_2$ together in Fig. 70, while $BC$ represents total reactive power. Then $A_1C$ represents total apparent power taken from line by the combination.

ent power of the combination then is represented by the vector $A_1C$, and the power factor of the combination is the fraction which the total effective power $A_1B$ is of the total apparent power $AC$.

$$\text{Total effective power} = A_1B_1 + B_1B$$
$$= 1650 + 880$$
$$= 2530 \text{ watts.}$$
$$\text{Total reactive power} = BC$$
$$= 660 \text{ vars.}$$

$$\text{Total apparent power}$$
$$= \sqrt{(\text{Total effective power})^2 + (\text{Total reactive power})^2}$$
$$= \sqrt{A_1B^2 + BC^2}$$
$$= \sqrt{2530^2 + 660^2}$$
$$= \sqrt{6{,}836{,}000}$$
$$= 2615 \text{ volt-amperes.}$$

$$\text{Power factor of the combination} = \frac{\text{Total effective power}}{\text{Total apparent power}}$$

$$= \frac{2530}{2615}$$

$$= 96.8 \text{ per cent.}$$

The rule for finding the power taken by a combination of appliances is:

**(1) Separate the apparent power of each load into its effective and reactive parts.**

**(2) Add all values of effective power to obtain the total effective power of the combination.**

**(3) Add all values of reactive power to obtain the total reactive power of the combination.**

**(4) Construct a right triangle with the above total values as sides. The hypothenuse of this triangle will represent the total apparent power of the combination.**

**(5) Divide total effective power of the combination by the total apparent power of the combination to get the power factor of the combination.**

There are no exceptions to the above rule, and it may be applied to any number of loads connected to the same system, either in series or in parallel.

Another way of looking at such a problem is as follows:

The problem is to add the 1650 watts taken by the lamps at unity power factor to the 1100 volt-amperes taken by the motor at 80 per cent power factor. Obviously, since the power factors of the two appliances are different, it is impossible to add the 1650 watts to the 1100 volt-amperes arithmetically. They must be added in such a way that their different power factors produce the proper effect upon the sum. This can be done by adding the two quantities **vectorially.** By this method, the 1650 watts of the lamp is represented by a line or vector as $A_1B_1$, Fig. 74, and the 1100 volt-amperes of the motor by another line or vector $B_1C$. If the power factor of both the 1650 watts and the 1100 volt-amperes were the same then the vector $B_1C$ would be added to the vector $A_1B_1$ in the same direction as $A_1B_1$. But since the 1100 volt-amperes has only 80 per cent power factor while the 1650 watts has unity power factor, the vector $B_1C$ representing the 1100 volt-amperes is drawn at an angle of 37°

to the vector $A_1B_1$ which represents the 1650 watts. This angle 37°, we have seen, is the angle corresponding to a power factor of 80 per cent as found by Table I.

To add vector $B_1C$ to vector $A_1B_1$, Fig. 74, we merely draw the line $A_1C$ from the tail of vector $A_1B_1$ to the head of vector $B_1C$. This vector $A_1C$ is the **vector sum** of the vectors $A_1B_1$ and $B_1C$.

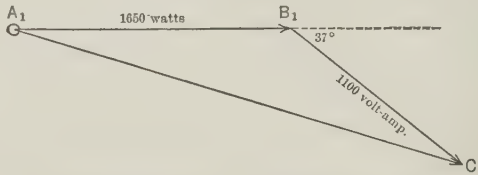



FIG. 74. $A_1C$, the total apparent power taken from the line by loads $A$ and $M_2$ together (Fig. 70), while not the arithmetical sum, is really the vector sum of apparent power to $A$ and apparent power to $M_2$, represented by vectors $A_1B_1$ and $B_1C$ respectively.

The value of $A_1C$ can be most easily found by extending the vector $A_1B_1$ and erecting a perpendicular from point $C$ to cross the extension of $A_1B_1$ at $B$. We then have Fig. 75, which is exactly like Fig. 73, and which we have solved; except that it has an extra line drawn from $B_1$ to $C$. It is solved in exactly the same way as Fig. 73. Thus in constructing and solving Fig. 73, we were really **adding vectorially** two quantities of power with different power factors.

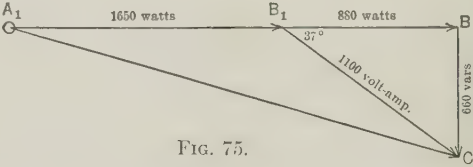



FIG. 75.

The case shown in Fig. 68, where both appliances had unity power factor, is not an exception to the rule but merely a special case. Here all the power is effective; thus there is no reactive power, and it is merely necessary to add the values of the effective power in both appliances together.

The method given for determining the total power in Fig. 69 is merely a short cut which is permissible only when the power factors of the appliances are the same. This is shown in the next problem.

**Prob. 27–4.** Using Fig. 69, find by the rules on page 101 the total effective power taken by $M_1$ and $M_2$, the total apparent power taken by $M_1$ and $M_2$, and compare the value found by the preceding rules with the sum of the values of the apparent power in each. Find the power factor of the combination and compare with the power factor of each appliance.

**Prob. 28–4.** Two induction motors are taking power from the same line. One motor takes 5 kv-a at 85 per cent power factor; the other takes 4 kv-a at 68 per cent power factor. Find the total effective and total apparent power drawn by the two motors from the line.

**Prob. 29–4.** Find the power factor of the total power drawn from the line by the two motors in Prob. 28.

**Prob. 30–4.** If the motors in Prob. 28–4 are operating on 220 volts, what current does the combination draw from the line?

**Prob. 31–4.** An induction motor drawing 2 kw at 70 per cent power factor is operating alone on a line. How much is the power factor of the line raised if ten 100-watt incandescent lamps are added to the line?

**Prob. 32–4.** How much would the power factor of the line in Prob. 31 be increased if ten 250-watt lamps had been added instead of ten 100-watt lamps?

**Prob. 33–4.** In a certain home the following loads are being supplied from the 115-volt lighting circuit. Electric refrigerator, 205 watts, 78 per cent power factor; radio receiver, 160 watts, 91 per cent power factor; eight 60-watt lamps.

(*a*) What total effective power is supplied by the line?
(*b*) What is the power factor of the total load?
(*c*) What total reactive power is supplied?

**Prob. 34–4.** What current is taken by each part of the load in Prob. 33? What is the total current in the line?

**40. Leading Power Factors.** Induction motors generally have a lagging power factor. But there are induction

motors on the market which (by certain construction and adjustment) have a leading power factor at all loads under full load, unity power factor at full load and a lagging power factor for all greater loads.

Synchronous motors, so-called because they operate at all loads exactly " in step " or, as it is called, " in synchronism " with the alternations of the current in the line, may be made to have a leading power factor. (See Chapter IX.) This characteristic has led to wide application of synchronous motors for " power factor correction." By this term, we mean the adjustment of the power factor of a total load so that it is as near unity power factor as can be produced at reasonable cost. A load with lagging power factor is "corrected " by substituting or adding sufficient leading reactive power to cancel out the lagging reactive power.

All of the power vector diagrams which we have thus far constructed have been for power with a lagging power factor. Note by the following example the difference between a power vector diagram with a leading power factor and one with a lagging power factor.

**Example:** A certain over-excited synchronous motor takes 50 kw from the line at 90 per cent power factor leading. Find:

(*a*) The apparent power taken from the line.
(*b*) The leading reactive power taken from the line.

**Solution:**

(*a*) **The apparent power taken.**

Since 50 kw is the effective power, and is 90 per cent of the apparent power, the apparent power therefore equals $\frac{50}{0.90} = 55.6$ kv-a.

(*b*) **The leading reactive power.**

Construct Fig. 76, drawing $AB$ to represent the 50 kw effective power. In Table I we see that the angle at $A$ must be 26° to correspond with a power factor of 90 per cent. But note that the vector $AC$ of apparent power must **lead** vector $AB$ of effective power, that

is, must be advanced in the direction of the rotation as shown by the arrow at the right.

In the table, the reactive factor corresponding to 90 per cent power factor is 0.438. Therefore the leading reactive power represented by vector $BC$ equals $0.438 \times 55.6$ kv-a, or 24.3 kvar.

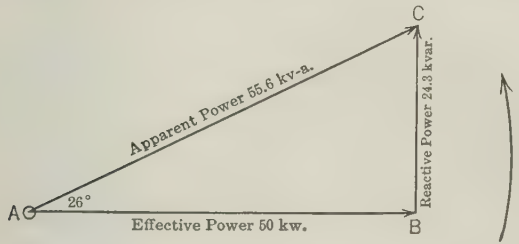



FIG. 76. Vector diagram for 50 kw effective power or 55.6 kv-a of apparent power, at 90 per cent power factor, the 24.3 kvar of reactive power being leading.

**Prob. 35–4.** How much leading reactive power does a synchronous motor take which is receiving 240 kv-a at 86 per cent leading power factor?

**Prob. 36–4.** An under-loaded compensated induction motor is drawing 475 vars leading reactive power from the line and 1230 volt-amperes apparent power.

(*a*) What is the power factor of this load?

(*b*) What effective power is the motor receiving?

**Prob. 37–4.** Two synchronous motors are drawing power from the same line. One receives 12.5 kv-a at 78 per cent leading power factor, the other receives 8.6 kv-a at 95 per cent leading power factor.

(*a*) What total apparent power do they receive from the line?

(*b*) What is the power factor of the total power received by the two motors?

(*c*) What total leading reactive power do they receive?

**41. Condensers. Condensive Reactance.** We found in Chapter III that a coil of wire which had a certain resistance to direct current, had a larger impedance to alternating current. This was found to be due to the inductive react-

ance caused by the changing magnetic field through the coil.

Exactly the opposite effect occurs in the case of a **condenser,** which in its simplest form consists merely of two conducting plates separated from each other by some kind of insulator. If we connect the two plates of a condenser to a source of direct current, we find that only a very small current flows, so small that practically speaking, the current is zero. The resistance of a condenser is hence very large if a good insulator is used between the plates.

Now, if we connect the same condenser to an alternating-current line, we find that an appreciable current flows which will cause an a-c ammeter to deflect. Evidently, the impedance of a condenser is **smaller** than its d-c resistance, which is just the opposite to the result found for a coil.

In order to understand this effect, we must consider briefly how electricity is conducted in various materials. According to modern theory, all matter is composed of atoms, which in turn consist of positive and negative charges of electricity. The positive charge, or **nucleus,** is relatively dense and heavy; the negative charge consists of one or more small particles called **electrons.** It is the **flow** of these **electrons** which constitutes an electric current. They are so small that 6,300,000,000,000,000,000 of them must flow by a point in one second to produce one ampere.

All materials contain large numbers of these electrons, more or less firmly attached to the atoms. If the bonds between the atoms and the electrons are weak, a voltage applied across part of the material will easily cause these negative particles to move, and we say the material is a good **conductor.** If, however, the electrons are firmly bonded to the atoms, it requires a very high voltage to cause any appreciable number of them to move, and we say this material is a good **insulator.**

When an insulator is placed between two conducting plates and a voltage is applied to the plates, the electrons in the insulator cannot move freely, but they can shift slightly in position because the bonds are more or less flexible. It is this property which permits an alternating current to flow in a condenser.

In Fig. 77 a simple condenser is shown with conducting plates 1 and 2 separated by a piece of insulating material. At *A* there is represented an atom with four electrons bonded to it; these electrons

are distributed at random as they would be with no voltage across the condenser. If now a source of voltage is connected to the plates and at a certain instant plate 2 is positive, the negative electrons will move toward plate 2 and take up positions as represented by atom *B* in Fig. 77. When all the electrons in the insulator have thus been shifted in position, we say the condenser is **charged.**

The electrons of the insulator moving toward plate 2 exert a repelling force on the free electrons in the plate, and these in turn move in the same direction, forcing all the electrons ahead of them to shift also. This process continues throughout the circuit until finally an excess of electrons appears on plate 1 to replace those which have shifted in the dielectric.

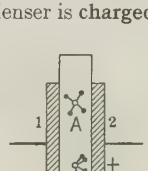



FIG. 77. A simple condenser composed of two metallic plates separated by insulation.

This motion of electrons around the circuit is, of course, an electric current. If a d-c voltage is applied to the condenser the current lasts only a short time and becomes zero after the electrons shift. There are so few free electrons in the insulating material that the steady current is very small and the resistance of the condenser is large. But when an alternating voltage is applied to the condenser, the electrons throughout the circuit shift back and forth with the alternations, and this alternating motion of the electrons is a true alternating current.

The impedance of a condenser is found by dividing the voltage across it by the current through it, just as we did for a coil. However, for most of the condensers used in alternating-current circuits, the impedance is practically equal to the condensive reactance*; that is,

$$\text{Impedance (ohms)} = \frac{\text{Voltage}}{\text{Current}},$$

or practically,

$$\text{Condensive reactance (ohms)} = \frac{\text{Voltage}}{\text{Current}}.$$

* There is no perfect insulator, so all condensers will conduct some direct current. Hence every condenser has a resistance, but we shall see in Chapter VI that the resistance has very little effect on the a-c impedance.

One of the most important properties of a condenser is that it draws leading apparent power at practically zero power factor. Hence these devices are extremely useful for power-factor control, as will be seen in the next paragraph. Such condensers are not of the simple type shown in Fig. 77; they consist of many plates and insulators, with the plates connected in parallel as shown in Fig. 78. The plates are commonly made of aluminum foil, and the insulation is waxed or oil-impregnated paper. Fig. 79 shows a modern condenser for power factor correction.

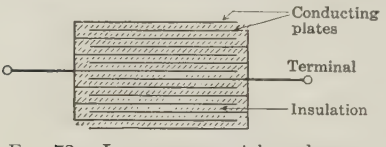



FIG. 78. Large commercial condensers contain numerous plates connected in parallel.

**42. Combination of Leading and Lagging Power Factors.** It is possible to have connected to the same line, some motors with lagging power factors and others with leading power factors. This is generally a desirable combination because the resulting power factor of the combination is usually better than that of either set of motors. This result is accomplished by the reactive power of one set being opposed to the other, so that at the instant one set of motors requires reactive power the other set is ready to give up reactive power and vice versa. Thus the two sets of motors supply each other with

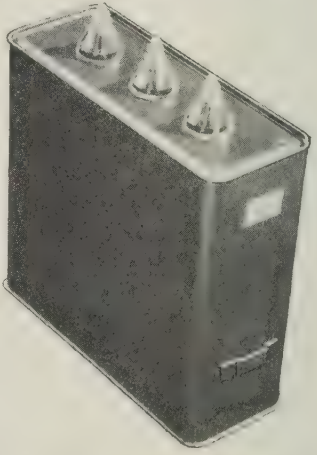


FIG. 79. Modern high-capacity, Pyranol*-insulated condenser. *General Electric Co.*

* General Electric trademark for chlorinated hydrocarbon mixture.

most of the needed reactive power, or the necessary reactive power merely circulates locally between the motors, and very little is drawn from the line.

It is not always desirable or possible to use a type of motor which will operate with a leading power factor. In such cases, the same result can be secured by the use of " static " condensers described in the preceding paragraph. These devices have several advantages which are leading to their wide application for power factor correction. They not only draw leading apparent power, but they require practically no real power. There is practically no maintenance cost because there are no moving parts and the units are sealed in metal containers. They are very compact and may be erected in otherwise wasted space. They are available in a wide range of sizes and ratings and may be applied economically to small installations as well as large ones.

The action of a synchronous condenser when connected in parallel with a lagging load is illustrated by the following example.

**Example.** A 130-kv-a synchronous motor operating at full load with 90 per cent leading power factor is used in the same shop with a 100-kv-a induction motor operating at full load with 85 per cent lagging power factor. Find:

(*a*) The total effective power taken from the line.

(*b*) The total reactive power taken from the line.

(*c*) The total apparent power taken from the line.

(*d*) The power factor of the power taken from the line.

Construct the vector diagram of Fig. 80 for the synchronous motor, which shows that the 130 kv-a at 90 per cent leading power factor is composed of 117 kw effective power and 56.7 kvar leading reactive power.

For the induction motor construct the diagram of Fig. 81, which shows that the 100 kv-a with an 85 per cent lagging power factor is composed of 85 kw effective power and 53 kvar lagging reactive power.

(*a*) The total effective power taken by the two motors is the alge-

braic sum * of the effective power taken by each motor, or 117 + 85 = 202 kw. This is represented in Fig. 82 by the vector $AB_1$, which is merely the sum of the vectors $AB$ of Fig. 80 and $A_1B_1$ of Fig. 81.

(*b*) The total reactive power taken by the two motors is the algebraic sum of the reactive power taken by each. Since 56.7 kvar of

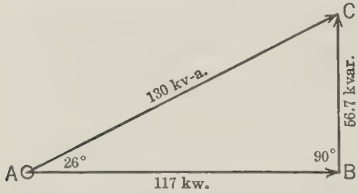



FIG. 80. Vector diagram for synchronous motor taking 130 kv-a of apparent power at 90 per cent leading power factor.

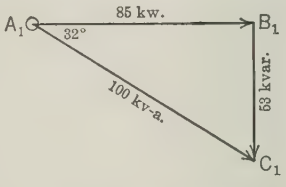



FIG. 81. Vector diagram for load taking 100 kv-a of apparent power at 85 per cent lagging power factor.

the reactive power is leading and 53 kvar is lagging, the algebraic sum is really the arithmetical difference or 56.7 − 53 = 3.7 kvar. Thus the vector $B_1C_2$ in Fig. 82 is merely the difference between the vectors $BC$ of Fig. 80 and $B_1C_1$ of Fig. 81. This means that at the

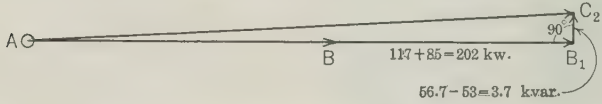



FIG. 82. Vector diagram showing result of putting the loads of Fig. 80 and 81 together on the same line. $AB_1$ is the total effective power, $B_1C_2$ is the total reactive power (leading), and $AC_2$ is the total apparent power taken from the line by the two loads together.

instant when the synchronous motor needs 56.7 kvar reactive power, the induction motor has 53 kvar reactive power just ready to be returned to the line, so it gives the 53 kvar to the synchronous

* The algebraic sum means merely the arithmetical sum or the arithmetical difference of the quantities as indicated by the direction of the vectors. Thus if the vectors point in the same direction, add them arithmetically; if they point in opposite directions, subtract one quantity from the other, their resultant or difference being leading or lagging the same as the larger of the two vectors.

motor instead. Thus the synchronous motor has to draw but 3.7 kvar reactive power from the line and this must be leading. At another instant the induction motor needs 53 kvar reactive power. The synchronous motor is just returning the 56.7 kvar to the line at this instant and gives 53 kvar to the induction motor and returns the rest, 3.7 kvar to the line, still leading. Thus the line has to carry but 3.7 kvar reactive power either way; and this is leading, because the motors need more leading than lagging reactive power.

The statement that one motor takes leading reactive power and the other lagging reactive power is merely another way of saying that one motor is returning its reactive power to the line at the instant that the other is drawing its reactive power from the line.

(*c*) The total apparent power taken by the two motors is represented by the vector $AC_2$, Fig. 82, the resultant of the total effective power, represented by vector $AB_1$ of Fig. 82 and the total reactive power represented by the vector $B_1C_2$.

$$\begin{aligned} AC_2 &= \sqrt{AB_1^2 + B_1C_2^2} \\ &= \sqrt{202^2 + 3.7^2} \\ &= \sqrt{40{,}819} \\ &= 202 \text{ kv-a (practically).} \end{aligned}$$

(*d*) The power factor of the total power taken by the two motors can be found as follows:

$$\begin{aligned} \text{Combined power factor} &= \frac{\text{Total effective power}}{\text{Total apparent power}} \\ &= \tfrac{2\,0\,2}{2\,0\,2} \\ &= 1, \text{ or unity.} \end{aligned}$$

**Prob. 38–4.** An induction motor taking 40 kv-a at 75 per cent lagging power factor is in parallel with a synchronous motor taking 65 kv-a at 90 per cent leading power factor. Find the total effective power taken by the two motors.

**Prob. 39–4.** Find the total apparent power and power factor taken by the two motors of Prob. 38.

**Prob. 40–4.** To what power factor would the synchronous motor of Prob. 38 have to be adjusted (by control of its field excitation) while still taking the same effective power from the line, in order to produce a total power factor of unity?

**Prob. 41–4.** A 2300-volt transmission line is delivering 150 kv-a to a load at 80 per cent lagging power factor. If a synchronous motor taking 75 kw effective power is added to the line, at what leading power factor must it operate so that the line will operate at 95 per cent lagging power factor?

**Prob. 42–4.** In Prob. 41, what current is supplied by the line before and after the synchronous motor is added?

**Prob. 43–4.** What is the apparent power taken from a 230-volt line when an induction motor is taking 45 amperes at 93 per cent lagging power factor and another running at light load takes 27 amperes at 60 per cent leading power factor?

**Prob. 44–4.** If a static condenser is used instead of the synchronous motor in Prob. 41, what reactive power must the condenser take at zero power factor, leading, in order to make the line operate at 95 per cent? What current is supplied by the line under this condition? Compare with the result of Prob. 42.

## SUMMARY OF CHAPTER IV

**APPARENT POWER** in any part of a circuit is the product of the volts across that part and the amperes flowing in that part, these quantities being measured by accurate voltmeters and ammeters properly connected. The amount of apparent power is expressed in terms of VOLT-AMPERES or of kilovolt-amperes (= volt-amperes ÷ 1000).

In a non-inductive circuit, or in a non-inductive part of any circuit, the power (watts) is exactly equal to the apparent power (volt-amperes), or kilowatts equals kilovolt-amperes. When the circuit is inductive, that part of the apparent power which goes to build up the magnetic field around the circuit is returned by means of the induced back-voltage when the current decreases and the field collapses. This component of apparent power is called REACTIVE POWER, and is expressed in terms of VARS or kilovars. The reactive power merely circulates between the electric circuit and the magnetic field, but is not consumed. The active or true power (watts) is transformed into mechanical energy or heat, and does not return to the electric circuit.

These relations are most conveniently represented by a VECTOR DIAGRAM, or geometrical figure in which the lengths

of lines (vectors) are proportional to watts, volt-amperes, and vars (or kilowatts, kilovolt-amperes, and kilovars). If power (kw) be represented by a horizontal line, then the apparent power is represented by a line swung ahead in the direction of rotation when the power factor is leading, and behind when the power factor is lagging, because the apparent power must lag behind the effective power with a lagging power factor and lead it when the power factor is leading. LAGGING kilovars would be represented by a vertical line pointing downward, and LEADING kilovars by a vertical line pointing upward, at the right-hand end of the power vector.

It follows that:

$$\text{Kv-a} = \sqrt{(\text{kw})^2 + (\text{kvar})^2},$$

or

$$\text{Kvar} = \sqrt{(\text{kv-a})^2 - (\text{kw})^2},$$

or

$$\text{Kw} = \sqrt{(\text{kv-a})^2 - (\text{kvar})^2}.$$

**POWER FACTOR** of any part of a circuit is the ratio of the power (watts, or kw) in that part to the apparent power (volt-amperes, or kv-a) in the same part, during the same period of time. The power factor of a non-inductive circuit, or non-inductive part of a circuit, is unity (1.00, or 100 per cent); the power factor of an inductive circuit is less than 1.00.

$$\text{Power factor} = \frac{\text{kilowatts}}{\text{kilovolt-amperes}}.$$

Similarly,

$$\text{Reactive factor} = \frac{\text{kilovars}}{\text{kilovolt-amperes}}.$$

and

$$\text{kilowatts} = (\text{kilovolt-amperes}) \times (\text{power factor})$$

or

$$\text{kilovolt-amperes} = \text{kilowatts} \div \text{power factor}.$$

When various loads are connected together either in series or in parallel, the total power that must be carried or delivered by the supply mains or by the generator, and the total power factor, may be found as follows:

$$\text{Total kv-a} = \sqrt{(\text{total kw})^2 + (\text{total kvar})^2}.$$

Total kw is the algebraic sum of the kw in each individual load or part of the circuit, power consumed being considered as positive and power generated as negative.

Total kvar is the algebraic sum of the kvar in each individual load or part of the circuit, leading kvar being considered as positive and lagging kvar as negative.

$$\text{Total power factor} = \frac{\text{total kw}}{\text{total kv-a}}.$$

Higher power factor is usually desired because it indicates a reduction in the amount of volt-amperes (and therefore in the size and cost of apparatus) required to handle a given amount of real power (watts). High power factor (not exceeding 1.00) may be obtained in inductive circuits by making one or more of the loads ANTI-INDUCTIVE or CONDENSIVE (so that they take leading kvar). Synchronous motors particularly are useful in this way; when the field magnets are strengthened they tend to draw leading kvar, and when the field magnets are weakened they tend to draw lagging kvar. Large numbers of static condensers are also being used for power-factor control, particularly where loads are scattered and too small to use synchronous condensers. When lagging and leading kvar are drawn in equal amounts from the same line, the power factor of the line is unity (1.00); the reactive power merely circulates between the inductive and the condensive loads, and none of it is drawn from the generator or line.

## PROBLEMS ON CHAPTER IV

**Prob. 45–4.** The ammeter shows that a certain generator is delivering 25.6 amperes. The voltmeter reads 230 volts. A wattmeter shows that 5.1 kw are being delivered. What is the power factor of the load?

**Prob. 46–4.** A certain single-phase induction motor operates at full load at 78 per cent power factor. How many amperes does it take from a 115-volt, a-c line if the power taken is 958 watts?

**Prob. 47–4.** What is the reactive factor of the motor of Prob. 46? What reactive power does it draw from the line?

**Prob. 48–4.** A generator is supplying two induction motors in parallel which take 92 kv-a each at 220 volts. Each has a lagging power factor of 80 per cent. What is the total kv-a output of the generator? Total watts output? Power factor of the line?

**Prob 49–4.** If one of the motors in Prob. 48 were replaced by a 120-kv-a synchronous motor with a leading power factor of 88 per cent, what would be:

(*a*) The total effective power taken from the line?

(*b*) The total apparent power?

(*c*) The power factor of the line?

**Prob. 50–4.** In order to improve the power factor of Prob. 48, one of the induction motors is exchanged for a synchronous motor which carries the same load but takes a leading current. To what power factor must the synchronous motor be adjusted and what apparent power in kv-a must it take in order to make the power factor of the line unity?

**Prob. 51–4.** Compare the apparent power supplied by the generator in Prob. 48 with that supplied in Prob. 50. What becomes of the difference?

**Prob. 52–4.** An induction motor taking a lagging line current of 22 amperes with a power factor of 75 per cent is connected in parallel on a 230-volt line with a static condenser taking a leading line current of 35 amperes at zero power factor.

(*a*) What total apparent power is drawn from the line?

(*b*) What is the power factor of the line?

**Prob. 53–4.** An induction motor takes a current of 18 amperes at a pressure of 220 volts. The power factor is 0.866. What is the angle between the effective and the apparent power of the motor at this load and how much power (watts) does it take?

**Prob. 54–4.** A synchronous motor is taking a leading current of 26 amperes at 220 volts. Power factor, 94 per cent. What is the angle between the effective and the apparent power of this motor under these conditions, and what power does it take?

**Prob. 55–4.** If the two motors of Prob. 53 and 54 are put in parallel on the same circuit:

(*a*) What current will flow in the line?

(*b*) What will be the power factor of the power drawn from the line?

(*c*) How much power will the line be delivering, assuming these two motors are alone on a short line?

**Prob. 56–4.** What power would be taken from a 230-volt, a-c line if the two coils of Prob. 5 and 6 were placed in parallel across the line?

**Prob. 57–4.** If a resistance of 50 ohms is added to the line in parallel with the coils of Prob. 56, what total power will be taken?

**Prob. 58–4.** What total current is drawn from the line and what is the power factor of the total load:

(*a*) In Prob. 56?

(*b*) In Prob. 57?

**Prob. 59–4.** In a certain circuit the effective power is 400 kw and the reactive power (lagging) is 300 kvar. What is the apparent power? What is the power factor?

**Prob. 60–4.** In Prob. 59, how much leading reactive power is required to raise the power factor of the load to 85 per cent, assuming that 300 watts of effective power must be added for every kvar of leading reactive power?

**Prob. 61–4.** Assume a " synchronous condenser " of zero power factor is connected to the line in Prob. 48. What leading kvar must it take to raise the power factor to 0.85? To 0.90? To 0.95? To 1.00? Notice the relative increase in size of the synchronous condenser required for each 5 per cent increase of line power factor and, assuming that the cost of such units is directly proportional to the kvar which they must take, discuss the advisability of attempting to raise the line power factor clear up to 100 per cent.

**Prob. 62–4.** A 500-kv-a synchronous motor is to be operated at various power factors (by adjusting the field current), but always it must drive such a load as will cause the total apparent kv-a input to have full rated value (500 kv-a). For each 10 per cent change of power factor from 0.10 to 1.00, calculate the power (watts), the reactive power (vars), and the arithmetical sum of these two quantities. From this information, decide at what power factor you would prefer to operate your synchronous motor, in order to get the greatest total benefit (ability to carry real power load plus ability to correct poor power factor) for the investment of money which you have in this motor.

**Prob. 63–4.** Synchronous converters, or " rotaries," give highest efficiency, greatest capacity and least trouble when the field excitation is adjusted so that they take their alternating-current power from the line at 100 per cent power factor. A 500-kw converter so adjusted is connected to a central station which already has a load of 1200 kw at 70 per cent power factor. What is the resulting power factor at the station, and the increase in total kv-a output of the generators?

**Prob. 64–4.** What is the least kv-a rating of an a-c generator which could supply the following loads all connected to the same circuit or mains: 430 kw in incandescent lamps, 580 kw to induction motors at 0.80 power factor lagging, 150 kw to arc lamps at 0.70 power factor lagging, and line loss equal to 10 per cent of the total load (kw), at 0.90 power factor lagging?

**Prob. 65–4.** If the least power factor at which a synchronous motor can operate (carrying no real load and serving merely to raise the line power factor, or acting as a " synchronous condenser ") is 10 per cent, what must be the least kv-a rating of such a synchronous motor added to the system of Prob. 64–4, in order to raise the power factor of the total load on the generators to 100 per cent?

**Prob. 66–4.** A bank of incandescent lamps takes 7.5 amperes at 115 volts. When a reactive dimmer is inserted in series, the circuit takes 6.2 amperes, at 73 per cent power factor.

(*a*) What power is taken by the lamps when no dimmer is used?

(*b*) What power is taken by lamps and dimmer?

**Prob. 67–4.** A non-inductive dimmer is used to dim a bank of lamps which normally takes 15 amperes from a 115-volt line. The dimmer cuts the current down to 8.2 amperes; at this reduced current the resistance of the lamps is only 62 per cent of its former value:

(*a*) How much power do the lamps take when the dimmer is not used?

(*b*) How much power is taken by the lamps when the dimmer is used?

(*c*) How much power is taken by the dimmer?

(*d*) How much power is taken by the lamps and the dimmer combined?

**Prob. 68–4.** If a reactive dimmer is used to reduce the current in the lamps of Prob. 67–4 to 8.2 amperes:

(*a*) How much power do the lamps take when the dimmer is used?

(*b*) If the power factor of the lamps and the dimmer combined is 70 per cent, how much power (watts) do the lamps and the dimmer together use?

(*c*) How much power (watts) does the dimmer take when used in connection with the lamps?

**Prob. 69–4.** (*a*) How many vars must be taken by the dimmer of Prob. 68–4?

(*b*) At what power factor is the dimmer itself operating?

# CHAPTER V

## CURRENT AND VOLTAGE RELATIONS IN SERIES AND IN PARALLEL CIRCUITS

The power taken by a combination of electric appliances can be determined best, as we have seen, by means of vector diagrams. The same method is used to determine the amount of current and voltage in any part of a series or parallel combination of appliances.

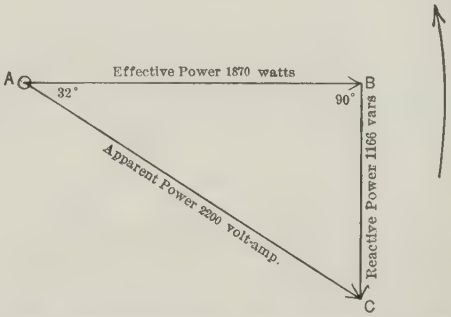



Fig. 83. Vector diagram showing power relations in circuit taking 1 ampere at 2200 volts, or 10 amperes at 220 volts, with power factor 85 per cent.

**43. Vector Diagram of Current.** When we wish to represent 2200 volt-amperes at 85 per cent lagging power factor, we have seen that we may construct a diagram like Fig. 83. We consider the apparent power $AC$ to be made up of two components, the effective power $AB$ and the reactive power $BC$. We represent the idea of lagging by drawing the apparent power vector $AC$ at an angle to $AB$ in such a manner

that it lags behind $AB$ when the direction of the rotation of the diagram is taken into consideration. The power factor is represented by the fraction which the length $AB$ is of the length $AC$. This fraction is always 0.85 when the angle between $AB$ and $AC$ is 32°.

If now we divide each of these power quantities by the voltage, we shall obtain the current corresponding to each power quantity. Thus if we assume that the voltage is 220 volts and divide the apparent power of 2200 volt-amperes by

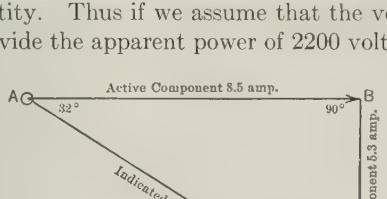



FIG. 84. Total or indicated current in a circuit may be analyzed into components just as the apparent power was analyzed in Fig. 83; each vector here equals corresponding vector in Fig. 83 divided by the line voltage.

220 we obtain 10 amperes. Since this current is a factor of the apparent power, we will represent it by the vector $AC$ in Fig. 84 which corresponds to the vector of apparent power $AC$ in Fig. 83. Similarly divide the effective power 1870 watts by 220 and we have the current 8.5 amperes, which is the current part of the effective power. This we will represent by the vector $AB$ in Fig. 84, corresponding to the vector $AB$ of Fig. 83. In the same way, vector $BC$, 5.3 amperes, in Fig. 84, corresponds to the vector $BC$ of Fig. 83 and represents the current part of the reactive power. It is found by dividing the reactive power 1166 vars by 220 volts. Note that Fig. 84 is exactly like Fig. 83, each vector representing current instead of power and each having a value $\frac{1}{220}$ of the

value of corresponding power vector. Just as the vector $AC$ of apparent power is thought of as consisting of the two components, $AB$, the effective power, and $BC$, the reactive power, so the vector of indicated current, $AC$, is thought of as consisting of the two component vectors $AB$, the **active** or **power component of current,** and $BC$, the **reactive* component of current.**

Similarly, just as in Fig. 83 the vector $BC$ of reactive power is drawn downward at an angle of 90° to the vector $AB$ of effective power, so in Fig. 84 the vector $BC$ of reactive current is drawn downward at an angle of 90° to the vector $AB$ of active current.

We have thus divided the current into two components, the active or power component and the reactive component, which are represented by vectors at 90° to each other.

The same relation exists between these current components and the indicated current, as between the two power components and the apparent power.

Thus,

Power component of current = Current × Power factor,

or
$$\begin{aligned} AB &= AC \times \text{power factor} \\ &= 10 \times 0.85 \\ &= 8.5 \text{ amperes;} \end{aligned}$$

and

Reactive component of current = Current × Reactive factor,

or
$$\begin{aligned} BC &= AC \times \text{reactive factor} \\ &= 10 \times 0.53 \\ &= 5.3 \text{ amperes;} \end{aligned}$$

and

$$\text{Indicated current} = \sqrt{(\text{Active current})^2 + (\text{Reactive current})^2}$$

or
$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ &= \sqrt{8.5^2 + 5.3^2} \\ &= \sqrt{100 \text{ (approx.)}} \\ &= 10 \text{ amperes.} \end{aligned}$$

* Sometimes (incorrectly) called the *wattless* component of current.

Draw rough diagrams and solve the following:

**Prob. 1–5.** A single-phase induction motor with lagging power factor of 79 per cent takes 52.5 amperes. What is:
(*a*) The active component of current?
(*b*) The reactive component of current?
(*c*) The angle of lag between the current in the line and the active component of current?

**Prob. 2–5.** If the pressure on the motor of Prob. 1 is 230 volts, what is:
(*a*) The effective power?
(*b*) The reactive power?
(*c*) The apparent power?

**Prob. 3–5.** A group of incandescent lamps takes 16.2 amperes. What is:
(*a*) The power component of current?
(*b*) The reactive component of current?
(*c*) The angle of lag of the current in the lamps behind the power component?

**Prob. 4–5.** A radio receiver takes 1.43 amperes at 93 per cent power factor, from a 115-volt line.
(*a*) What is the power component of the current?
(*b*) What is the effective power?
(*c*) What is the apparent power?
(*d*) Compute the reactive component of current.
(*e*) What is the reactive power?

**Prob. 5–5.** An electric refrigerator motor takes a current of 1.72 amperes which lags 20° behind its active or power component:
(*a*) What is the power component of current?
(*b*) What is the reactive component?
(*c*) What is the power factor? the reactive factor?

**Prob. 6–5.** If the motor of Prob. 5 operates on 115 volts, what is:
(*a*) The effective power?
(*b*) The apparent power?
(*c*) The power factor?
(*d*) The reactive power?

**Prob. 7–5.** If we put an ammeter in one of the leads of the motor in Prob. 5, what current would it indicate?

**44. Current in Series and in Parallel Circuits.**

**(1) Series circuits.** A series circuit is one in which all the electrical devices are arranged in **tandem.** Consider the simple series circuit of Fig. 85, consisting of an ammeter $A$,

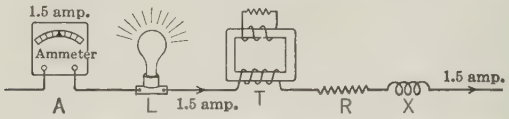



FIG. 85. Each part of a series circuit carries the same current as in every other part, regardless of whether it is a-c or d-c.

a lamp $L$, transformer $T$, a resistance $R$, and a reactance $X$. If we find that an alternating current of 1.5 amperes flows through the ammeter, we know that an alternating current of 1.5 amperes flows through the entire circuit. The law is exactly the same as the law for a direct current flowing through a series circuit.

**The alternating current flowing through each part of a series circuit is the same as that which flows through every other part.**

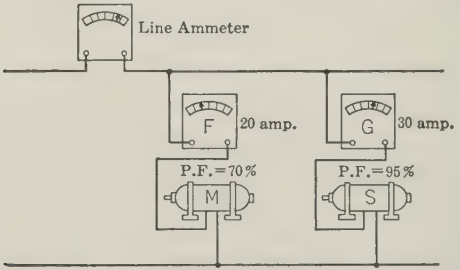



FIG. 86. Each of several circuits in parallel receives the same voltage; the line current is the vector sum (not the arithmetical sum) of the currents in the branches.

**(2) Parallel circuit.** In a parallel circuit the electrical devices are so arranged that the current is divided among them. Consider the parallel circuit of Fig. 86, consisting of

two induction motors $M$ and $S$ in parallel across the line. The motor $M$ has a lagging power factor of 70 per cent and the current through it as indicated by the ammeter $F$ is 20 amperes. The motor $S$ has a lagging power factor of 95 per cent and the current through it as indicated by the ammeter $G$ is 30 amperes. How much is the line current as indicated by the line ammeter?

(1) Construct rough current diagram, Fig. 87, for motor $S$. By Table I:

Indicated current vector $A_SC_S$ lags 18° behind active current vector $A_SB_S$ when the power factor is 95 per cent lagging. The reactive factor is 31.2 per cent for 95 per cent power factor. Thus,

$$\text{Power current } A_SB_S = 0.95 \times 30 = 28.5 \text{ amp.}$$
$$\text{Reactive current } B_SC_S = 0.312 \times 30 = 9.4 \text{ amp.}$$

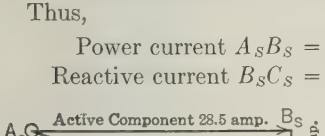



FIG. 87. We may consider that two currents flow in $S$, Fig. 86, at the same time; namely 28.5 amp in phase with line voltage and 9.4 amp at 90°. Their resultant is the 30 amp as indicated.

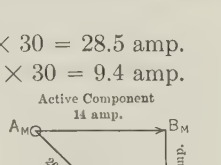



FIG. 88. The 20 indicated amperes to $M$, Fig. 86, consists of 14 amp in phase with line voltage, and 14.3 amp at 90° to it.

(2) Construct rough current diagram, Fig. 88, for motor $M$. By Table I:

Apparent current vector $A_MC_M$ lags 46° behind the active current vector $A_MB_M$, when the power factor is 70 per cent lagging. The reactive factor is 71.4 per cent for a 70 per cent power factor. Thus,

$$\text{Power current } A_MB_M = 0.70 \times 20 = 14.0 \text{ amp.}$$
$$\text{Reactive current } B_MC_M = 0.714 \times 20 = 14.3 \text{ amp.}$$

(3) Construct rough diagram, Fig. 89, for the line current feeding the parallel combination of motors $M$ and $S$, as follows:

The vector $A_LB_L$ represents the power component of the line current and is equal to the sum of the active components of motors $M$ and $S$.

$$A_LD = A_SB_S \text{ (Fig. 87)} = 28.5 \text{ amp.}$$
$$DB_L = A_MB_M \text{ (Fig. 88)} = 14 \text{ amp.}$$
$$A_LB_L = A_SB_S + A_MB_M = 42.5 \text{ amp.}$$

Similarly, the vector $B_LC_L$, Fig. 89, represents the reactive component of line current feeding the two motors.

$$B_LF = B_SC_S \text{ (Fig. 87)} = 9.4 \text{ amp.}$$
$$FC_L = B_MC_M \text{ (Fig. 88)} = 14.3 \text{ amp.}$$
$$B_LC_L = B_SC_S + B_MC_M = 23.7 \text{ amp.}$$

The vector $A_LC_L$, Fig. 89, must, therefore, represent the indicated line-current since it represents a total current of

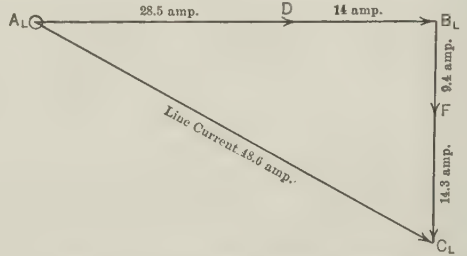



FIG. 89. Line ammeter in Fig. 86 indicates resultant of total active current to both $M$ and $S$, and total reactive current which is at 90°. This resultant is $A_LC_L = 48.6$ amp.

which $A_LB_L$ is the power component and $B_LC_L$ is the reactive component.

$$A_LC_L = \sqrt{A_LB_L{}^2 + B_LC_L{}^2}$$
$$= \sqrt{42.5^2 + 23.7^2}$$
$$= \sqrt{2368}$$
$$= 48.6 \text{ amp.}$$

The line ammeter in Fig. 86 would, therefore, indicate a line-current of 48.6 amperes.

This line current of 48.6 amperes as found above is actually the combined currents of 30 amperes of motor $S$ and 20 amperes of motor $M$. Since these two currents have different power factors it is necessary to add the 30 amperes and the 20 amperes vectorially and not arithmetically. The vector $A_L C_L$, Fig. 89, therefore represents the vectorial sum of the vectors $A_S C_S$ of Fig. 87, and $A_M C_M$ of Fig. 88.

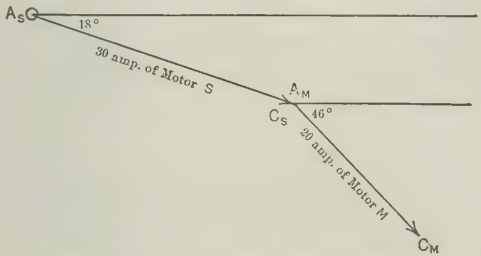



FIG. 90. The 20 amperes of $M$, Fig. 86, is added *vectorially* to the 30 amperes of $S$ which is in parallel, to obtain the line current.

This can be seen more clearly if we construct Fig. 90, in which vector $A_S C_S$ drawn at a lagging angle of 18° to the horizontal, represents the 30 amperes of motor $S$ at a lagging power factor of 95 per cent. The vector $A_M C_M$ drawn at a lagging angle of 46° to the horizontal represents the 20 amperes of motor $M$ at a lagging power factor of 70 per cent. To add the two vectors $A_S C_S$ and $A_M C_M$ we merely attach the tail of one vector to the head of the other, keeping them both at their respective angles to the horizontal. The vector sum of the two will then be represented by the vector drawn from the tail of the first to the head of the last vector. Vector $A_S C_M$ in Fig. 91 is so drawn and represents the vector sum of $A_S C_S$ and $A_M C_M$.

To find the numerical value of this vector $A_S C_M$ we may complete the triangle $A_L B_L C_L$ of Fig. 92 in which the vector $A_L C_L$ is the same as the vector $A_S C_M$ of Fig. 91.

The triangle of Fig. 92 is exactly the same as the triangle of Fig. 89, and can be solved in the same way. In Fig. 89 we con-

structed the triangle from the active and reactive components of the motor currents, while in Fig. 92 we have drawn in the total motor currents first and then resolved them into their active and reactive components. Of course the final results are the same.

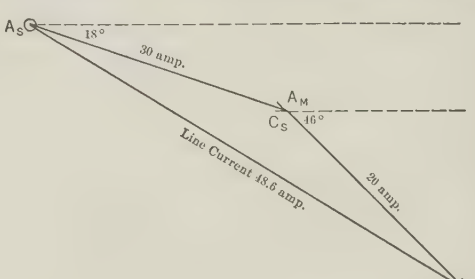



FIG. 91. Note that the vector sum of currents to $M$ and $S$ in parallel, Fig. 86, is always less than the arithmetical sum unless the power factors of $M$ and $S$ happen to be equal. In such case vector $A_M C_M$ would be in same straight line with $A_S C_S$.

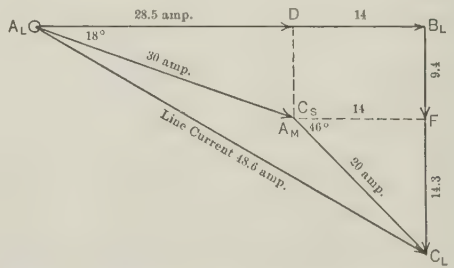



FIG. 92. Showing how Fig. 91, as usually drawn, is really derived from Fig. 87, 88 and 89.

The rules for finding the current in a parallel combination may be stated as follows:

**(1) Resolve the indicated current taken by each appliance into its active and reactive components.**

**(2) Add (algebraically) the active components together and the reactive components together.**

**(3) The indicated current in the parallel combination equals the square root of the sum of the squares of the total active component and the total reactive component.**

**Prob. 8–5.** What is the power factor of the line current in Fig. 86?

**Prob. 9–5.** What line current would the ammeter in Fig. 86 indicate if the power factor of motor $M$ were 95 per cent lagging?

**Prob. 10–5.** If the power factor of motor $M$, Fig. 86, were 50 per cent lagging and of motor $S$, 90 per cent lagging, what current would the line ammeter indicate?

**Prob. 11–5.** How much current would the line ammeter indicate if the power factors of both motors in Fig. 86 were leading instead of lagging?

**Prob. 12–5.** If the power factor of motor $S$, Fig. 86, were lagging and of motor $M$ were leading, what current would flow in the line?

**Prob. 13–5.** Two motors are in parallel on the same transformer. Motor No. 1 draws 62 amperes at 76 per cent lagging power factor and motor No. 2 draws 95 amperes at 90 per cent leading power factor. What current flows in the secondary coil of the transformer?

**Prob. 14–5.** What is the power factor of the secondary current of the transformer in Prob. 13?

**Prob. 15–5.** How much current would flow in the transformer secondary if the power factor of Motor No. 1, Prob. 13, were leading and if the power factor of Motor No. 2 were lagging?

**Prob. 16–5.** A certain house has fourteen 50-watt, 115-volt Mazda lamps and a motor taking 360 watts at 65 per cent power factor. How much current do the main leads carry into the house when all appliances are being used?

**45. Vector Diagram of Voltage.** Just as alternating currents and alternating-current power are represented by vector diagrams, so also we may represent alternating voltage. Thus Fig. 93 is the power vector diagram for 2200 apparent volt-amperes at 85 per cent lagging power factor, the vector $AB$ representing the true power of 1870 watts and

*BC* the reactive power of 1166 vars. If we assume 10 amperes to flow, then the voltage of the apparent power must be $\frac{2200}{10}$, or 220 volts, the voltage of the effective power $\frac{1870}{10}$, or 187 volts, and the voltage of the reactive power $\frac{1166}{10}$, or 116.6 volts. The voltage of the apparent power is called the indicated voltage because it is the voltage which a voltmeter would indicate if put across the circuit. The voltage

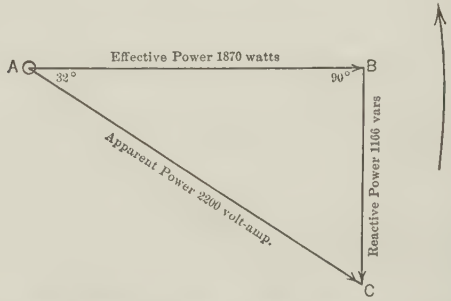



FIG. 93. Vector diagram showing power relations in circuit taking 1 ampere at 2200 volts, or 10 amperes at 220 volts, with power factor 85 per cent.

of the effective power is called the active component of the voltage, and the voltage of the reactive power is called the reactive voltage.*

The active component of voltage, 187 volts, is represented by the vector *AB*, Fig. 94, just as the effective power is represented in Fig. 93 by the vector *AB*. But the reactive component of voltage, 116.6 volts is represented by the vector *BC* in Fig. 94, drawn **upward** at an angle of 90° to the vector *AB* of the effective component, while the reactive power is represented by the vector *BC* drawn **downward** at an angle of 90° to the vector *AB* representing the effective power.

* The reactive voltage is sometimes (incorrectly) called the *wattless* component of the voltage.

The reason why the vector of reactive voltage is drawn upward while the reactive vectors of power and current are drawn downward will be explained later. The fact, however, must be carefully observed.

The active voltage of 187 volts is then represented by the vector $AB$, Fig. 94, lagging 32° behind the indicated voltage $AC$, while in Fig. 93, the active power is represented by a vector $AB$ leading the vector of apparent power $AC$ by 32°.

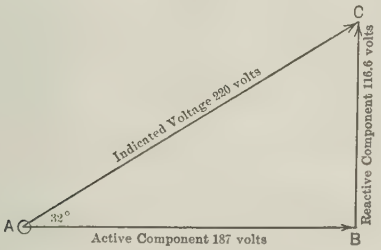



FIG. 94. An "active" voltage of 187 in phase with the current, added vectorially to a "reactive" voltage of 116.6 at 90° with the current, gives a resultant or total voltage of 220, which the voltmeter indicates.

This apparent or indicated voltage of 220 volts with a lagging power factor of 85 per cent, or having an active component lagging 32°, may be likened to a 220-lb pull on a car, at an angle of 32° to the direction in which it is desired to move the car. The active pull in the desired direction would be only 187 lb. This is shown in Fig. 95, in which the vector $AC$ represents the apparent or indicated pull on the car, but at an angle of 32° to the proper direction. The vector $AB$ of 187 lb represents the active component of this pull, since it is in the direction of motion, along the track. The vector $BC$ of 116.6 lb represents the reactive component since it merely pulls the car sideways against the rails and not forward. Thus while there is a total or apparent pull of 220 lb on the car there is only an active pull of 187 lb, because of the angle at which the pull is acting.

Thus the same relations exist among the indicated voltage, the active component of voltage and the reactive component

of voltage as among the apparent power, the effective power and the reactive power. This may be stated as follows:

**The active component of voltage**

**= Indicated voltage × power factor.**

In this case,

$$AB = 220 \times 0.85$$
$$= 187 \text{ volts.}$$

**The reactive component of voltage**

**= Indicated voltage × reactive factor.**

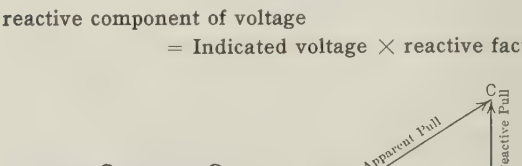



FIG. 95. If a car is pulled with 220 lb at 32° to the track, only 187 lb of this is active in producing motion along the track. The other component, of 116.6 lb, merely forces the car sidewise against the track.

In this case,

$$BC = 220 \times 0.53$$
$$= 116.6 \text{ volts.}$$

**Indicated voltage**

$$= \sqrt{(\text{active component})^2 + (\text{reactive component})^2}.$$

$$AC = \sqrt{187^2 + 116.6^2}$$
$$= 220 \text{ volts.}$$

**Prob. 17–5.** In a 230-volt, single-phase induction motor operating at 82 per cent lagging power factor, find:

(*a*) The power component of voltage.

(*b*) The reactive component of voltage.

(*c*) The angle between the indicated voltage and its power component.

**Prob. 18–5.** If the motor of Prob. 17 takes 14.5 kilowatts from the line, find:

(*a*) The effective power.

(*b*) The reactive power.

(*c*) The angle of lag between the apparent power and the effective power.

**Prob. 19–5.** In the motor of Prob. 18, what is:

(*a*) The indicated current?

(*b*) The power component of current?

(*c*) The reactive component of current?

(*d*) The angle of lag between the indicated current and the power component of current.

**Prob. 20–5.** What is the power component of the current in a 110-volt induction motor taking 1.5 kw and operating at 92 per cent power factor leading?

**Prob. 21–5.** How large is the reactive component of current in the motor of Prob. 20?

**Prob. 22–5.** A house has eighteen 60-watt, 115-volt, Mazda lamps. What is the current in the house mains?

**Prob. 23–5.** What is the power component of current in the mains of Prob. 22?

**Prob. 24–5.** How large is the reactive component of the current in the house mains of Prob. 22?

### 46. Voltage in Parallel and in Series Combinations.

(1) **Parallel Circuit.** If we place a coil and a relay in parallel across a 30-volt line as in Fig. 96, a voltmeter, *V*, placed

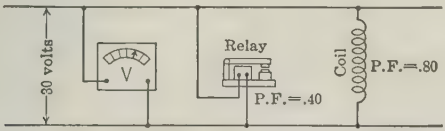



FIG. 96. Voltage across a parallel combination is the same as the voltage across each part of the combination.

as in the diagram will show the voltage across both the coil and the relay, since it measures the voltage between the two points where the coil and the relay are connected. The volt-

age across the relay and the voltage across the coil is the voltage across the line. In other words, the rule for the voltage across a parallel combination is the same for alternating current as it is for direct current.

**The voltage across a parallel combination is the same as the voltage across each part of the combination.**

(2) **Series Circuit.** Suppose that we put the relay and the coil of Fig. 96 in series on a line as in Fig. 97. If the voltmeter $V_1$ across the coil indicates 25 volts and the voltmeter

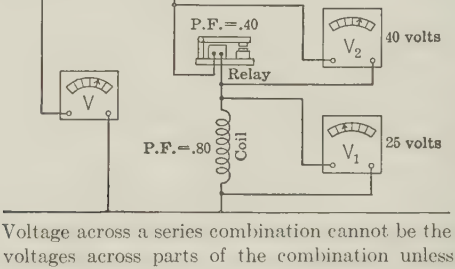



FIG. 97. Voltage across a series combination cannot be the arithmetic sum of voltages across parts of the combination unless all power factors are equal.

$V_2$ across the relay indicates 40 volts, what should we expect the voltmeter $V$ across the series combination of the two to read?

Since the power factors of the two appliances are different, we know that the voltage across the two in series cannot be the arithmetical sum of the voltage across the relay and the voltage across the coil.

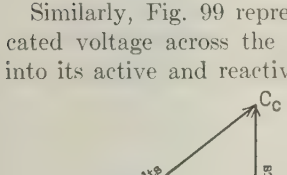
The simplest method of obtaining the voltage across a series combination is to resolve the separate voltages into their active and reactive components, combine them and find their resultant indicated voltage as in Fig. 98, 99 and 100.

The vector $A_C C_C$, Fig. 98, represents the indicated voltage of 25 volts across the coil. As the power factor of the coil is

80 per cent, the angle between the indicated voltage and the active component of voltage $A_CB_C$ must be 37° according to Table I.* The vector $B_CC_C$ then represents the reactive component of the voltage across the coil, and is drawn upward at 90° to $A_CB_C$. By the table, the reactive factor is 60 per cent for a power factor of 80 per cent.

$$A_CB_C = 25 \times 0.80 = 20 \text{ volts},$$
$$B_CC_C = 25 \times 0.60 = 15 \text{ volts}.$$

Similarly, Fig. 99 represents the indicated voltage across the relay, resolved into its active and reactive components.



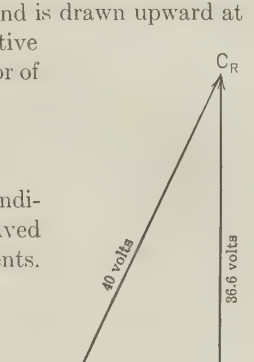




FIG. 98 and FIG. 99. The 25 volts across the coil at 0.80 power factor, in Fig. 97, consists of 20 active volts in phase with current, and 15 reactive volts at 90° to the current. Similarly the 40 volts across the relay with 0.40 power factor consists of 16 volts in phase and 36.6 volts at 90° with the same current.

The angle between the vector $A_RC_R$ of the indicated voltage and the vector $A_RB_R$ of the active component, by Table I, is approximately 66° for a power factor of 40 per cent. The corresponding reactive factor is 0.916

The active component $A_RB_R = 40 \times 0.40 = 16$ volts.
The reactive component $B_RC_R = 40 \times 0.916 = 36.6$ volts.

* It will be noted that the angle 37° does not exactly correspond to a power factor of 80 per cent, but rather to that of 79.9 per cent. Note also that the reactive factor for 37° is 60.2 per cent rather than 60 per cent as used in the above computation. For most problems it is precise

In Fig. 100, $A_L B_L$ represents the active component of the voltage across the series combination of the relay and coil and is equal to the sum of the active component (20 volts) of the voltage across the coil plus the active component (16 volts) of the voltage across the relay.

Active component of voltage across series combination = 20 + 16 = 36 volts.

The vector $B_L C_L$ drawn upward at an angle of 90° to the active component $A_L B_L$ represents the reactive component of the voltage across the series combination and equals the sum of the reactive component (15 volts) of the voltage across the coil plus the reactive component (36.6 volts) of the voltage across the relay.

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enough to work to the nearest degree or half degree for the angles corresponding to various power and reactive factors.

It is not necessary, however, to use Table I for corresponding power factors and reactive factors. It will be noted that if we square the power factor and the reactive factor corresponding to any angle, the sum of the squares always equals 1.00. For instance, the power factor corresponding to 30° is 0.866, and the reactive factor is 0.500.

$$0.866^2 = 0.750$$
$$0.500^2 = 0.250$$
$$\text{Sum of the squares} = 1.000$$

Thus, in any case, we have merely to subtract the square of the power factor from 1 in order to obtain the square of the reactive factor. Accordingly, when in the above example we use the power factor of 80 per cent in order to find the corresponding reactive factor, we merely subtract the square of 80 per cent (or 0.640) from 1.00; that is, 1.00 − 0.640 = 0.360. The square root of 0.360 is 60 per cent, which is the reactive factor used.

The rule is generally stated by the equation:

$$\text{Reactive factor} = \sqrt{1.00 - (\text{power factor})^2}.$$

Note also that:

$$\text{Power factor} = \sqrt{1.00 - (\text{reactive factor})^2}.$$

Reactive component of voltage across the series combination,

$$B_L C_L = 15 + 36.6 = 51.6 \text{ volts.}$$

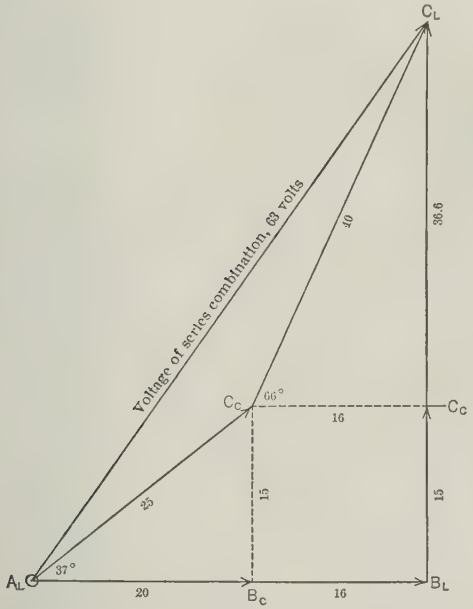



FIG. 100. Total voltage ($V = 63$) across the series combination of Fig. 97 is the vector sum or resultant of total active voltage in relay and coil, in phase with current, and total reactive voltage at 90° to the current. Compare Fig. 98 and 99.

The vector $A_L C_L$ represents the indicated voltage across the series combination.

$$\begin{aligned} A_L C_L &= \sqrt{(A_L B_L)^2 + (B_L C_L)^2} \\ &= \sqrt{36^2 + 51.6^2} \\ &= 63 \text{ volts.} \end{aligned}$$

The voltage is thus 63 volts across a series combination of 25 volts with 80 per cent lagging power factor and 40 volts with 40 per cent lagging power factor. The voltmeter $V$, Fig. 97, would, therefore, indicate 63 volts.

**Prob. 25–5.** What would be the voltage across the series combination of Fig. 97 if the coil had a power factor of 70 per cent, and the relay a power factor of 50 per cent?

**Prob. 26–5.** If the impedance of the coil in Fig. 97 is 20 ohms, how much current flows:

($a$) Through the relay?

($b$) Through the coil?

($c$) Through the combination of coil and relay?

**Prob. 27 5.** What is the indicated voltage across the series combination of a set of Mazda lamps and a dimmer or choke coil? The voltage across the lamps is 62 volts and across the dimmer is 88 volts. The power factor of the dimmer is 25 per cent lagging.

**Prob. 28–5.** What would be the indicated voltage across the combination of Prob. 27 if the dimmer had a power factor of 95 per cent?

**Prob. 29 5.** Two coils of 63 per cent power factor each are joined in series. The voltage across each is 115 volts. What is the voltage across the combination?

**Prob. 30 5.** What would be the voltage across the series combination of Prob. 25–5 if the coil were replaced by a condenser taking the same voltage but with a leading power factor of 15 per cent?

**47. Similarity of Diagrams for Power, Current and Voltage.** It will be noted from the foregoing paragraphs that the vector diagrams for power, current and voltage are exactly similar. All are right triangles, having the apparent (or indicated) values as the hypothenuse, and the active (or effective, or power) components at right angles to the reactive components.

Note particularly that in every case of lagging power factor in the power diagram and the current diagram, the reactive vector is drawn downward at an angle of 90° to the

active or power vector. In the voltage diagram it is drawn upward. When the power factor is leading, the reverse is true.

In all cases,

The effective (or active) component
= apparent (or indicated) value × power factor.
The reactive component
= apparent (or indicated) value × reactive factor.

The apparent (or indicated) value always equals the square root of the sum of the squares of the active and reactive components.

**48. Do not Resolve Both the Voltage and Current Vectors into Active and Reactive Components.** While it is possible to resolve either the current or the voltage into their active and reactive components, no advantage is gained in resolving both the voltage and current of a single problem into their components, and it usually results in confusion to do so. Thus, if an appliance of 4 ohms impedance and 80 per cent lagging power factor is placed across a 20-volt circuit, we know that an indicated current of $\frac{20}{4}$, or 5 amperes will flow.

In order to find the indicated current, we divide the indicated voltage by the impedance. Or, in order to find the indicated voltage, we may multiply the indicated current by the impedance, $5 \times 4$, or 20 volts.

To find the power, we have our choice of three methods:

(1) We may draw a power diagram as in Fig. 101. The vector $A_PC_P$ represents the apparent power, $20 \times 5$, or 100 volt-amperes. The vector $A_PB_P$ represents the effective power. The vector $A_PC_P$ is drawn lagging at an angle of 37° behind $A_PB_P$ because the angle 37° corresponds to a power factor of 80 per cent.

The value of the vector $A_PB_P$ of effective power may be found from the equation:

Effective power = apparent power × power factor
= (20 × 5) × 0.80
= 80 watts.

(2) Or we may resolve the current of 5 amperes into its active and reactive components as in Fig. 102. The vector $A_C C_C$ represents the indicated current of 5 amperes. The vector $A_C B_C$ represents the active component of the current, $5 \times 0.80$, or 4 amperes. $A_C C_C$ is drawn lagging an angle of 37° behind $A_C B_C$ because this angle corresponds to the power factor of 80 per cent.

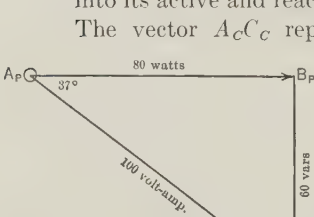



FIG. 101. Power diagram for 5 amperes at 20 volts, with 80 per cent power factor, showing relations between active, reactive, and total apparent power.

The effective power always equals the product of the active component of current times the indicated voltage. Thus,

Effective power = active component of current × indicated volts
= (5 × 0.80) × 20
= 80 watts.

(3) Or we may resolve the indicated voltage of 20 volts into its active and reactive components as in Fig. 103. The vector $A_V C_V$ represents the indicated voltage of 20 volts. The vector $A_V B_V$ represents the active component of voltage, 20 × 0.80, or 16 volts, in phase with the current, and is drawn lagging by an angle of 37° behind $A_V C_V$ because the angle of 37° corresponds with this power factor. The current and the active component of voltage $A_V B_V$ lag 90° behind

$B_VC_V$, which is consumed in overcoming the back voltage produced by inductance.

The effective power always equals the product of the active component of voltage times the indicated current. Thus,

$$\begin{aligned}\text{Effective power} &= \text{active component of voltage} \times \text{indicated current} \\ &= (20 \times 0.80) \times 5 \\ &= 80 \text{ watts.}\end{aligned}$$

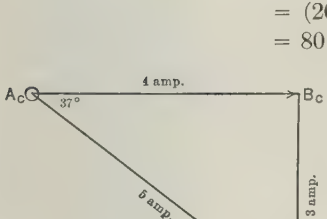



FIG. 102. Current diagram corresponding to Fig. 101.

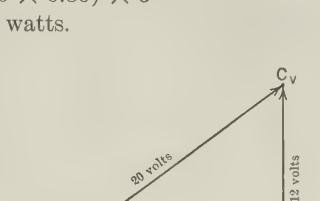



FIG. 103. Voltage diagram corresponding to Fig. 101.

(1) Note that we may multiply the indicated volts by the indicated amperes and obtain the apparent power. This multiplied by the power factor gives us the effective power. Or,

(2) We may multiply the indicated current by the power factor and obtain the active component of current. This multiplied by the indicated voltage gives us the effective power. Or,

(3) We may multiply the indicated voltage by the power factor and obtain the active component of voltage. This multiplied by the indicated current gives us the effective power.

Thus we either multiply the active component of current by the indicated voltage, or the active component of voltage

by the indicated current. **We never use the active component of current and the active component of voltage in the same equation to obtain effective power.**

**49. Real Meaning of Lead and Lag. Phase.** The reason why we do not use the active component of current and the active component of voltage in the same equation may be explained as follows.

The voltage (20 volts) of the preceding example is being used to force a current of 5 amperes through an appliance. But the only power consumed by the appliance is the power consumed by the active component of the 20 volts in forcing the current through the appliance. The remainder of the power is returned to the line in the same way that a flywheel, using up only the power necessary to overcome the resistance to motion, returns the rest of the power to the engine to carry it over the dead centers.

Therefore, although the 20-volt pressure was forcing 5 amperes through the appliance, the active component of voltage consisted of only $20 \times 0.80$, or 16 volts. Thus only $16 \times 5$, or 80 watts of power were being used. The reactive power represented by the product of the reactive component of voltage (12 volts) times the current, or $12 \times 5 = 60$ vars, was returned to the line.

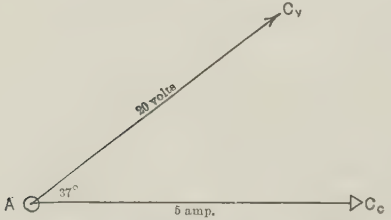



FIG. 104. Five amperes current at 20 volts pressure at 80 per cent power factor, the current lagging behind the voltage.

For this reason it is customary to represent the current and voltage conditions by a current diagram like Fig. 104, when a current of 5 amperes is forced through a circuit by a pressure of 20 volts with a lagging power factor of 80 per cent. Since

the angle corresponding to 80 per cent is 37°, we draw the current vector $AC_C$ representing 5 amperes lagging 37° behind the voltage vector $AC_V$ representing 20 volts. It is not even necessary to draw them both to the same scale.

We then resolve the voltage into its two components as in Fig. 105, $AB_V$ representing the 16 volts active component of voltage and $B_VC_V$ representing the reactive component of voltage.

Since the vector representing active component of voltage, $AB_V$, lies along the same line as the indicated current vector $AC_C$, we say that the active component of voltage is **in phase**

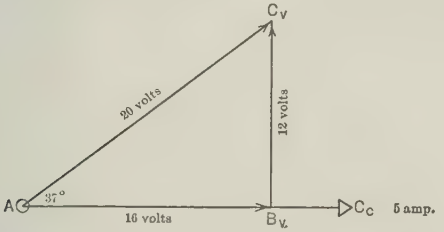



FIG. 105. Voltage resolved into its active and reactive components. Compare Fig. 104.

with the (indicated) current, and the power consumed is only that power represented by the product of the current and as much of the voltage as is in phase with the current.

Thus the effective power in this case equals the product of the indicated current (5 amperes) times that component of the voltage which is in phase with the current (16 volts), that is, $16 \times 5$, or 80 watts.

The reactive component of voltage is represented by a vector $B_VC_V$ drawn up at right angles to both the power component of voltage and the indicated current $AC_C$. The reactive volt-amperes then equals the product of the indicated current times this reactive component of voltage. In this case the reactive power equals the product of the indicated

current (5 amperes) times the reactive component of voltage (12 volts), that is, $5 \times 12$, or 60 vars.

This phase relation of the current to the voltage when the power factor is 80 per cent lagging can also be represented as in Fig. 106. Note that the voltage starts at zero, grows to a maximum, dies out to zero again, grows to a maximum in the opposite direction and dies to zero again. The current curve goes through an exactly similar cycle. But note that

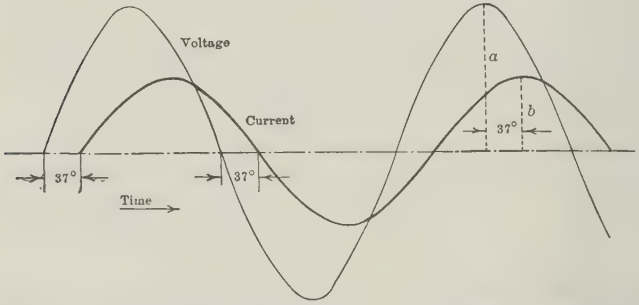



FIG. 106. Current reaches its maximum value (*b*) 37° (or $\frac{37}{360}$ of the time required for one complete cycle) after the voltage has passed its maximum value (*a*) in the same direction through the circuit. This corresponds to a power factor of 80 per cent lagging.

at all times the values of the current **are 37° behind the corresponding values of the voltage** because we have seen from Table I that the current lags 37° behind the voltage when the power factor is 80 per cent lagging. Thus the current curve does not start up from zero until the voltage curve has gone through 37° of its cycle. The current does not become zero again until 37° after the voltage has become zero. Similarly, the current does not reach its maximum values until 37° after the voltage has reached its maximum in the same direction as shown at (*a*) and (*b*). This method of representing a current lagging behind a voltage gives a little clearer mental picture of what lag means but is not so

useful for obtaining derived numerical values, as in the computation of power.

A leading current is represented in the same way except that the current curve is drawn so that it reaches its values ahead of, instead of behind, the corresponding values of the voltage.

**Prob. 31–5.** (*a*) Represent by diagrams similar to Fig. 105 and 106 the current and voltage relations in an appliance through which 230 volts forces a current of 14 amperes at 84 per cent lagging power factor.

(*b*) What is the apparent power?

(*c*) What is the effective power?

**Prob. 32–5.** Repeat Prob. 31–5 with a leading power factor of 84 per cent.

**Prob. 33–5.** In a certain reactive dimmer the current of 15 amperes lags practically 90° behind the pressure of 72 volts.

(*a*) Draw diagrams of these conditions, similar to Fig. 105 and 106. State the power factor and the reactive factor.

(*b*) Compute the apparent power.

(*c*) Compute the effective power.

(*d*) Compute the reactive power.

**Prob. 34–5.** (*a*) Represent the conditions in a Mazda lamp circuit taking 5.5 amperes at 115 volts, by diagrams similar to Fig. 105 and 106.

(*b*) Compute the apparent power.

(*c*) Compute the effective power.

(*d*) Compute the reactive power.

**Prob. 35–5.** An induction motor takes 16 kw at 230 volts and 70 per cent lagging power factor. Represent the conditions by diagrams similar to Fig. 105 and 106 and compute:

(*a*) Apparent power.

(*b*) Indicated current.

(*c*) Reactive power.

**Prob. 36–5.** Repeat Prob. 35–5, using a leading power factor of 92 per cent.

**50. Relation of the Induced Back Voltage to the Impressed Voltage.** It will be remembered that the voltage applied to the primary coils of a transformer may be divided

into two parts as stated in paragraph 27, one part being needed to overcome the induced back voltage, the other part being used to overcome the resistance of the primary coils. The voltage necessary to overcome the resistance is composed entirely of active voltage. The induced back voltage in an unloaded transformer is entirely reactive voltage. Thus we see that when we send an exciting current through the primary coils, that part of the impressed voltage needed to overcome the induced back voltage of the coil leads 90° that part of the voltage needed to overcome the resistance of the coils.

FIG. 107. With low power factor, practically all of the voltage is reactive, and current lags nearly 90° behind indicated voltage.

The problem in paragraph 27 was to find the value of the induced back pressure when 115 volts were applied to a 0.55-ohm coil, and forced 0.15 ampere through it.

The voltage necessary to force 0.15 ampere against 0.55 ohm resistance is only $0.15 \times 0.55$, or 0.08 volt. This is represented in Fig. 107 by the vector $AB$, which is drawn much longer than it really should be because it is impossible to represent so small a quantity on the same scale as the vector $AC$ of 115 volts. The vector $AC$ represents the indicated 115 volts, which is made up of the power component $AB$ and the reactive component $BC$. The vector $AB$ must lag behind the vector $AC$ by an angle corresponding to the power factor, $\frac{0.08}{115}$, or 0.000696. From Table I this angle is seen to be almost 90°, as 89° corresponds to a power factor of 0.017 which is still over 20 times as large a power factor as 0.000696.

The vector $BC$, representing the reactive component, or

the voltage used in overcoming the induced back voltage, must, therefore, be practically equal to the vector $AC$ or 115 volts. Even in Fig. 107, this is apparent, but if vector $AB$ could be drawn small enough in comparison with $AC$, the practical equality of $AC$ and $BC$ would be still more apparent.

Thus in paragraph 27 when we made the assumption in the case of a coil possessing only 0.55 ohm resistance, that the induced back voltage was practically equal to the applied or indicated voltage of 115 volts, we were entirely justified.

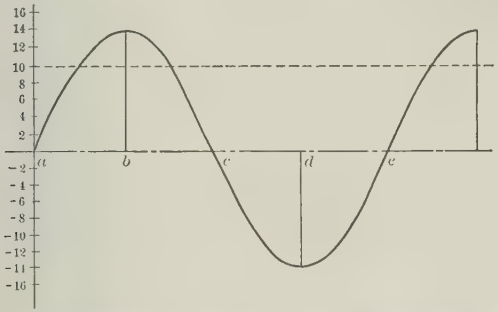



FIG. 108. An alternating current (or voltage) continually changes in value and in direction as time progresses. The particular manner of change is represented by the "wave form" of the current (or voltage). For the usual mode of variation, shown here, the meter indicates a value of 0.707 times the maximum value attained during each cycle.

**51. Relation between the Effective and the Maximum Values of Current and Voltage.** When we wish to describe the value of a given alternating current, we have seen that we state it in terms of amperes; thus, we speak of an alternating current of 10 amperes, or of 50 amperes. But an alternating current is continually changing in value, as is shown in Fig. 108. For instance, in Fig. 108, at given instants, marked $a$, $c$, $e$, it has a value of zero, while at other instants marked $b$ and $d$ it has a value of 14.1 amperes. But if we were asked what was the value of the alternating cur-

rent represented by Fig. 108 we would say neither zero nor 14.1 amperes, but 10 amperes. The 10 amperes is called the **effective** value of the alternating current and is the value which an a-c ammeter would indicate. It is the value which a direct current would have if it produced the same amount of heat in the same time in the same circuit. It is slightly greater than the average of all the values which the current goes through in a cycle. Thus, if we had a steady current of 10 amperes going through a circuit it would produce the same heating effect as an alternating current of 10 amperes in the same circuit, although the alternating current would at some instants be less than 10 amperes and at other instants greater. However, it would be said to have an effective value of 10 amperes.

The greatest value which a standard alternating current will have at any instant is called its maximum value and is always equal to 1.41 times the effective value. This may be written in the form of an equation:

**Maximum value of current = 1.41 × effective value of current.**

Thus, if an alternating current is stated as being of 10 amperes, it will twice during each cycle reach a value of 1.41 × 10, or 14.1 amperes. These instants are represented by the points *b* and *d* in Fig. 108. There are also two points in every cycle, represented by *a* and *c* in Fig. 108, when this 10-ampere current has a value of zero. At all other instants the current has a value somewhere between 0 and 14.1 amperes. If the number of amperes is not expressly stated to be the maximum value or an instantaneous value, it is always assumed to be the effective value.

The same relation exists between the effective voltage and the maximum voltage. When we speak of an alternating voltage of 110 volts, we always mean an effective voltage of 110 volts. But there are two instants in each cycle when the voltage has a maximum value of 1.41 × 110, or 155 volts.

This is one of the reasons why an alternating voltage of 110 volts seems to produce so much more of a shock than a direct voltage of 110 volts. A person getting across a 110-volt alternating-current circuit of 60 cycles is subjected to the maximum voltage of 155 volts 120 times during each second he remains in contact with the circuit. The fact that at 120 instants during the same second he has no voltage across his body only makes the effect at the instants of 155 volts seem the more violent.

## SUMMARY OF CHAPTER V

The INDICATED CURRENT, which would be measured by an ammeter in series with a circuit, is conveniently considered as being composed of two parts: an ACTIVE COMPONENT, or power component, which delivers the power that is consumed in the circuit, and a REACTIVE COMPONENT (sometimes incorrectly called "wattless" component) which delivers the power that is merely stored in the magnetic field around the circuit to be returned later to the generator.

Active component = indicated current × power factor.

Reactive component = indicated current × reactive factor.

These currents are related to one another as the sides of a right triangle, the length of one side being proportional to the active component, of the other side to the reactive component, and of the hypothenuse to the actual resultant or indicated current. It follows, then, from the geometrical properties of a right triangle, that

$$\text{Indicated amperes} = \sqrt{(\text{active amperes})^2 + (\text{reactive amperes})^2}.$$

The reactive component of the (indicated) current may either lag or lead by 90° (or one-quarter cycle) with respect to the active component, depending upon the nature of the circuit — its composition, arrangement, and surroundings; in an inductive circuit the reactive component lags 90°, and in a condensive (anti-inductive) circuit the reactive component leads 90°, just as we assumed the active power to lag behind the apparent power in an inductive circuit, and to lead it in a condensive circuit.

Similarly the INDICATED VOLTAGE between any two points in a circuit, which would be measured by a voltmeter connected to those points, is conveniently considered as being composed of two parts, an active component (power component) and a reactive component ("wattless" component). The reactive component is equal to the induced back voltage. The relations between indicated voltage and its components, and the power factor and reactive factor, are exactly similar to the relations already stated to exist in the case of currents. The effect of a given indicated current is exactly the same as if its active and reactive components were forced through the same circuit at the same time, but from separate generators or sources of power; and the effect of a given indicated voltage is exactly the same as if its active and reactive components were added together in the same circuit but originated in separate generators or sources of power.

Study of the relations between indicated or apparent values of current, pressure and power, and their respective active and reactive components, discloses the following relations:

Watts = apparent volt-amperes × power factor
= indicated volts × active component of amperes
= indicated amperes × active component of volts.

Vars = apparent volt-amperes × reactive factor
= indicated volts × reactive component of amperes
= indicated amperes × reactive component of volts.

Reactive factor may be found directly from the power factor, or vice versa, by a simple calculation without the use of a table of factors. Thus,

$$\text{Reactive factor} = \sqrt{1.00 - (\text{power factor})^2}.$$

A table is necessary only to find the corresponding angle, but most of the usual practical calculations can be made by means of these factors, without knowing the angles. The reason for the relation stated above will be apparent from careful study of current and power relations previously stated.

In SERIES CIRCUITS, every part carries the same amperes, but the voltages across the various parts must be added together by a vector diagram to find the total voltage across the whole circuit. Multiply the indicated voltage across each part by the power factor of that part to obtain the active component

of voltage in that part; similarly, multiply the indicated voltage by the reactive factor to find the reactive component of voltage in that part. Add together all active components to find active component of the total voltage; add together (algebraically) all reactive components to find reactive component of the total voltage. These components of the total voltage are at right angles to each other; therefore,

$$\text{Indicated total voltage} = \sqrt{(\text{sum of active components})^2 + (\text{sum of reactive components})^2}.$$

$$\text{Power factor of whole circuit} = \frac{\text{sum of active components}}{\text{indicated total voltage}}.$$

In PARALLEL CIRCUITS, each of the parallel parts receives the same voltage, but the currents in these parts must be added together by a vector diagram to find the total or indicated current in the mains. Multiply the indicated value of current in each path by the power factor of that path, and add together the active components of current so obtained to find the active component of the total current. Multiply the indicated value of current in each path by the reactive factor of that path, and add together the reactive components so obtained to find the reactive component of the total current. Then,

$$\text{Indicated total current} = \sqrt{(\text{sum of active components})^2 + (\text{sum of reactive components})^2}.$$

$$\text{Power factor of whole circuit} = \frac{\text{sum of active components}}{\text{indicated total current}}.$$

In making these summations, an active component of current or voltage or power is considered as positive when it is generated in or supplied to the circuit, and as negative when it is consumed by or dissipated in the circuit. Similarly, a reactive component is considered as positive when it is leading, and as negative when it is lagging. It is the oppositeness of these signs which is of greatest significance.

When two alternating currents, or two voltages, or a current and a voltage, reach their maximum values in the same direction at the same instant, and their zero values at the same instant, then these two quantities are said to be IN PHASE with each

other. Thus, a circuit has 100 per cent power factor when the voltage and the current are in phase with each other. When quantities are in phase with each other, the vectors representing them are in line with each other, and pointing in the same direction.

An alternating current or voltage passes regularly through various INSTANTANEOUS VALUES between zero and the MAXIMUM VALUE. When the value is stated in amperes or in volts without qualification, we mean the EFFECTIVE VALUE which would be indicated by a correct ammeter or voltmeter. The effective value of any alternating current is the value which a direct current or unvarying current would have if it produced the same amount of heat in the same time in the same circuit. When a current or voltage alternates (at any frequency) in the manner which has been adopted as standard, we have the relation:

Maximum instantaneous value = 1.41 × effective value.

## PROBLEMS ON CHAPTER V

**Prob. 37 5.** (*a*) What total effective power is being delivered to the two motors in Fig. 86? Line volts = 230.

(*b*) What total apparent power?

(*c*) What is the power factor of the total power delivered to the motors, and how does it compare with the power factor of the line current as found in Prob. 8–5?

**Prob. 38 5.** If motor *M* of Fig. 86 takes 35 kw at 220 volts with a lagging power factor and motor *S* takes 50 kw with a leading power factor, what total effective power do they both take? Power factors are as indicated in Fig. 86.

**Prob. 39 5.** What total apparent power do the motors of Prob. 38 take from the line?

**Prob. 40 5.** What current flows in the line in Prob. 38?

**Prob. 41 5.** If motor *M* in Fig. 86 takes 22 kw at 440 volts with lagging power factor of 76 per cent, at what leading power factor must motor *S* operate in order to draw 18 kw from the line and produce unity power factor for the combination of the two motors?

**Prob. 42 5.** What is the impedance of the coil in Fig. 96 if it draws 0.8 ampere from the line?

**Prob. 43 5.** The impedance of the relay in Fig. 96 is 60 ohms. How much current does the combination of the coil of Prob. 42 and the relay draw from the line, the power factors being as indicated in Fig. 96, and both lagging?

**Prob. 44–5.** The transformer $T$ in Fig. 85 has an impedance of 26 ohms, the lamp 40 ohms, $R = 36$ ohms, and $X = 20$ ohms. The power factor of the transformer is 92 per cent, of $R$, unity, and of $X$, 25 per cent.

(*a*) What is the voltage across $L$?
(*b*) What is the voltage across $T$?
(*c*) What is the voltage across $R$?
(*d*) What is the voltage across $X$?

**Prob. 45 5.** What is the total voltage across the series combination of Prob. 44?

**Prob. 46 5.** What apparent power is consumed by each of the appliances in Prob. 44?

**Prob. 47 5.** What is the total effective power consumed by the series combination of Prob. 44?

**Prob. 48 5.** What is the power factor of the series combination of Prob. 44–5?

**Prob. 49 5.** Two inductive coils are connected in series; coil $A$ has a power factor of 50 per cent, and coil $B$ a power factor angle of 30° (lagging). With 6 amperes flowing in the series circuit, the voltage across coil $A$ is 82 volts, and across coil $B$ 64 volts. What are the impedances of the coils?

**Prob. 50 5.** What is the power taken by each coil of Prob. 49?

**Prob. 51 5.** What reactive power is taken by each coil in Prob. 49?

**Prob. 52 5.** What power is taken by the series circuit in Prob. 49?

**Prob. 53 5.** What is the voltage across the series circuit in Prob. 49?

**Prob. 54 5.** At what line voltage will the power supplied to coil $B$ in Prob. 49 equal 50 watts.

**Prob. 55 5.** What power will coil $A$ take in Prob. 54?

**Prob. 56 5.** What current will flow in the series circuit of Prob. 54?

**Prob. 57–5.** What voltage will there be across each coil in Prob. 54?

**Prob. 58–5.** If the two coils of Prob. 49 are connected in parallel across a 115-volt line, what power will each coil take?

**Prob. 59–5.** What current will each coil take in Prob. 58?

**Prob. 60–5.** What is the line current in Prob. 58?

**Prob. 61–5.** What is the total reactive power taken by the two coils in Prob. 58?

**Prob. 62–5.** At what line voltage will the parallel coils of Prob. 58 take the same total power as the series circuit of Prob. 49?

**Prob. 63–5.** What is the power factor of the parallel combination in Prob. 58?

**Prob. 64–5.** A 10-ohm resistor is connected in series with the series coils of Prob. 49 and the combination is connected across a 100-volt line. What power will be supplied to this circuit?

**Prob. 65–5.** What current will flow in Prob. 64?

**Prob. 66–5.** What power is supplied to each part of the circuit of Prob. 64?

**Prob. 67–5.** What is the power factor of the series combination of Prob. 64?

**Prob. 68–5.** Two voltages are impressed upon a circuit, in series. One voltage is 170 volts and lags 42° behind the other, which equals 214 volts. What is the voltage across the circuit?

**Prob. 69–5.** Two alternating currents are flowing in parallel branches of a circuit. The first equals 65 amperes, the second equals 40 amperes and lags 38° behind the first. (*a*) What is the resultant of the two currents?

(*b*) What is the phase relation between the resultant current and the first current?

**Prob. 70–5.** If the voltage across the parallel circuits in Prob. 69 is 115 volts and is in phase with the resultant current, find:

(*a*) Power in branch carrying the 65 amperes current.

(*b*) Power in branch carrying the 40 amperes current.

(*c*) Total power in parallel circuit.

**Prob. 71–5.** How many volts are necessary to force 30 amperes alternating current through 12 ohms resistance?

**Prob. 72–5.** (*a*) How many watts are consumed in the resistance of Prob. 71?

(*b*) How much direct current would be necessary to cause the same heating effect as this alternating current, in the same circuit?

**Prob. 73–5.** If a coil of 12 ohms inductive reactance and of negligible resistance is used instead of the resistance of Prob. 71:

(*a*) How many volts are necessary to force 30 amperes through it?

(*b*) How many watts are consumed by the coil?

**Prob. 74–5.** A generator is to deliver 60 amperes at 115 volts to supply power to incandescent lamps, which are non-inductive. If the line wires have 0.4 ohm resistance and 0.2 ohms reactance, what must the brush voltage of the generator be?

**Prob. 75–5.** What is the voltage drop in the line impedance of Prob. 74–5?

**Prob. 76–5.** A synchronous motor takes a leading current of 52 amperes when the fields are over-excited. An induction motor takes a lagging current of 90 amperes. Power factor of synchronous motor is 0.85; of induction motor 0.75. If the two motors are operated in parallel on a 230-volt line, what current does the generator supply?

**Prob. 77–5.** What is the power factor of the load on the generator in Prob. 76?

**Prob. 78–5.** Prove, by aid of a vector diagram, that if each of a number of loads connected in parallel has the same power factor, the power factor of the total load in the mains has the same value, and the total current in the mains is the arithmetical sum of all the load currents.

**Prob. 79–5.** Extend the proof of Prob. 78–5 for the voltages in series circuits.

**Prob. 80–5.** An alternating-current generator which was delivering 400 kv-a at 70 per cent power factor lagging to a load of induction motors, had its power factor raised (while still delivering 400 kv-a) to 95 per cent by adding an over-excited synchronous motor to the line. Calculate:

(*a*) Apparent power (volt-amperes) taken by the synchronous motor.

(*b*) Power (watts) taken by synchronous motor.

(*c*) Power factor of synchronous motor.

**Prob. 81–5.** An incandescent lamp in series with a choke coil carries alternating current. The voltage across the lamp is 115 volts, across the coil 115 volts and across the two in series it is 180 volts.

(*a*) What is the power factor of the coil?

(*b*) What is the power factor of the lamp and coil together?

**Prob. 82–5.** If the lamp in Prob. 81 is consuming 60 watts, how many watts is the coil consuming?

**Prob. 83–5.** An alternating-current generator supplies three feeders, one of which takes 100 kw at 0.80 power factor, another 250 kw at 0.70 power factor and the third 175 kw at 0.95 power factor, all lagging. What load, in kw and in kv-a, is the generator delivering, and at what power factor?

**Prob. 84–5.** By what percentage would the current in the generator of Prob. 83 be reduced if the circuit breaker on the first feeder were opened, the voltage of the generator being maintained constant meanwhile by an automatic voltage regulator?

**Prob. 85–5.** A soldering iron built for 110 volts and 2.4 amperes is to be used on a 220-volt circuit of the same frequency. What must be the impedance of a choke coil to be connected in series with the soldering iron so as to prevent the current exceeding 2.4 amperes, if the winding of the soldering iron is non-inductive and the choke coil has a power factor of 0.35?

**Prob. 86–5.** What per cent of the total power supplied in Prob. 85 is lost in the choke coil?

**Prob. 87–5.** What is the power factor of the series circuit in Prob. 85?

**Prob. 88–5.** What is the maximum voltage across a 230-volt alternating-current line?

**Prob. 89–5.** If the maximum voltage across a line is 1620 volts, what is the effective voltage?

**Prob. 90–5.** What is the maximum voltage on a 2300-volt alternating-current line?

**Prob. 91–5.** A man gets across a 25-cycle 115-volt alternating-current line. What maximum voltage is he subjected to and how many times a second is he subjected to this maximum value?

## CHAPTER VI

## RELATION BETWEEN IMPEDANCE, RESISTANCE AND REACTANCE

Impedance diagrams may be drawn to show the relation between impedance, resistance and reactance, just as voltage diagrams are drawn to show the relation of the indicated voltage to its active and reactive components. Such diagrams are often very convenient.

**52. Impedance Diagrams.** Vector $AC$, Fig. 109a, represents the indicated voltage across an appliance. Vector $AB$ represents the active component of voltage, and $BC$, the reactive component of voltage, where the power factor is 80 per cent lagging, corresponding to a lag of 37°. If we assume the current through the appliance to be 10 amperes, then,

$$\text{The impedance} = \frac{\text{indicated voltage}}{\text{current}}$$
$$= \tfrac{5}{1}\tfrac{0}{0}$$
$$= 5 \text{ ohms.}$$

$$\text{The resistance} = \frac{\text{active voltage component}}{\text{current}}$$
$$= \tfrac{4}{1}\tfrac{0}{0}$$
$$= 4 \text{ ohms.}$$

$$\text{The reactance} = \frac{\text{reactive voltage component}}{\text{current}}$$
$$= \tfrac{3}{1}\tfrac{0}{0}$$
$$= 3 \text{ ohms.}$$

Note that just as

Indicated voltage

$$= \sqrt{(\text{active component})^2 + (\text{reactive component})^2}$$

$$= \sqrt{40^2 + 30^2}$$

$$= 50 \text{ volts},$$

so, also,

$$\text{Impedance} = \sqrt{\text{resistance}^2 + \text{reactance}^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= 5 \text{ ohms}.$$

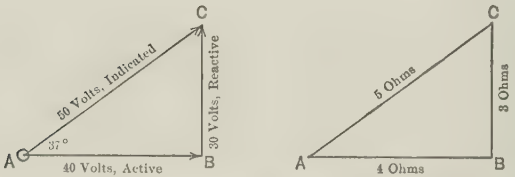



FIG. 109a and FIG. 109b. Impedance, resistance, reactance bear to each other exactly the same numerical relations as the corresponding sides of the voltage triangle $ABC$. If each side of the impedance triangle be multiplied by the number of amperes, we have the voltage triangle.

Thus, the 5 ohms impedance may be represented by the hypothenuse of a right triangle, the resistance (4 ohms) and the reactance (3 ohms) being represented by the other two sides, as in Fig. 109b. This diagram is merely the voltage diagram of Fig. 109a, drawn to a different scale. Each side of Fig. 109b represents the corresponding side of Fig. 109a divided by the number of amperes, 10.

Note also that just as the angle between the lines representing the power component of voltage and the indicated voltage is 37°, so the angle between the lines representing the resistance and the impedance is also 37°.

Similarly, just as the

$$\text{Power factor} = \frac{\text{Active component of voltage}}{\text{Indicated voltage}}$$
$$= \tfrac{4}{5}\tfrac{0}{0}$$
$$= 80 \text{ per cent},$$

so the

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}}$$
$$= \tfrac{4}{5}$$
$$= 80 \text{ per cent.}$$

And as

$$\text{Reactive factor} = \frac{\text{Reactive component of voltage}}{\text{Indicated voltage}}$$
$$= \tfrac{3}{5}\tfrac{0}{0}$$
$$= 60 \text{ per cent},$$

so the

$$\text{Reactive factor} = \frac{\text{Reactance}}{\text{Impedance}}$$
$$= 60 \text{ per cent.}$$

Therefore, if the impedance and the resistance of a piece, or the impedance and the reactance are known, the power factor may be found directly by dividing the resistance by the impedance; or the reactive factor may be found by dividing the reactance by the impedance and the corresponding power factor determined by reference to Table I, or by other means already explained.

From the above, it can be seen that the same relations exist between impedance, resistance and reactance as between indicated voltage, active component of voltage and reactive component of voltage. Thus,

Active component of voltage
$$= \text{indicated voltage} \times \text{power factor}$$

and $\textbf{Resistance} = \textbf{Impedance} \times \textbf{power factor.}$

Reactive component of voltage

= indicated voltage × reactive factor

and **Reactance = Impedance × reactive factor.**

Construct impedance diagrams for the following, and solve:

**Prob. 1–6.** The resistance of a transmission line, including return wire, is 1.2 ohms. The impedance is 12.9 ohms. What is the power factor of the line alone?

**Prob. 2–6.** What is the reactance of the line in Prob. 1?

**Prob. 3–6.** How much voltage is required to force 8 amperes through the line of Prob. 1?

**Prob. 4–6.** What power is lost in the line of Prob. 3?

**Prob. 5–6.** The 60-cycle impedance of a certain device is 23 ohms, and the power factor 82 per cent. What is the resistance of the appliance?

**Prob. 6–6.** What is the reactance of the device of Prob. 5?

**Prob. 7–6.** How much does the current lag behind the voltage in a reactive dimmer, the resistance of which is 2 ohms and the impedance 15 ohms?

**Prob. 8–6.** Find the power factor and the reactive factor of the dimmer in Prob. 7.

**Prob. 9–6.** How much power will the dimmer of Prob. 7 consume when the voltage across it is 115 volts?

**53. Impedance of Series Combinations.** Finding the impedance of a series combination is quite similar to finding the voltage across a series combination. The impedance of each appliance is resolved into its resistance and reactance components by multiplying the impedance of the appliance by its power factor and its reactive factor, respectively. The sum of the resistance components is found and the sum of the reactance components. If these sums are now made the two legs of a right triangle the hypothenuse will represent the

impedance of the combination: or,

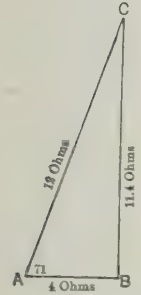
Impedance of series combination =

$$\sqrt{(\text{sum of resist. components})^2 + (\text{sum of react. components})^2}.$$

Notice also that

Power factor of series combination =

$$\frac{\text{sum of resistance components}}{\text{impedance of series combination}}.$$




**Example 1.** A certain appliance No. 1, having 12 ohms impedance and 4 ohms resistance, is connected in series with an appliance No. 2 of 10 ohms impedance with 90 per cent power factor. What is the impedance of the series combination?

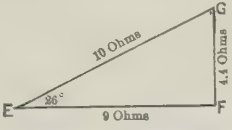



FIG. 110 and FIG. 111. Impedance triangles for the two loads shown connected in parallel in Fig. 113.

**Solution.** The power factor of the first appliance equals $\frac{4}{12} = 33$ per cent.

From Table I the angle of 71° corresponds to a power factor of 33 per cent and a reactive factor of 95 per cent.

Construct Fig. 110, letting $AC$ represent the impedance of 12 ohms and $AB$ the resistance of 4 ohms. The angle of lag or the angle between $AB$ and $AC$ is 71°, corresponding to the power factor $\frac{4}{12}$ or 33 per cent. The line $BC$ now represents the reactance and equals $0.95 \times 12$, or 11.4 ohms.

The resistance of the second appliance equals $0.90 \times 10$, or 9 ohms. By Table I the reactive factor for a power factor of 90 per cent is 44 per cent and the corresponding angle of lag is 26°.

The reactance equals $0.44 \times 10 = 4.4$ ohms.

Construct Fig. 111, in which $EG$ represents the impedance of 10

ohms, $EF$ the resistance of 9 ohms and $FG$ the reactance of 4.4 ohms. The angle between $EF$ and $EG$ is the angle of lag, 26°.

Now construct Fig. 112 making

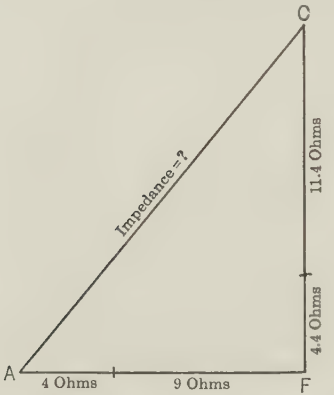



Fig. 112. For a series combination of the loads represented separately in Fig. 110 and 111, total resistance equals arithmetical sum of resistances and total reactance equals arithmetical sum of reactances (if both are inductive).

$$AF = AB + EF$$
$$= 4 + 9$$
$$= 13 \text{ ohms},$$

and

$$FC = FG + BC$$
$$= 4.4 + 11.4$$
$$= 15.8 \text{ ohms}.$$

$AF$ represents the resistance of the series combination and $FC$ the reactance of the series combination.

$AC$ represents the impedance of the series combination.

$$AC = \sqrt{AF^2 + FC^2}$$
$$= \sqrt{13^2 + 15.8^2}$$
$$= 20.5 \text{ ohms}.$$

Power factor of series combination

$$= \frac{AF}{AC} = \frac{13}{20.5} = 0.634$$
$$= 63.4 \text{ per cent.}$$

**Prob. 10–6.** What will be the impedance of a series combination of the dimmer of Prob. 7 and a bank of lamps having 13 ohms resistance?

**Prob. 11–6.** What is the power factor of the combination in Prob. 10?

**Prob. 12–6.** The power factor of appliance $A$ is 80 per cent and the reactance is 25 ohms. The resistance of appliance $B$ is 24 ohms and the impedance is 40 ohms. What is the impedance of a series combination of $A$ and $B$? 80.82

**Prob. 13–6.** What is the power factor of the combination of Prob. 12?

**Prob. 14–6.** How much voltage is required to force 3.2 amperes through the combination of Prob. 12?

**Prob. 15–6.** What is the value of reactive power in Prob. 14?

**Prob. 16–6.** An impedance of 150 ohms at 88 per cent lagging power factor is joined in series with an impedance of 90 ohms at 60 per cent lagging power factor. What is the impedance and power factor of the series combination?

**54. Impedance of Parallel Combinations.** The impedance of a parallel combination is found as follows:

(*a*) Find the current through each path of the combination. (If no voltage is given, assume any convenient voltage.)

(*b*) Compute the current through the combination by the method explained in Paragraph 44.

(*c*) Divide the voltage across the combination by the current through the combination.

**Example 2.** What would be the impedance of a parallel combination of the appliances in Example 1?

**Solution.** Assume for convenience a voltage of 60 volts across the parallel combination as in Fig. 113.

(*a*) Current through I $= \dfrac{\text{Voltage across I}}{\text{Impedance of I}}$

$= \frac{60}{12}$

$= 5$ amperes.

Current through II $= \dfrac{\text{Voltage across II}}{\text{Impedance of II}}$

$= \frac{60}{10}$

$= 6$ amperes.

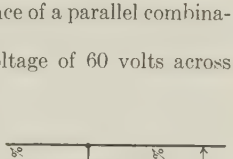



FIG. 113.

(*b*) Resolve the current of 5 amperes through I into its active and reactive components as in Fig. 114. Power factor = $33\frac{1}{3}$ per cent, reactive factor = 0.944.

Power component $AB = AC \times$ power factor

$= 5 \times 33\frac{1}{3}$ per cent

$= 1.67$ amp.

$$\text{Reactive component } BC = AC \times \text{reactive factor}$$
$$= 5 \times 0.944$$
$$= 4.72 \text{ amp.}$$

Resolve current of 6 amperes through II into its active and reactive components as in Fig. 115. Power factor = 90 per cent, reactive factor = 0.438.

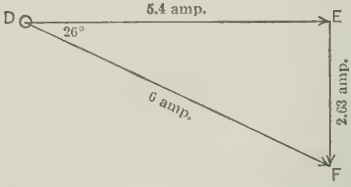



FIG. 114 and FIG. 115. For each of the parallel loads of Fig. 113, calculate the current and by means of power factor resolve into its active and reactive components, in phase with and at 90° to the line voltage respectively.

$$\text{Power component } DE = DF \times \text{power factor}$$
$$= 6 \times 0.90$$
$$= 5.4 \text{ amperes}$$
$$\text{Reactive component } EF = DF \times \text{reactive factor}$$
$$= 6 \times 0.438$$
$$= 2.63 \text{ amperes.}$$

Combine the power components $AB$ and $DE$ into $AE$ as in Fig. 116.

$$\text{Active current through combination} = AE = AB + DE$$
$$= 1.67 + 5.4$$
$$= 7.07 \text{ amperes.}$$

Reactive current through combination $= EG = BC + EF$
$= 4.72 + 2.63$
$= 7.35$ amperes.

Current through combination $AG = \sqrt{AE^2 + EG^2}$
$= \sqrt{7.07^2 + 7.35^2}$
$= 10.2$ amperes.

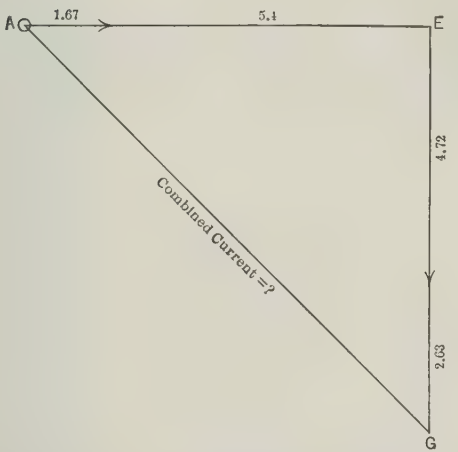



FIG. 116. Line current of Fig. 113 is vector sum of active components of individual load currents added at 90° to the sum of reactive components of individual load currents. Total impedance equals line voltage divided by line current.

(*c*) Impedance of combination $= \dfrac{\text{voltage across combination}}{\text{current through combination}}$

$= \dfrac{60}{10.2}$

$= 5.88$ ohms.

Power factor of combination $= \dfrac{AE}{AG} = \dfrac{7.07}{10.2} = 0.693 = 69.3$ per cent.

Note that the value finally obtained for the impedance of this combination, namely 5.88 ohms, would have been exactly the same if we had selected any other pressure than 60 volts to impress upon the combination of parallel circuits. Thus, doubling the voltage would double all values of current, but would not change the value of impedance which is the ratio existing between voltage and current. The power factor also would not be affected by change of assumed voltage, being a ratio between quantities which change always in the same proportion. The amount of power in the circuit would, however, vary greatly with the voltage impressed; doubled voltage would produce doubled current, and doubled amperes with doubled volts would mean that the volt-amperes, watts, and vars would be increased to four times their respective former values, or in proportion to the square of the voltage.

**Prob. 17–6.** What is the power (watts) consumed by the combination in Example 2 at 230 volts?

**Prob. 18–6.** If the appliances of Prob. 12 are joined in parallel, what will be the impedance of the combination?

**Prob. 19–6.** What would be the impedance and power factor of a parallel arrangement of the impedances of Prob. 16?

**Prob. 20–6.** How much power would be consumed by the combination of Prob. 19 if it was on a 115-volt line?

**55. Condensive Reactance.** The impedance of series and parallel circuits containing condensers * is found by the methods described in Paragraphs 53 and 54. However, we must remember that the current in a condenser **leads** the voltage instead of lagging.

In many static condensers, the insulation is so good that practically no power is lost, and the condenser operates at zero power factor. If we have a condenser drawing 5 amperes from a 230-volt line at zero power factor, we see from the diagram, Fig. 117, that the voltage is all reactive voltage,

* The term "condenser" used alone usually means a "static" condenser of the type described in Paragraph 41. Machines which operate as condensers are called "synchronous" or "rotary" condensers, or synchronous motors.

since it is 90° out of phase with the current. Hence, when operating at zero power factor,

$$\text{Impedance} = \text{Condensive reactance}$$
$$= \frac{\text{Reactive voltage}}{\text{Current}}$$
$$= \tfrac{230}{5} = 46 \text{ ohms.}$$

The condenser in this case would take $230 \times 5$, or 1150 vars, or 1.15 kvars. Commercial power condensers will be found to be rated both in kilovars and kilovolt-amperes; in either case, the condenser is assumed to operate at practically zero power factor.

Occasionally, condensers will develop leaky insulation due to faulty materials, mechanical damage, moisture, etc., and on test will be found to conduct an appreciable current when a d-c voltage is applied. The condenser then behaves as it would if a resistance were connected in parallel with it. When the voltage, current, and power relations are computed for such "leaky" condensers, it is more convenient, however, to assume that the faulty condenser is represented by a perfect condenser in *series* with a small resistance instead of by a perfect condenser in *parallel* with a large resistance.

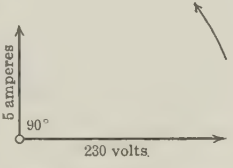



FIG. 117. The current in a condenser operating at zero power factor leads the voltage by 90°.

Suppose, for example, we have a condenser which draws 1 ampere from a 230-volt line, and takes an effective power of 15 watts. Then,

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$
$$= \frac{15}{230 \times 1} = 0.0652$$
$$= 6.5 \text{ per cent.}$$
$$\text{Impedance} = \tfrac{230}{1} = 230 \text{ ohms.}$$

And since,

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}},$$

$$\text{Resistance} = 0.0652 \times 230 = 15 \text{ ohms.}$$

$$\text{Reactance (condensive)} = \sqrt{\text{Impedance}^2 - \text{Resistance}^2}$$

$$= \sqrt{230^2 - 15^2}$$

$$= 229.6 \text{ ohms.}$$

The resistance of 15 ohms found above is the equivalent series resistance of the leaky condenser. We can readily see that it is a series resistance because the total current of 1 ampere must flow through it in order to consume the effective power of 15 watts.

The angle which corresponds to a power factor of 0.0652 is 86.3°. If this were a perfect condenser, the power-factor angle would be 90°. The difference, or 3.7°, is called the "phase defect angle" of the condenser. There is, of course, no such thing as a perfect insulator, and all condensers therefore have a phase defect angle. In practice, however, this angle is so small that it can usually be neglected.

**56. Capacitance of a Condenser.** In most of the work done with condensers, it is more important to know the capacitance (or the **capacity**) of a condenser than the amount of reactive power it takes. The **capacitance,** or capacity of a condenser, is the **quantity** of electricity which the condenser will hold per volt (d-c) applied. The unit of quantity is the **coulomb,** which is the amount of electricity represented by a steady current of 1 ampere flowing for 1 second. The unit of capacitance is the **farad;** a condenser which will store 1 coulomb when 1 volt is applied across its terminals has a capacity of 1 farad. If the voltage across the condenser is doubled, the amount of electricity stored is doubled; the capacitance is simply the amount of electricity per volt.

In actual practice, we rarely deal with condensers as large as 1 farad, because a condenser of this capacitance would be

extremely large in size.* The practical unit of capacitance is the **microfarad,** abbreviated $\mu$f, which is one millionth of a farad. In much of modern radio practice, even this unit is too large, and very small condensers are measured in micro-microfarads ($\mu\mu$f), or a millionth of a millionth of a farad.†

**57. Effect of Change of Frequency on Impedance.** We have seen that impedances are formed by combinations of resistance, inductive reactance, and condensive reactance. All three of these factors will change in magnitude when the frequency of the supply voltage changes.

(*a*) Resistance. At the frequencies commonly used in power systems, the resistance of lines, transformers and other devices is practically constant except for changes due to temperature variation. In general, however, the resistance of wires and windings does increase if the frequency is increased. Thus a No. 0000 copper conductor carrying a 60-cycle current has an effective resistance equal to about 1.005 times its d-c resistance. This change is so small that usually it is neglected, but in high-frequency circuits such as are encountered telephone and radio work, the a-c resistance may be much greater than the d-c resistance, and the increase must be taken into account.

This change of resistance with frequency is due to the so-called "skin effect." A conductor carrying an alternating current sets up a weak magnetic field **inside** the conductor as well as outside. The lines of force cutting the conductor itself cause reactive voltages to be induced within the conductor, but this induced voltage is slightly

* To get some idea of the physical size of such a unit, a certain paper condenser rated at 1 microfarad, 600 volts, has a volume of 4.5 cubic inches. A condenser of this type with a capacitance of 1 farad would fill a room approximately 20 feet long, 10 feet wide, and 10 feet high.

† For example, in circuits containing vacuum tubes, the condenser effect between the elements of the tube is often very important. In a type 76 tube, there is a capacitance of about 3 $\mu\mu$f between the grid and the plate.

greater at the center than at the outer surface of the conductor. This causes the current to be somewhat more concentrated near the outside of the conductor (hence the name "skin effect") and the resistance drop is therefore greater than it would be if the current distribution were uniform.

(*b*) Inductive reactance. The inductive reactance of a device changes in direct ratio with the frequency. Thus an appliance which has an inductive reactance of 10 ohms on a 60-cycle circuit will have only 5 ohms reactance if put on a 30-cycle circuit.

An explanation of how a change in frequency changes the inductive reactance can be given as follows. The inductive reactance of a circuit is due to the lines of the magnetic field cutting and recutting the conductors of the circuit as the current changes in value. This cutting, it will be remembered, sets up a back voltage which limits the current. If the frequency is lowered, then the current changes at a slower rate and both the cutting and the induced back voltage is smaller in value. Thus the reactance is less, and more current can flow than at a higher frequency. If the frequency is raised, the current changes faster, the magnetic lines cut the circuit a correspondingly greater number of times each second and the back voltage is increased in proportion. Thus the reactance of the circuit becomes greater and the current less.

(*c*) Condensive reactance. The reactance of a condenser changes in inverse ratio with the frequency, decreasing as the frequency increases. Thus a condenser which has a reactance of 1000 ohms at 60 cycles has a reactance of 2000 ohms at 30 cycles. The current through a condenser depends not only on the amount of charge which it will hold but also on the rate at which the condenser is charged and discharged. As this rate increases due to an increase of frequency the current through the condenser increases, and the reactance therefore decreases.

The reactance of a condenser also changes in inverse ratio with the capacitance; a condenser of 10 microfarads capacitance has half the reactance of a 5-microfarad condenser.

The relation between reactance, frequency, and capacitance of a condenser can be written as a formula, namely

$$\text{Condensive reactance} = \frac{159{,}200}{\text{Frequency} \times \text{Capacitance}},$$

where reactance is in ohms, frequency is in cycles per second, and capacitance is in microfarads. Thus, at 60 cycles, a condenser of 10 microfarads capacitance has a reactance of $\frac{159{,}200}{60 \times 10} = 265.3$ ohms.

The most common frequencies are 60 and 25 cycles. The inductive reactance of an appliance on a 25-cycle circuit is only $\frac{2}{6}\frac{5}{0}$ or $\frac{5}{12}$ of what it is on a 60-cycle circuit. However, at 25 cycles, the reactance of a condenser is $\frac{12}{5}$ of the reactance at 60 cycles.

**Example 3.** A dimmer has an impedance of 18 ohms at 40 per cent power factor on a 60-cycle circuit. What will be the impedance and power factor of the dimmer on a 25-cycle circuit?

**Solution.** On a 60-cycle circuit:

Reactive factor for 40 per cent power factor = 0.916.

$$\begin{aligned}\text{Resistance} &= \text{Impedance} \times \text{power factor}\\ &= 18 \times 0.40 = 7.2 \text{ ohms.}\end{aligned}$$

$$\begin{aligned}\text{Reactance} &= \text{Impedance} \times \text{reactive factor}\\ &= 18 \times 0.916\\ &= 16.5 \text{ ohms.}\end{aligned}$$

On a 25-cycle circuit, the resistance will be the same or 7.2 ohms.

$$\begin{aligned}\text{Reactance} &= 16.5 \times \tfrac{2}{6}\tfrac{5}{0}\\ &= 6.87 \text{ ohms.}\end{aligned}$$

$$\begin{aligned}\text{Impedance} &= \sqrt{\text{Resistance}^2 + \text{Reactance}^2}\\ &= \sqrt{7.2^2 + 6.87^2}\\ &= 9.96 \text{ ohms.}\end{aligned}$$

$$\text{Power factor} = \frac{7.2}{9.96} = 0.723 = 72.3 \text{ per cent.}$$

Thus the impedance is reduced from 18 ohms on a 60-cycle circuit to about 10 ohms on a 25-cycle circuit, and the power factor is increased from 40 per cent to about 72 per cent.

**Prob. 21–6.** A coil has an impedance of 35 ohms at 70 per cent power factor on a 25-cycle circuit. What impedance will the coil have on a 60-cycle circuit?

**Prob. 22–6.** A dimmer connected directly across a 115-volt 60-cycle line takes 650 volt-amperes at 45 per cent power factor. What current will it take on a 25-cycle line at the same voltage?

**Prob. 23–6.** What will be the power and power factor of the dimmer in Prob. 22 on the 25-cycle line?

**Prob. 24–6.** How much current will an induction coil take from a 40-cycle line, if it takes 0.40 ampere at 60 per cent power factor on a 60-cycle line at the same voltage?

**Prob. 25–6.** A 4-$\mu$f condenser is connected in series with a 50-ohm resistance across a 1000-cycle supply. What is the line voltage if this combination draws 0.040 ampere?

**Prob. 26–6.** What is the power factor of the circuit in Prob. 25?

**Prob. 27–6.** At what frequency will the power factor of the circuit in Prob. 25 be 80 per cent? What will be the impedance at that frequency?

**58. Resonance. Tuned Circuits.** Suppose we connect a coil with an impedance (at 60 cycles) of 2000 ohms, 0.2 power factor lagging, in series with a 1-$\mu$f condenser. We can find the impedance of this combination by the method given in Paragraph 53, noting however that inductive reactance and condensive reactance are of opposite sign and must hence be subtracted in order to get the total reactance.

For the coil,

$$\begin{aligned}\text{Resistance} &= \text{Impedance} \times \text{Power factor},\\ &= 2000 \times 0.2,\\ &= 400 \text{ ohms.}\end{aligned}$$

$$\begin{aligned}\text{Reactance} &= \sqrt{\text{Impedance}^2 - \text{Resistance}^2},\\ &= \sqrt{2000^2 - 400^2},\\ &= 1960 \text{ ohms.}\end{aligned}$$

For the condenser, the resistance is zero, hence:

$$\text{Reactance} = \frac{159{,}200}{\text{Frequency} \times \text{Capacitance}}.$$

At 60 cycles, this has the value $\frac{159{,}200}{60 \times 1} = 2653$ ohms.

The total resistance of the series circuit is 400 ohms. Since the reactances are of opposite sign, the total reactance is $1960 - 2653 = -693$ ohms. Hence for the series circuit, we have

$$\text{Impedance} = \sqrt{400^2 + (-693)^2}$$
$$= 800 \text{ ohms}.$$

Note that the impedance of the series circuit is **less** than the impedance of either part. Fig. 118 shows the vector diagram for this circuit. The line $OA$ represents the coil resistance of 400 ohms, $AB$ the inductive reactance of 1960 ohms, and $OB$ the coil impedance of 2000 ohms. To the same scale, $OD$ represents the condenser reactance of 2653 ohms which must be combined vectorially with $OB$. Hence $DC$ is laid off equal to $OB$ and parallel to it. The sum of the impedances is then $OD$ plus $OB$ which is the line $OC$, and the latter has the computed value of 800 ohms, made up of the resistance $OA$ of 400 ohms and the total reactance $AC$ of $-693$ ohms.

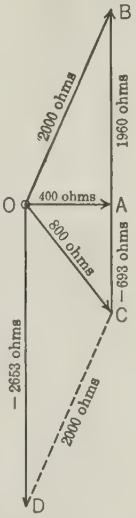



FIG 118. Vector diagram for an inductive impedance in series with a condenser.

Now let us change the frequency, remembering that as the frequency increases, the inductive reactance **increases** and the condensive reactance **decreases** in direct ratio. We will try to find a frequency at which the inductive reactance equals the condensive reactance.

As the frequency changes,

$$\text{Inductive reactance} = 1960 \times \frac{\text{Unknown frequency}}{60 \text{ (cycles)}},$$

and

$$\text{Condensive reactance} = 2653 \times \frac{60 \text{ (cycles)}}{\text{Unknown frequency}}.$$

When the inductive reactance is equal to the condensive reactance,

$$1960 \times \frac{\text{Unknown frequency}}{60} = 2653 \frac{60}{\text{Unknown frequency}},$$

and $$(\text{Unknown frequency})^2 = \tfrac{2653}{1960} \times 60 \times 60$$
$$= 69.9 \text{ cycles}.$$

At a frequency of 69.9 cycles per second, the inductive reactance is $1960 \times \frac{69.9}{60}$, or 2280 ohms, and the condensive reactance is $2653 \times \frac{60}{69.9}$, or $-2280$. These reactances in a series circuit operating at 69.9 cycles combine to produce zero reactance and the circuit acts as though it consisted only of the resistance of 400 ohms. Hence, at this frequency, the circuit operates at unity power factor and we say that we have adjusted the circuit to **resonance.**

Fig. 119 shows the impedance diagram for the resonant circuit. The coil impedance $OB$ is composed of the resistance $OA$ and the inductive reactance $AB$. The condensive reactance is given by the line $OD$ equal in magnitude to $AB$ but drawn in the opposite direction. When we add $OB$ and $OD$ we find that the sum $OC$ lies exactly on the resistance $OA$. In a series resonant circuit, therefore, the current which flows will be exactly in phase with the total voltage. However, the voltage across each part of the circuit will be out of phase with the current by the angle between the impedance and the resistance of the part

Suppose we apply 400 volts at 69.9 cycles to the above

series circuit. The current is $\frac{400}{400}$ or 1 ampere. The impedance of the coil is 2315 ohms and the voltage across the coil is $2315 \times 1$, or 2315 volts, and the voltage across the condensive reactance of 2280 ohms is $2280 \times 1$, or 2280 volts. Hence we have the remarkable result that the voltage across separate parts of the series resonant circuit is over five times as great as the applied voltage. This apparent contradiction is explained if we remember that both the applied voltage and current are parallel with *OA* in Fig. 119, while the voltages across the coil and condenser are parallel with *OB* and *OD*, respectively. The two large voltages when added vectorially produce the relatively small terminal voltage.

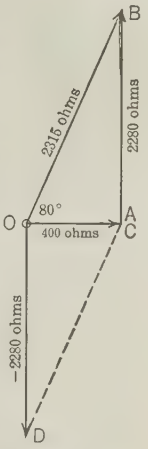



FIG. 119. Vector diagram of series circuit at resonance.

In the above example, we found the frequency for which the series circuit was resonant; that is, the frequency at which the power factor became unity. Instead of changing the frequency, we could have produced resonance by changing one or both of the reactances. By this means we can produce resonance at any frequency.

The same situation is true for parallel combinations of inductive and condensive impedances, and resonance is produced in this case also by adjusting the reactances until the power factor is unity. Any series or parallel circuit adjusted to resonance at any frequency is said to be tuned to that frequency. In all modern radio receivers, tuned circuits of adjustable resonant frequencies are provided to permit the selection of various stations. These are commonly parallel circuits tuned by variable condensers. In addition, superheterodyne receivers also contain resonant circuits of fixed resonant frequency in the intermediate-frequency amplifier.

The transformer of Fig. 46 is part of such a circuit and the "trimmer" condensers for tuning can be observed at the top of the case.

The total impedance of a series circuit containing inductive and condensive reactance is a minimum at resonance, while the impedance of a similar parallel circuit is a maximum at resonance. In either case, the power factor of the combination is unity at resonance.

**Prob. 28–6.** What capacity is required for the condenser in series with the coil in Paragraph 58 in order to produce resonance at 60 cycles?

**Prob. 29–6.** The coil described in Paragraph 58 is connected in series with a condenser of 0.06 $\mu$f capacity. What is the resonant frequency of this circuit?

**Prob. 30–6.** What would be the impedance of the series circuit of Prob. 25 at 60 cycles.

## SUMMARY OF CHAPTER VI

**If RESISTANCE (ohms) be represented by one side of a right triangle and REACTANCE (ohms) by the other side, then the hypothenuse of the triangle represents to the same scale the IMPEDANCE. It follows that:**

$$\text{Impedance} = \sqrt{\text{Resistance}^2 + \text{Reactance}^2}.$$

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}}.$$

$$\text{Reactive factor} = \frac{\text{Reactance}}{\text{Impedance}}.$$

$$\text{Resistance} = \text{Impedance} \times \text{power factor}.$$

$$\text{Reactance} = \text{Impedance} \times \text{reactive factor}$$

$$= \text{Impedance} \times \sqrt{1 - (\text{power factor})^2}.$$

**The angle between the resistance and impedance legs of the right triangle is the angle representing the amount (of time) by which the current in the circuit lags or leads with respect to the**

alternating voltage — LAG if the circuit has INDUCTIVE REACTANCE, and LEAD if the circuit has CONDENSIVE REACTANCE.

To find the TOTAL IMPEDANCE and TOTAL POWER FACTOR of a number of parts arranged in SERIES, proceed as follows:

First, find the resistance and reactance of each part from the known impedance and power factor or from quantities which determine them (as volts, amperes, watts).

Second, add the resistances of the parts in series to obtain total resistance of the combination, and add reactances of parts in series to obtain total reactance. Inductive reactance and condensive reactance are of opposite sign. Then,

$$\text{Total impedance of a series circuit} = \sqrt{(\text{total resistance})^2 + (\text{algebraic sum of reactances})^2}.$$

$$\text{Power factor of a series circuit} = \frac{\text{sum of resistances}}{\text{total impedance}}$$

The power factor of a series circuit may be adjusted to unity or to any desired value, by using some condensive reactance to compensate excessive inductive reactance if necessary, or vice versa. Condensive reactance is opposite in sign to inductive reactance, and must be subtracted to find the algebraic sum. It is convenient to call inductive reactance positive, and condensive reactance negative.

To find the TOTAL IMPEDANCE and TOTAL POWER FACTOR of a number of parts arranged in PARALLEL, proceed as follows:

First, assume that some particular voltage is impressed on all the parallel parts; usually this is taken as 1 volt, but the results will be the same if any value is chosen. From its impedance find how many amperes each part will be forced by this voltage to carry.

Second, find total current for all parts in parallel, by method explained in Chapter V. Thus,

$$\text{Total current in parallel combination} = \sqrt{(\text{sum of power components})^2 + (\text{sum of reactive components})^2}.$$

Then,

$$\text{Impedance of parallel combination} = \frac{\text{voltage applied}}{\text{total current in combination}}.$$

Power factor of parallel combination

$$= \frac{\text{sum of power components}}{\text{total current in combination}}.$$

Power factor of a parallel combination may be adjusted to any desired value in same way as for series combination. In order to secure the effect of large condensers in power systems, it is customary to use over-excited synchronous motors or synchronous condensers to draw sufficient leading current.

The total impedance and total power factor of any circuit may be calculated from the known properties of the individual parts by considering it as a series of groups, some of which consist of parts in parallel, or by considering it as a number of groups in parallel, some of which consist of parts in series. By careful application of the laws already stated any such problem may be solved accurately.

Change of FREQUENCY changes the INDUCTIVE REACTANCE in DIRECT PROPORTION, but changes the CONDENSIVE REACTANCE in INVERSE PROPORTION. Resistance increases as frequency increases, due to skin effect, but in power work this change is usually neglected.

## PROBLEMS ON CHAPTER VI

**Prob. 31–6.** An inductive appliance has a 60-cycle impedance of 125 ohms and a resistance of 75 ohms. How much is the reactance at this frequency?

**Prob. 32–6.** What is the 25-cycle impedance of the appliance in Prob. 31?

**Prob. 33–6.** What current will the appliance in Prob. 31 take from a 230-volt 40-cycle circuit?

**Prob. 34–6.** What will be the power factor of the appliance in Prob. 22 if it is placed on a 133-cycle circuit?

**Prob. 35–6.** What is the 60-cycle impedance of an appliance, the resistance and the inductive reactance of which are 12.8 ohms and 4.6 ohms respectively?

**Prob. 36–6.** What is the power factor of the appliance in Prob. 35?

**Prob. 37–6.** What is the 25-cycle impedance of the appliance in Prob. 35?

**Prob. 38–6.** If the appliances of Prob. 31 and 35 are placed in series on a 115-volt 60-cycle circuit how much current will flow through the combination?

**Prob. 39–6.** What will be the power factor of the combination of Prob. 38?

**Prob. 40–6.** If the appliances of Prob. 31 and 35 are placed in parallel what will be the impedance of the combination?

**Prob. 41–6.** What will be the power factor of the combination in Prob. 40?

**Prob. 42–6.** What maximum current will 115 volts at 25 cycles force through a series combination of two appliances, one having a 60-cycle impedance of 40 ohms at 50 per cent power factor lagging, the other a 60-cycle impedance of 50 ohms at 30 per cent power factor leading?

**Prob. 43–6.** How much power will each of the appliances take in Prob. 42?

**Prob. 44–6.** How much more power will a coil consume on a 40-cycle 115-volt circuit than it consumes on a 60-cycle 115-volt circuit if it has an impedance of 25 ohms at 85 per cent power factor at the latter frequency?

**Prob. 45–6.** The impedance of a parallel combination of two appliances is 16.4 ohms with a lagging power factor of 85 per cent. One of the appliances has an impedance of 35 ohms with a lagging power factor of 62 per cent. What are the impedance and the power factor of the other appliance?

**Prob. 46–6.** Tests on a single-conductor steel-taped cable showed that 1000 feet of No. 6 copper wire so protected gave a voltage drop of 6.7 volts of which 3 volts were due to resistance. The resistance of No. 6 copper wire of ordinary grade at ordinary temperatures is about 0.395 ohm per 1000 feet. Calculate the reactance of this cable, in ohms per 1000 feet, for the frequency at which this test was made.

**Prob. 47–6.** A cable similar to that specified in Prob. 46 is used to connect into a single series circuit 80 series incandescent street lamps, each lamp being rated (and operated) 6.6 amperes, 37.1 volts. The average distance between these lamps is 250 feet. Calculate how many volts must be impressed upon the whole circuit in order to force 6.6 amperes through it.

**Prob. 48–6.** (*a*) How much power (kw) is delivered to all the lamps together in the series circuit of Prob. 47?

(*b*) How much power (kw) is lost in the cable of Prob. 47?

**Prob. 49–6.** What is the power factor of the entire series circuit of Prob. 47, the lamps being of course non-inductive?

**Prob. 50–6.** A certain alternating-current transmission line is tested for impedance by short-circuiting it perfectly at the outer end, and applying a relatively low voltage at the station end. Under these conditions, the instruments on this line at the station indicate as follows: wattmeter 5.8 kw, voltmeter 120 volts, ammeter 75 amperes. Station frequency is 60 cycles per second. Calculate the values of impedance, resistance and reactance for this line.

**Prob. 51–6.** When 250 kw are being delivered at 2300 volts and 0.80 lagging power factor at the outer end of the line of Prob. 50, how many volts are consumed in overcoming the impedance of the line itself?

**Prob. 52–6.** With the line of Prob. 50 loaded as in Prob. 51, how many volts must be impressed upon the station end of the line?

**Prob. 53–6.** What per cent of the power (kw) delivered by the generator into the loaded line of Prob. 52, is consumed in overcoming the resistance of the line, or in heating the line?

**Prob. 54–6.** The load at the end of the line in Prob. 52 consists of motors and lamps operating in parallel. The lamps take altogether 130 kw non-inductive. Calculate how many amperes are delivered to the motors, and at what power factor.

**Prob. 55–6.** If the lamp load of Prob. 54 is suddenly removed from the line while the motors continue at the same current input, calculate: (*a*) What voltage at the station end of the line would now be required in order to keep the voltage at the motors unchanged at its former value of 2300 volts? (*b*) To what value will the voltage at the motors rise if the station voltage remains at the value calculated in Prob. 52?

**Prob. 56–6.** (*a*) If to a load having a power factor of 0.60 a non-inductive load of 20 per cent of the original amount (kv-a) be added, what is the resultant power factor?

(*b*) If to a load having a power factor of 0.90 a non-inductive load of 20 per cent of the original amount (kv-a) be added, what is the resultant power factor?

(*c*) What conclusions can you draw from a comparison of the results of parts (*a*) and (*b*)?

**Prob. 57–6.** What is the reactance of a $\frac{1}{4}$-microfarad condenser (*a*) at 60-cycles, (*b*) at 1000 cycles, (*c*) at 1500 kilocycles (a kilocycle is 1000 cycles)?

**Prob. 58–6.** A 2-microfarad condenser is connected in series with a 200-ohm resistor. What is the impedance of the combination at 500 cycles?

**Prob. 59–6.** What is the power factor of the series circuit of Prob. 58?

**Prob. 60–6.** A resistor of 250 ohms is connected in parallel with the series circuit of Prob. 58. What is the impedance of the series-parallel combination?

**Prob. 61–6.** What is the power factor of the circuit of Prob. 60?

**Prob. 62–6.** A coil with a 60-cycle impedance of 35 ohms at 0.72 power factor is connected in parallel with the series circuit of Prob. 58. What is the 60-cycle impedance of the series-parallel combination?

**Prob. 63–6.** What is the power factor of the circuit of Prob. 62?

**Prob. 64–6.** If the coil of Prob. 62 is connected in series with the resistance and condenser of Prob. 58, what is the impedance of the entire series circuit? What is the power factor?

**Prob. 65–6.** At what frequency will the series circuit of Prob. 64 be resonant?

**Prob. 66–6.** An induction motor draws 6.35 amperes at 71 per cent power factor from a 230-volt, 60-cycle line. What condenser capacitance is required in parallel with the motor so that the line load will be at 90 per cent power factor, lagging?

# CHAPTER VII

## POLYPHASE CIRCUITS

Thus far we have considered single-phase alternating-current power only. Because of the several decided advantages which polyphase systems possess, they are in general use where large quantities of power are utilized. It is necessary, therefore, to make a special study of these systems.

**59. Distinction Between Single-phase and Polyphase Systems.**

**Single-phase.** In a single-phase system power is obtained from a single set of armature or transformer windings or the equivalent. This system is much like a direct-current system. The power may be distributed as in a direct-current system, by two or three wires. When three wires are used, the voltage between one outside wire and the neutral is in phase with the voltage between the other outside wire and the neutral, and also in phase with the voltage between the two outer wires. Thus the voltage between the two outside wires is equal to the arithmetical sum of the voltages between either outside wire and the neutral, as in a direct-current system.

**Polyphase.** When the power is derived from more than one set of armature windings or the equivalent, the system is said to be polyphase. The polyphase systems in general use are the two-phase (sometimes called the quarter-phase) and the three-phase.

(*a*) **Two-phase.** Two-phase power is derived from two sets of armature windings and is generally distributed by four wires, each phase taking two wires. **Each phase is usually distinct from the other and the voltage between two wires of one phase always lags behind or leads the voltage**

between the two wires of the other phase by 90°. This latter fact is the reason for sometimes calling the system "quarter-phase," 90° being a quarter of a cycle. Either phase may be regarded alone as a single phase, and the voltages of the two phases are normally equal. The two-phase installations using three wires are so few that this scheme will not be considered in this book.

(*b*) **Three-phase.** The power in a three-phase system is derived from three sets of armature windings or their equivalent, and is usually transmitted by three wires, though four-

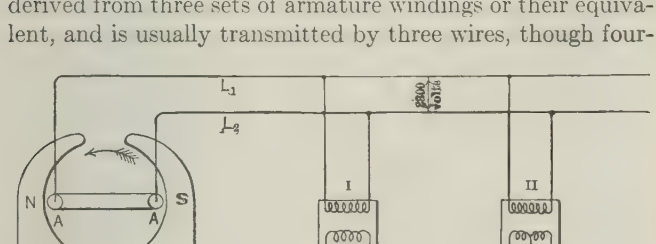



FIG. 120. Single-phase a-c system consisting of generator delivering power through transmission line and transformers to loads. Transformer II serves three-wire single-phase low-tension mains, motor *M* being connected between outer wires and lamps between neutral (*n*) and either outer wire.

wire systems are not uncommon. **The voltage across any one phase always differs in phase from the voltage across either of the other two phases by 120°. The voltages across the three phases are equal.**

**60. Single-Phase.** Fig. 120 represents a typical single-phase system. The power is taken from the single armature winding and transmitted by the two line wires $L_1$ and $L_2$. We will assume the line pressure to be obtained directly from the generator at 2300 volts. In the diagram the armature

is represented as being the revolving element of the generator and the fields $N$ and $S$ as stationary. It is the usual practice to construct an alternator, especially in the larger sizes, with the armature stationary and the fields revolving.

An illustration of such a generator of the three-phase type is seen in Fig. 137. Of course the electrical result is the same in either case, as it makes no difference whether the magnetic field moves and cuts the armature wires or whether the armature wires move and cut the magnetic lines of force. In either case the same voltage is induced in the armature coils.

Returning to Fig. 120, it will be seen that the voltage of the line is stepped down by distributing transformers at points where power is to be used. Thus transformer I steps down the line voltage from 2300 volts to 115 volts for a two-wire line, to be used with the lamps shown in the diagram. Transformer II steps down the voltage to 230 volts and 115 volts to be used on the three-wire line, for the lamps and the single-phase motor $M$.

The power, voltage and current distribution in such a system has been discussed fully in the previous chapters.

**61. Polyphase Systems.**

**Two-phase System.** A typical two-phase system is represented by Fig. 121. The generator now has two armature windings $A_1A_2$ and $B_1B_2$ independent of each other, instead of the single armature winding of Fig. 120. These windings are so placed that the voltages across them are at 90° to each other, that is, when the voltage in one winding is at its greatest value, the voltage in the other is zero and vice versa. It will be seen from a study of Fig. 121 that coil $A_1A_2$, at the instant shown, is sweeping across the magnetic lines perpendicularly and cutting them at the fastest rate and the voltage of coil $A_1A_2$ is, therefore, at its greatest value at this instant. Coil $B_1B_2$, however, which of course is revolving at the same speed as $A_1A_2$, is moving along the magnetic lines and not cutting them. The voltage in coil $B_1B_2$ is, therefore, zero

at this instant. Thus the voltage of coil $B_1B_2$ is zero at the instant the voltage of coil $A_1A_2$ has the greatest value. We have seen that when this condition is true, the voltages differ in phase with each other by 90°, or are 90° "out of phase." Accordingly the voltage across the line wires $A_1$ and $A_2$ connected to coil $A_1A_2$ is 90° out of phase with the voltage across the line wires $B_1$ and $B_2$ connected to coil $B_1B_2$. The line wires $A_1$ and $A_2$ are generally called phase $A$ of the line, while $B_1$ and $B_2$ are called phase $B$ of the line.

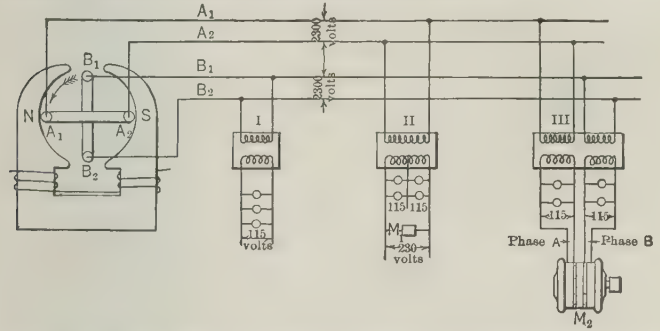



FIG. 121. Two-phase a-c system consisting of generator with two armature-windings at 90° to each other, connected to two independent load circuits. Single-phase transformers or loads, as I or II, may be connected to either phase. III is a "bank" of two transformers feeding the phases of a two-phase motor $M_2$.

Each phase of the transmission line may now be considered and used as a single-phase line, with single-phase transformers like connected transformers I and II in Fig. 121. Single-phase transformer I is connected to phase $B$ and steps down the voltage of that phase from 2300 to 115 volts for use with the lamps shown. Single-phase transformer II is connected to phase $A$ and steps down the 2300 volts to 230 volts for use with the single-phase motor $M$, and to 115 volts for use with the lamps, exactly as transformer II does in Fig. 120. Thus the distributing system from transformer I is

**a two-wire single-phase system,** and from transformer II is **a three-wire single-phase system.**

Transformers I and II make no use of the two-phase character of the transmission line. Transformer "bank" III, however, utilizes both phases and produces a four-wire two-phase distributing system, cutting down the voltage of both phases to 115 volts and making two phases, each of 115 volts, available for the two-phase motor $M_2$ as well as for the lamps. This transformer may consist of two separate single-phase transformers or it may be a single two-phase transformer. The result is the same, — a four-wire two-phase distributing system of 115 volts. This system has a decided advantage over a single-phase system in the operation of induction motors inasmuch as a two-phase induction motor is naturally self-starting, while a single-phase induction motor must be supplied with some special starting device.

In using a two-phase system it is always highly desirable to so "balance" the loads that the same amount of current flows in each wire of the transmission line. The two windings, called phase $A$ and phase $B$, of a two-phase motor are so constructed that the phases receive equal voltages and amounts of current and power from the line. Thus it is necessary only to see that not many more lamps are used on one phase than on the other. Unbalancing a four-wire two-phase line causes the voltage of the two phases to be unequal, and results in lowering the capacity of the line and generator.

**Example 1.** How much power must each phase of the generator in Fig. 121 deliver? Each lamp takes 500 watts, motor $M_1$ takes 1 kw and motor $M_2$ takes 3 kw.

Phase A

| | | |
|---|---|---|
| Through Transformer II | Motor $M_1$ takes | 1000 watts |
| | 4 lamps take | 2000 watts |
| Through Transformer III | $\frac{1}{2}$ Motor $M_2$ takes | 1500 watts |
| | 2 lamps take | 1000 watts |
| | Total | 5500 watts |

Phase B

| | | |
|---|---|---|
| Through Transformer I | 3 lamps take | 1500 watts |
| Through Transformer III | $\frac{1}{2}$ Motor $M_2$ takes | 1500 watts |
| | 2 lamps take | 1000 watts |
| | Total | 4000 watts |

The generator is unbalanced, Phase A delivering 1500 watts more than Phase B.

**Prob. 1–7.** If the power factor of motor $M_1$, Fig. 121, is 65 per cent, how much current does the secondary winding of transformer II carry? Use data of Example 1.

**Prob. 2–7.** If one wire of each of the two phases in motor $M_2$, Fig. 121, became grounded, what voltage would then exist between the remaining two wires?

**Prob. 3–7.** The power factor of motor $M_2$, Fig. 121, is 88 per cent. How much current does each low-voltage coil of transformer bank III carry? Use data of Example 1.

**Prob. 4–7.** How much current does each coil of the alternator of Fig. 121 carry? Use data of Probs. 1 and 3. Neglect transformer losses.

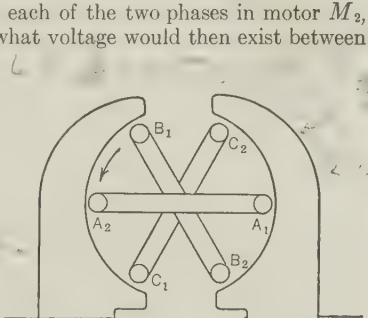



FIG. 122. Three-phase alternator. Voltage $A_1$ to $A_2$ reaches its maximum value 120° later than voltage $B_1$ to $B_2$, and the latter 120° later than the voltage $C_1$ to $C_2$.

**Three-phase System.** If we put three equally spaced coils or windings as in Fig. 122 on the armature of Fig. 120, we have a three-phase generator. The voltage in coil $B_1B_2$ reaches its maximum instantaneous value 120° ahead of the voltage in coil $A_1A_2$, and the voltage of $C_1C_2$ 120° ahead of that of $B_1B_2$. That is, the voltage in $B_1B_2$ had passed through 120° of its cycle

when the voltage of $A_1A_2$ is at zero and about to begin its cycle, just as the voltage in coil $A_1A_2$, Fig. 121, has passed through 90° at the instant the voltage in coil $B_1B_2$ is zero and about to begin its cycle.

Thus the voltage relations of the two coils in Fig. 121 are represented by the curves in Fig. 123, where $B_1B_2$ is 90°

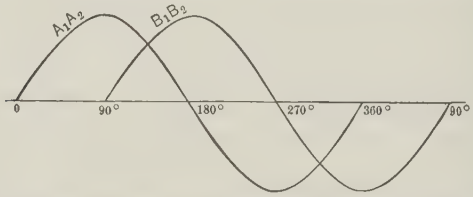



FIG. 123. Voltage curves for two phases of Fig. 121, $B_1B_2$ lagging 90° after $A_1A_2$.

behind $A_1A_2$ and has a zero value when $A_1A_2$ has a maximum value. Fig. 124 shows similar curves of the voltages in the three coils of Fig. 122. The voltage in $C_1C_2$ has passed through 120° of its cycle just as the voltage in $B_1B_2$ is about

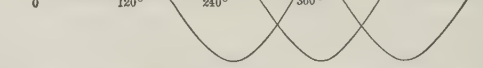



FIG. 124. Voltage curves for three phases of Fig. 122; three distinct voltages equal in value but 120° apart as to phase.

to begin its cycle, and the voltage of $B_1B_2$ has passed through 120° of its cycle at the instant the voltage in $A_1A_2$ is just beginning its cycle.

Similarly, Fig. 125 is a vector diagram of the conditions in Fig. 121, showing that the voltage in $A_1A_2$ is equal to the voltage in $B_1B_2$ but 90° ahead of it, and Fig. 126 is a vector

diagram of the condition of the three-phase generator of Fig. 122, showing that the voltage in $C_1C_2$ has passed through 240° of its cycle, when $B_1B_2$ has passed through but 120° and $A_1A_2$ is just beginning its cycle. It also shows that the voltages in all three phases are equal.

The three phases of a three-phase generator are rarely ever run out on separate circuits of two wires each, as are the phases of a two-phase generator. They are generally con-

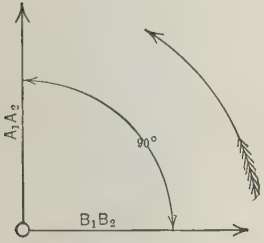



FIG. 125. Vector diagram of voltage relations in Fig. 121 and 123.

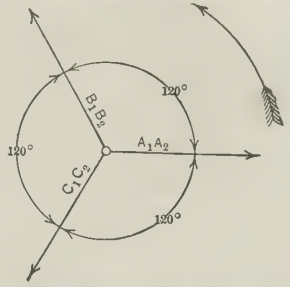



FIG. 126. Vector diagram of voltage relations in Fig. 122 and 124.

nected together on the inside of the machine and run out on three or sometimes four wires.

There are two regular ways of making these connections in a three-phase generator.

**62. Delta or Mesh Connection.** The terminal $A_2$ may be connected to $B_1$, $B_2$ to $C_1$, and $C_2$ to $A_1$. Fig. 127 shows these coils so connected. The name Delta is applied to this method of connection because the diagram resembles the Greek letter delta, made like this, — Δ, and corresponding to the English letter D. The vector diagram of this connection is represented in Fig. 132.

Three wires are usually brought out from a generator so connected as 1, 2 and 3, in Fig. 127 and 128. Fig. 128 represents conventionally a generator delta-connected, though it

must not be supposed that the armature or coils have this appearance. This is merely a convenient method of representing an armature connected in this manner.

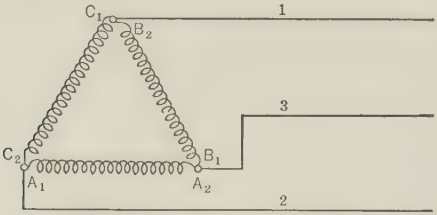



FIG. 127. Delta connection of the coils of Fig. 122. Line wires 1, 2, 3 carry current to the loads.

It will be noted that the voltage between the line wires 1 and 2, for instance, is the voltage in the coil $C_1C_2$. But it is

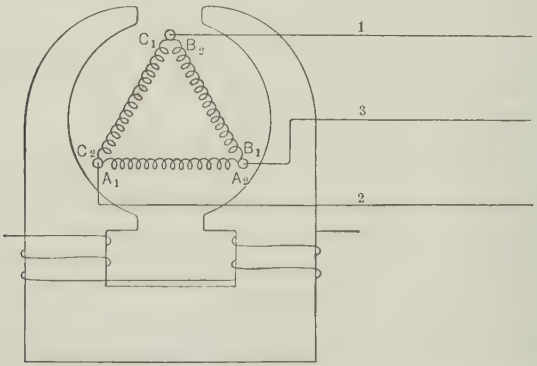



FIG. 128. Voltages between line wires are called the "delta voltages"; they equal the voltage in delta-connected phases.

also the resultant voltage of the series connection of the coils $A_1A_2$ and $B_1B_2$, since these coils are connected in series between the line wires 2 and 1.

Assuming that the voltage across each armature coil is 2300 volts, let us see what value the voltage between the line wires 3 and 1 has, in Fig. 128. From the fact that the line wires 1 and 2 are connected to the ends of the coil $C_1C_2$ we

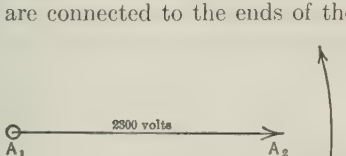



FIG. 129.

see that the voltage between 2 and 1 should be 2300 volts. But we have seen in Fig. 128 that the wires 1 and 2 are also across the series combination of the coils $A_1A_2$ and $B_1B_2$. Let us therefore also determine the voltage of the series combination of $A_1A_2$ and $B_1B_2$.

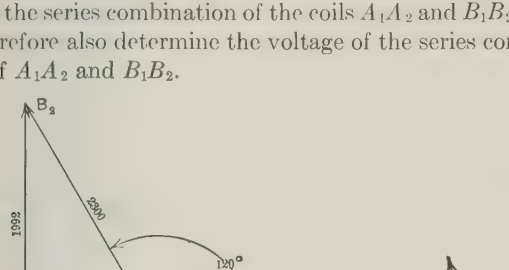



FIG. 130. Voltage $B_1B_2$ (2300) consists of $B_1X$ (1150) in phase opposition with voltage $A_1A_2$, and $XB_2$ (1992) in quadrature with $A_1A_2$.

Fig. 129 represents the voltage of 2300 volts in coil $A_1A_2$. In Fig. 130 the vector $B_1B_2$ is drawn at an angle of 120° ahead of $A_1A_2$, that is, swung around on the pivot $B_1$ to a position 120° from the direction of $A_1A_2$ in Fig. 129. We may now resolve the vector $B_1B_2$, Fig. 130, into two components, one

in phase with $A_1A_2$ and one in "quadrature" or at 90° to $A_1A_2$. These components correspond to the active and reactive components into which we have heretofore resolved any vector when we wished to add it to another. The vector $B_2X$ drawn from $B_2$ perpendicularly to the line of $A_1A_2$ represents the quadrature component, and the vector $B_1X$ represents the in-phase component. That is, we may consider the voltage of the coil $B_1B_2$ made up of the in-phase component $B_1X$ and the quadrature component $XB_2$.

The value of the in-phase component may be found from the equation:

In-phase component = voltage × (power factor corresponding to angle of phase difference)

or $$B_1X = B_1B_2 \times \text{"power factor."}$$

From Table I the power factor corresponding to an angle of 120° is −0.500.

Therefore, $$B_1X = 2300 \times (-0.500)$$
$$= -1150 \text{ volts.}$$

The minus sign merely indicates that the in-phase component of the voltage $B_1B_2$ is in the direction opposite to the voltage $A_1A_2$. This is also shown by the fact that the vector $B_1X$ points in the direction opposite to the vector $A_1A_2$.

The value of the quadrature component of $B_1B_2$ may be found from the equation

Quadrature component = voltage × (reactive factor corresponding to angle of phase difference)

or $$XB_2 = B_1B_2 \times \text{"reactive factor."}$$

The reactive factor corresponding to an angle of 120° is 0.866. Therefore,

$$XB_2 = 2300 \times 0.866$$
$$= 1992 \text{ volts.}$$

If now we combine the in-phase and quadrature components of $B_1B_2$ with the corresponding components of $A_1A_2$ we have Fig. 131.

$A_1A_2$ is entirely composed of in-phase component because we assumed it as a basis; thus to obtain the total in-phase component of the two voltages $A_1A_2$ and $B_1B_2$ we combine the whole of $A_1A_2$ with $B_1X$, the in-phase component of $B_1B_2$. Since $B_1X$ is in the reverse direction to $A_1A_2$ we subtract $B_1X$ from $A_1A_2$ to get the in-phase component $A_1X$ of the series combination.

$$\begin{aligned} \text{Thus} \quad A_1X &= A_1A_2 - B_1X \\ &= 2300 - 1150 \\ &= 1150 \text{ volts.} \end{aligned}$$

The quadrature component of the combination consists entirely of the quadrature component $XB_2$ of the voltage $B_1B_2$, since the voltage $A_1A_2$ which was taken as the base has no quadrature component. This is represented in Fig. 131 by the vector $XB_2$ of 1992 volts.

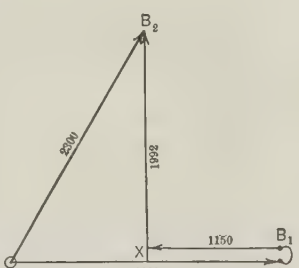



FIG. 131. When components of $B_1B_2$ are added vectorially to $A_1A_2$, the resultant is $A_1B_2$, equal and opposite to $C_1C_2$.

The resultant of the series combination of $A_1A_2$ and $B_1B^2$ is represented by the vector $A_1B_2$, Fig. 131. Its value may be found as follows:

$$\begin{aligned} A_1B_2 &= \sqrt{(A_1X)^2 + (XB_2)^2} \\ &= \sqrt{1150^2 + 1992^2} \\ &= 2300 \text{ volts.} \end{aligned}$$

Thus the voltage across the series combination of the coils $A_1A_2$ and $B_1B_2$ is 2300 volts, the same in value but opposite in phase to the voltage in coil $C_1C_2$. Therefore, it makes no difference whether we consider the voltage between the line wires 1 and 2 in Fig. 128 to be the voltage across the series combination of coils $A_1A_2$ and $B_1B_2$ or the voltage across the coil $C_1C_2$ alone. Either way we view it, the voltage between the two wires is 2300 volts. Similarly, the voltage between

the line wires 3 and 1 may be considered either as voltage across the single coil $B_1B_2$ or across the series combination of the coils $C_1C_2$ and $A_1A_2$. But the voltage across the series combination of the coils $A_1A_2$ and $C_1C_2$ may be shown in the same way as above to be equal to the voltage across the single coil $B_1B_2$ or 2300 volts.

Similarly, the voltage between the line wires 2 and 3 of Fig. 128 is the voltage across the series combination of the coils $B_1B_2$ and $C_1C_2$, or the voltage of the single coil $A_1A_2$, each of which equals 2300 volts. Thus it is seen that the voltage between any two of the three line wires is 2300 volts.

It may be stated as a rule that:

**In any three-wire three-phase system the voltage between any two wires should be the same.**

The voltage between any two line wires, however, is 120° out of phase with the voltage between any other two line wires. This is the main difference between a three-wire three-phase system and a three-wire single-phase system, which it will be remembered has the same voltage relations as a three-wire direct-current system.

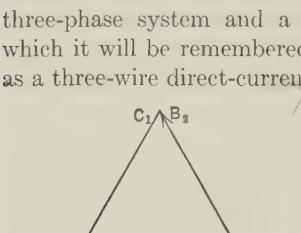



FIG. 132. Vector diagram of voltages in delta.

**Prob. 5–7.** If the voltage between line wires 1 and 2 in the three-wire three-phase system of Fig. 128 were 600 volts, what would be the voltage between the wires 2 and 3, and between 3 and 1? 600 V.

**Prob. 6–7.** What would be the voltage of the series combination of the armature windings $C_1C_2$ and $A_1A_2$ of generator in Prob. 5?

**Prob. 7 7.** From an inspection of Fig. 128, note that the armature windings form a closed circuit. Study Fig. 132 and 131 and state the reason why no current will circulate in the armature windings delta connected, when no current is being delivered to the line.

**63. Star or Y-Connection.** The other common way of connecting the armature coils of the three-phase generator shown in Fig. 122 is as shown diagrammatically in Fig. 133.

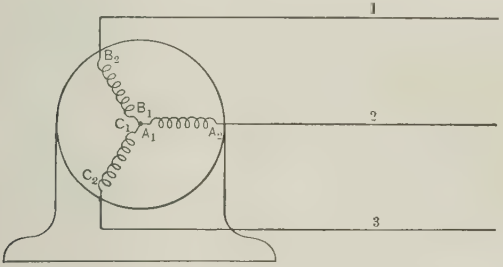



FIG. 133. Star or Y-connection of three phases.

The ends $A_1$, $B_1$ and $C_1$ of the three coils are connected and wires brought out from the free ends $A_2$, $B_2$ and $C_2$ to the line wires 1, 2 and 3. This is called the **star** or Y-connection because of the resemblance of the diagram to a star or to the letter Y.

The voltage between the line wires 1 and 2 is the voltage across the series combination of armature coils $B_2B_1$ and $A_1A_2$. We shall assume that the voltage in each armature coil is the same as in each coil of Fig. 128, and determine the voltage between line wires as we did before.

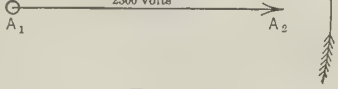



FIG. 134.

The vector in Fig. 134 represents the 2300 volts of the coil $A_1A_2$.

In drawing the vector for the voltage in coil $B_2B_1$, care must be taken to note the difference in the method of connection between these two coils in Fig. 128 and 133. In Fig. 128 the two coils are connected so that the current flows in a

positive direction through both coils $A_1A_2$ and $B_1B_2$, but in Fig. 133 note that the current must flow in the reverse direction through coil $B_1B_2$, that is, from $B_2$ to $B_1$, in order to get into coil $A_1A_2$. In other words, the coil $B_1B_2$ has been in effect reversed by connecting it in this manner to coil $A_1A_2$. Thus if the dotted line $B_1B_2$, Fig. 135, represents the voltage

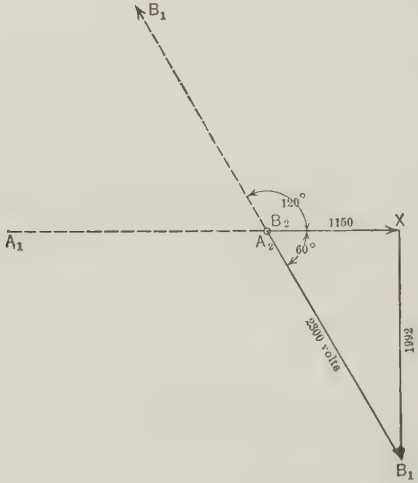



FIG. 135. The 2300 volts of phase $B$, Fig. 133, consist of 1150 volts in phase with $A_1A_2$ and 1992 volts in quadrature with $A_1A_2$. $A$ and $B$ pull in opposite directions with a phase difference of 120°, which is equivalent to pulling in the same direction with phase difference of 60°.

in coil $B_1B_2$ when connected as in Fig. 128 (compare Fig. 130), then the full line $B_2B_1$, Fig. 135, in the reverse direction, must represent the voltage in the coil when its connection relative to $A_1A_2$ is reversed. Note that this reversing of direction causes the voltage in coil $B_1B_2$ to have the effect of lagging 60° behind the voltage in coil $A_1A_2$ instead of leading it by 120° as before. The vector $B_2X$ thus represents the in-phase

component of the reversed voltage in $B_1B_2$ and the vector $XB_1$ represents its quadrature component.

The in-phase component may be found by the equation

$$B_2X = A_2B_1 \times \text{(power factor for 60° phase difference)}.$$

The power factor corresponding to 60° is 0.500 (Table I).

Thus,

$$B_2X = 2300 \times 0.500$$
$$= 1150 \text{ volts}.$$

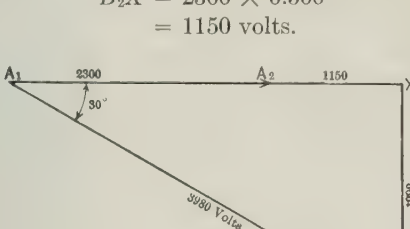



FIG. 136. Line voltage from 1 to 2 in Fig. 133 is vector sum of voltage $B_2$ to $B_1$ and voltage $A_1$ to $A_2$, or $A_1A_2$ added to $B_1B_2$ reversed. The line voltage is thus 3984, or 1.73 times the voltage across each of the star-connected phases.

The quadrature component may be found by the equation $XB_1 = B_2B_1 \times$ (reactive component for 60° phase difference). The reactive factor corresponding to an angle of 60° is 0.866.

$$XB_1 = 2300 \times 0.866$$
$$= 1992 \text{ volts}.$$

Combining, in Fig. 136, that component of voltage in $B_2B_1$ which is in phase with $A_1A_2$, with the voltage in $A_1A_2$ itself, we get the vector $A_1X$, which represents the in-phase component of the series combination of coils $A_1A_2$ and $B_2B_1$.

Draw $XB_1$ the quadrature component of voltage in $B_2B_1$ equal to the vector $XB_1$ of Fig. 135.

The voltage across the series combination may now be represented by the vector $A_1B_1$, Fig. 136, which is the resultant of the total in-phase and the total quadrature components.

$$\begin{aligned} A_1B_1 &= \sqrt{(A_1A_2 + A_2X)^2 + XB_1^2} \\ &= \sqrt{(2300 + 1150)^2 + 1992^2} \\ &= \sqrt{3450^2 + 1992^2} \\ &= 3984 \text{ volts (practically 3980).} \end{aligned}$$

This is the voltage between the line wires 1 and 2. Similarly, it can be shown that the voltage between any two line wires is also 3980 volts. Thus, when the voltage in each armature coil of a three-phase star-connected winding is 2300 volts, the volts between any two of the line wires is 3980 volts.

Note that the line voltage 3980 is $\frac{3980}{2300}$, or 1.73 times the coil voltage of the generator.

**Thus in a delta-connected three-phase generator, the voltage between any two line wires is the same as the voltage in each phase of the armature winding.**

**In a star-connected three-wire three-phase generator, the voltage between any two line wires is 1.73* times the voltage in each phase of the armature winding.**

Note also that the voltage between any two line wires of a three-wire line from a star-connected generator is 120° out of phase with the voltage between any other two line wires.

This is evident from an examination of the following. In Fig. 136, the vector $A_1B_1$ represents the voltage between the wires 1 and 2 of Fig. 133. Note that this vector $A_1B_1$ lags 30° behind the vector $A_1A_2$, which represents the voltage across the coil $A_1A_2$ of the generator of Fig. 133.

Similarly, the voltage between the wires 2 and 3 of Fig. 133 would be found to lag 30° behind the voltage across the

* Sometimes written $\sqrt{3}$, since $1.73 = \sqrt{3}$. For more accurate calculations use $\sqrt{3} = 1.732$.

generator coil $C_1C_2$, and the line voltage between 3 and 1 to lag 30° behind the voltage across coil $B_1B_2$. Thus, as the coil voltages are 120° apart in phase, so the line voltages, each of which lags 30° behind one of the coil voltages taken in order, must also be 120° apart.

The same generator can be used delta-connected to supply 2300 volts to a line, or star-connected to supply 3980 volts. The total power that could be delivered by the generator would be the same in either case, as the current per line wire would have to be correspondingly smaller at the higher voltage in order not to allow the current in the armature coils to become excessive. Note that in a star-connected armature, the windings carry the same current as the line wires, while in a delta-connected armature the line current is divided between two armature coils, allowing more current than that flowing in each armature coil to be delivered to the line.

For various technical reasons three-phase alternators are usually star-connected.

**Prob. 8–7.** If the coil voltage of the generator in Fig. 133 were 580 volts, what would be the voltage between line wires 2 and 3?

**Prob. 9–7.** In order that the line voltage in a Y-connected three-wire system be 6000 volts, what must the coil voltage of the generator be?

**Prob. 10–7.** What would be the line voltage of the generator in Prob. 9 if it were mesh-connected?

**Prob. 11–7.** How could a star-connected three-wire three-phase induction motor built to run on 220 volts be reconnected to operate on 125 volts?

**Prob. 12–7.** If a mesh-connected three-phase three-wire 550-volt motor were changed to the star connection, on what line voltage should it operate?

**64. Construction of a Polyphase Generator.** As was previously stated, the figures representing the coils of an alternator do not in any way represent the mechanical

appearance of the coils, but merely show the electrical and magnetic effects produced in the armature and fields. The appearance of a three-phase generator of the revolving field type is shown in Fig. 137. The field *F* consists of spools of wire wound on soft steel cores, and is shown more in detail

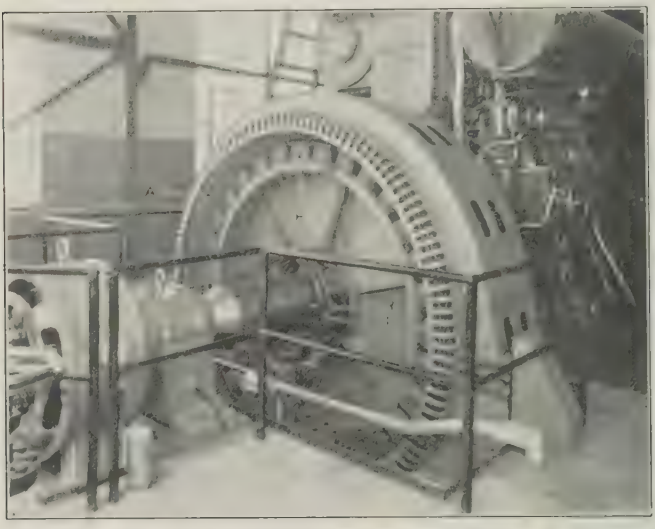


FIG. 137. Three-phase alternator, engine type, 2812 kv-a, 240 rpm, 2400 volts. Direct current from "exciter" generator at left end of main shaft passes through rings *R* into field magnets *F*. Stationary armature winding *A* connects to load. *Westinghouse Elec. and Mfg. Co.*

in Fig. 138. The field coils are supplied with direct current from an outside source through the collecting rings *R*. In this case, the source of field current is a d-c generator or "exciter" *E* mounted on the main shaft (Fig. 138). The field poles of the alternator are alternate north and south poles which revolve within the stationary armature.

The armature of the alternator is illustrated by Fig. 139, in which the armature windings and terminals are clearly shown. Note that it is impossible to tell by an examination of the picture of the armature windings whether the armature windings are connected in star or in delta. But whether it

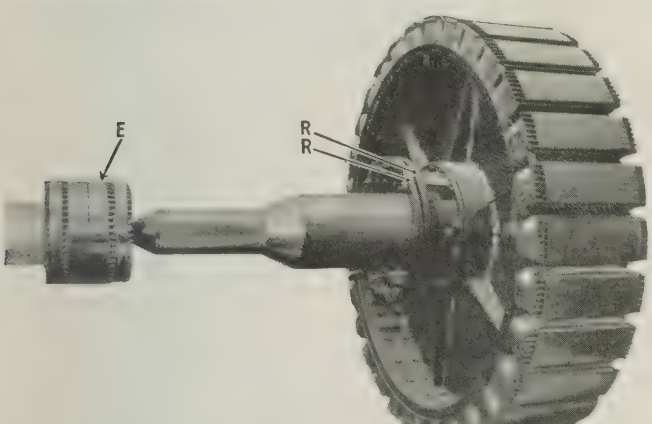



FIG. 138. Rotor of alternator shown in Fig. 137. Direct current supplied from exciter $E$ through rings $R$ causes adjacent poles of field magnets to be of north and south polarity. *Westinghouse El. c. and Mfg. Co.*

is star- or delta-connected, the armature coils would be put into the frame in exactly the same way. The only difference would be in the manner in which the coils were connected to each other after they were in place. There would be no difference in the appearance of the coils themselves nor in their relative positions on the frame, although electrical representations of the two methods of connection are entirely different as is shown in Fig. 128 and 133.

**65. Current in Line of Three-phase System.** It is often necessary to determine the amount of current which will flow

from each terminal of a three-phase generator, in order to run a line wire of the proper size for carrying this current.

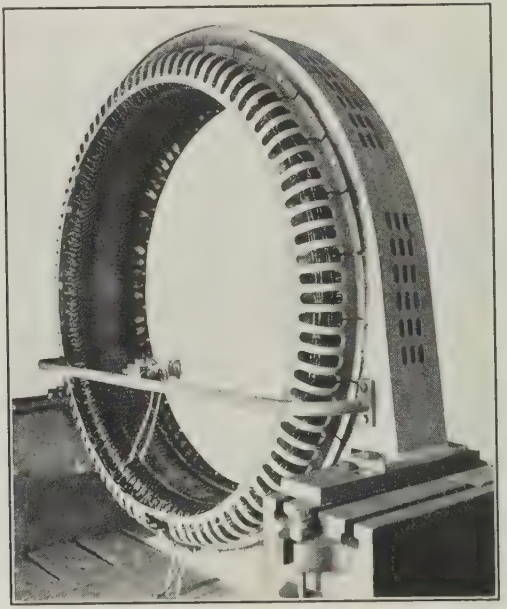


FIG. 139. Stator of an alternator similar to that of Fig. 137. The three wires at bottom carry the output of the alternator to a three-phase load.

Let us assume that a three-wire system having 110 volts between each pair of wires, *A*–*B*, *B*–*C* and *C*–*A* as in Fig. 140 is to supply a set of 15 incandescent lamps each taking 2 amperes. We should distribute the lamps so that an equal number is on each phase as in Fig. 140, in order to **balance** the system or to equalize the amount of current flowing in the various line wires and parts of the generator winding.

Each phase must, therefore, supply 5 lamps with a total current of 10 amperes. Thus the wires $D$, $E$, $F$, $G$, $H$ and $I$ must each carry 10 amperes. It is necessary now to determine what current the main wires $A$, $B$ and $C$ must carry.

Note that as each main wire is connected to two of the distributing wires it has to carry enough current to feed two sets of lamps. Thus Main $A$ feeds Groups II and III;

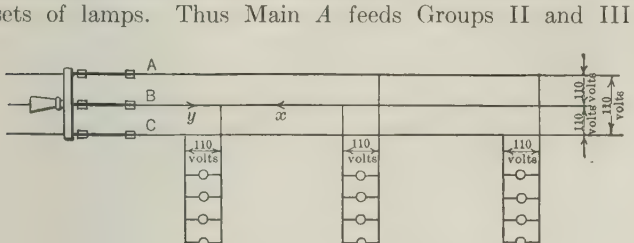



Fig. 140. Balanced load delta-connected on three-phase line.

Main $B$ feeds Groups I and II; Main $C$ feeds Groups III and I. We will study the current conditions in Main $B$, feeding I and II, by means of the wires $E$ and $F$.

As we have seen, in any balanced three-wire three-phase system, the voltage across Mains $AB$, $BC$ and $CA$ differ in phase by 120°. Therefore, let the voltage across $CA$ lead the voltage across $BC$ by 120°, and the voltage across $BC$ lead the voltage across $AB$ by 120°. The voltage across each pair of mains, as we have seen, is 110 volts. These facts are represented by the vector diagram Fig. 141.

The current in Group I of the lamps is in phase with the voltage across $BC$, since the lamps are non-inductive, and the current in Group II is in phase with the voltage across $AB$. Thus Fig. 142 will represent the current conditions in the lead wires $E$ and $F$ in which the vector $BF$ represents the

10 amperes flowing in wire $F$ and in phase with the voltage across $AB$. Note that accordingly, vector $BF$, Fig. 142, is drawn in the same direction as vector $AB$, Fig. 141. Since vector $BE$, Fig. 142, represents the current of 10 amperes in wire $E$, which is in phase with the voltage across $BC$, it is drawn in the same direction as the vector $BC$ in Fig. 141. Since the current in Main $B$ is the resultant of the currents flowing in wires $F$ and $E$, it can be represented as the resultant of the vectors $BF$ and $BE$ of Fig. 142. We have, therefore, merely to combine these vectors in order to find the current in Main $B$.

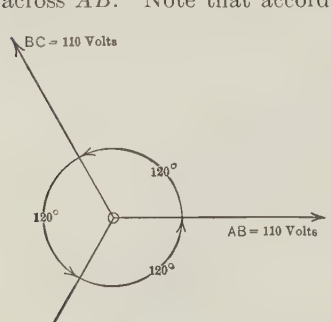



FIG. 141. Voltage relations in three-phase line of Fig. 140.

But note, in Fig. 141, that the positive direction across $AB$ is from wire $A$ to wire $B$. Thus the positive direction of the current in Group II is from wire $A$ to wire $B$, and, therefore, the positive direction of the current flowing into the wire $B$ from Group II would be toward the switch and could be indicated by the arrowhead $x$ in Fig. 140.

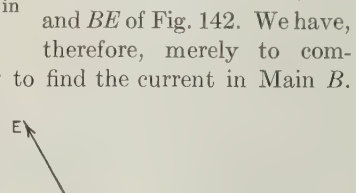



FIG. 142. Current from $A$ to $B$ in Group II, Fig. 140, is 120° out of phase with current from $B$ to $C$ in Group I.

Similarly, the positive direction of the voltage across $BC$ is from wire $B$ to wire $C$. Thus the positive direction of the current in Group I is from wire $B$ to wire $C$, and, therefore,

the positive direction of that component of the line current in $B$ which feeds Group I must be away from the switch as represented by the arrowhead $y$ in Fig. 140. Note from these arrowheads $x$ and $y$, Fig. 140, that the two components of the current in Main $B$ oppose each other. Thus if we consider the arrowhead $y$ as representing the positive direction of the current in Main $B$ then the arrowhead $x$ represents the direction of a reverse current, and vice versa. The result is the same, whichever we consider as the positive direction.

For convenience we will consider the direction indicated by $x$ as the positive direction. Therefore, the current of 10 amperes flowing from Group II into Main $B$ is in the positive direction of the current in the Main $B$ or in the same direction as the positive direction of the voltage from $A$ to $B$. Thus the vector $BF$, Fig. 142, represents that part of the current flowing in Main $B$ which is supplied from Group II.

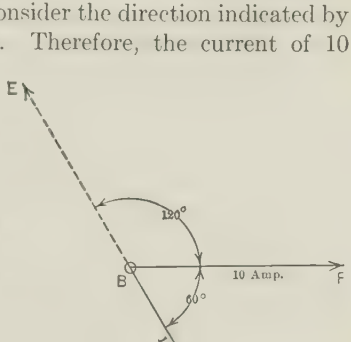



Fig. 143. Main $B$, Fig. 140, carries vector sum of currents $A$ to $B$, and $C$ to $D$. But current $C$ to $B$ is reverse of current $B$ to $C$. Compare Fig. 142.

But since the part of the current flowing from Main $B$ into Group I is in the direction opposite to the positive direction ($x$) of the current in Main $B$, therefore, the vector $BE$, Fig. 142, must be reversed and become $BE_1$ to represent the reverse direction ($y$) of that part of the current in Main $B$. The vector diagram of the parts of the current flowing in Main $B$ is then represented by Fig. 143, in which vector $BF$ represents the

10 amperes due to Group II in the positive direction, and the vector $BE_1$ represents the 10 amperes due to Group I leading the current $BF$ from Group II, by 120°, **but in the reverse direction.**

Note that in determining the method of constructing the current diagram for Main $B$, we

**First:** Decide upon the positive direction of the voltage across the groups feeding Main $B$.

**Second:** Make the positive direction of the currents in the groups the same as the positive directions of the voltages across the groups.

**Third:** Determine from the above the positive directions in Main $B$ for each of the currents flowing into Main $B$.

**Fourth:** Choose **one** of these as **the** positive direction for **all** currents in the Main $B$, and reverse the vector of any current which feeds into Main $B$ and has a positive direction opposite to that chosen for Main $B$.

In order to determine the current flowing in Main $B$, we have now only to solve the vector diagram of Fig. 143, for which we proceed as follows:

Construct Fig. 144, in order to find the two components of the current $BE_1$ in Group I, which are respectively in phase and in quadrature with the current $BF$ in Group II.

$BE_1$ represents the current of 10 amperes flowing from the line into Group I.

$KE_1$ represents the quadrature component of this current.*

$BK$ represents the in-phase component.

Power factor corresponding to angle of 60° = 0.500.

Reactive factor corresponding to angle of 60° = 0.866.

* Since the current in Group I is in phase with the voltage across Group I, it has no quadrature component with regard to the voltage across Group I. But since the current in Group I is out of phase with the current in Group II, it must have a quadrature component with respect to Group II. This component, represented by the vector $KE_1$, is the quadrature component of the current in Group I *with respect to the current in Group II.*

Quadrature component = indicated current × reactive factor.

$$KE_1 = BE_1 \times 0.866$$
$$= 10 \times 0.866$$
$$= 8.66 \text{ amperes.}$$

In-phase component = indicated current × power factor

$$BK = BE_1 \times 0.500$$
$$= 10 \times 0.500$$
$$= 5.00 \text{ amperes.}$$

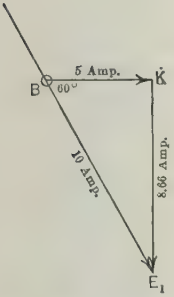



FIG. 144. Components of load I, Fig. 140, are 5 amperes in phase with II and 8.66 amperes at 90° to II.

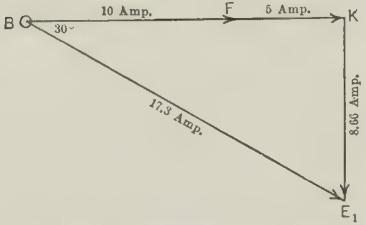



FIG. 145. Total current in Main *B*, Fig. 140, composed of currents in I and II, equals 1.73 times current in each phase when load is balanced.

Construct Fig. 145, in which $BK$ equals the combined in-phase components of the currents in Groups II and I (that is, in phase with the current $x$ of Group II which is chosen for reference).

$BF$ = in-phase component of Group II = 10 amperes.
$FK$ = in-phase component of Group I = 5 amperes.
$BK = 10 + 5$
$= 15$ amperes.

$KE_1$ represents the sum of the quadrature components of Groups II and I.

Quadrature component of Group II = 0.
Quadrature component of Group I = 8.66.

$$KE_1 = 0 + 8.66 = 8.66 \text{ amperes.}$$

$BE_1$ represents the total current in Main $B$, being a combination of the currents in Groups II and I.

$$BE_1 = \sqrt{BK^2 + KE_1^2}$$
$$= \sqrt{15^2 + 8.66^2}$$
$$= 17.3 \text{ amperes.}$$

Thus the line wire $B$ carries a current of 17.3 amperes, made up of the currents of 10 amperes from Group I and 10 amperes from Group II, the current in Group I leading the current in Group II by 120°, but in the reverse direction within Main $B$.

In the same manner the currents in Main $A$ and Main $C$ can be determined. It will be found that each main is carrying 17.3 amperes. In fact, since the system is balanced, we know that the current in all mains must be the same.

**66. Relation of Current in Mains to Current in Groups.** We have seen that when the current in each of the three groups of appliances on a balanced three-wire three-phase line is 10 amperes, a current of 17.3 amperes flows in each of the mains. In other words, $\frac{17.3}{10}$, or 1.73 times as much current flows in the mains as in each of the groups. This is always the relation of the current in each of the mains to the current in each group on a balanced three-wire three-phase system. It holds true regardless of what may be the power factor of the current in the three groups, inasmuch as the power factors, voltages and currents of all the groups on a balanced system must be the same, in order to maintain a true balance.

**Rule. To find the current in each main of a balanced three-wire, three-phase system, multiply the current in each phase or group by 1.73.**

**Example.** An auditorium is supplied with two hundred forty 100-watt lamps, on a three-wire three-phase system of 110 volts. How much current flows in the mains?

**Solution.**

The lamps would be divided into three groups of 80 lamps each.

$$\text{Current in each lamp} = \tfrac{100}{110} = 0.909 \text{ amperes.}$$

$$\text{Current taken by each group} = 80 \times 0.909 = 72.7 \text{ amperes.}$$

$$\text{Current in Main} = 72.7 \times 1.73 = 125.8 \text{ amperes.}$$

**Prob. 13–7.** A building is to be supplied with three-phase current for four hundred and fifty 150-watt 115-volt lamps. If the lamps are so grouped that the building load is balanced, how much current must each main carry?

**Prob. 14–7.** If the lamps in Prob. 13 were supplied with single-phase current, how much current would each main carry?

**Prob. 15–7.** What current would each main in the building of Prob. 13 carry if it were supplied with a four-wire two-phase system?

**Prob. 16–7.** Assume that the lamp groups of Fig. 140 are replaced by three 110-volt single-phase induction motors, each taking 2 kw at 70.7 per cent power factor. What current must each main carry in this case?

**Prob. 17–7.** Three single-phase transformers are put on a three-wire three-phase 2300-volt line so as to balance the system. Each transformer is delivering 60 kw at 90 per cent power factor. What current must the mains carry?

**Prob. 18–7.** A 2300-volt three-wire three-phase line is loaded with eighteen 20-kw single-phase distributing transformers operating at 94 per cent power factor. If line is balanced what current must be supplied to each main line wire, with all transformers fully loaded?

**Prob. 19–7.** What current would flow in mains of Prob. 18 if only 15 of the transformers were in operation at a single time, and these at a power factor of 82 per cent? The line is still balanced.

**67. Power in a Balanced Three-wire Three-phase System at Unity Power Factor.** Refer again to the balanced three-wire three-phase system of Fig. 140. Each

group of lamps receives $10 \times 110 \times 1$, or 1100 watts from the line, since the power factor of the lamps is unity. As there are three groups:

$$\text{Total power received from line} = 3 \times 1100$$
$$= 3300 \text{ watts.}$$

We know that the main line current is 17.3 amperes. If we multiply this current by the voltage between line wires we obtain only $17.3 \times 110$, or 1903 watts. Thus it is evident that the product of the line current times the line voltage does not equal the total power carried by the line. In fact the power carried by the line, 3300 watts, is $\frac{3300}{1903}$, or 1.73 times as great as the product of the line amperes times the line volts. Hence the rule:

**To find the total power carried at unity power factor by a balanced three-wire three-phase system, multiply by 1.73 the product of the line current times the line voltage.**

**Example.** How much power can be transmitted at 220 volts by a balanced three-wire three-phase system at unity power factor if each wire can carry safely 25 amperes?

**Solution.** Power, at unity power factor, in a balanced three-wire three-phase system equals 1.73 × line amperes × line current, or

$$1.73 \times 25 \times 220 = 9520 \text{ watts.}$$

**Prob. 20–7.** How much power can be distributed by a balanced three-wire three-phase system at 115 volts, unity power factor, if each wire can carry 55 amperes?

**Prob. 21–7.** How many 60-watt 115-volt lamps can be put on a balanced three-wire three-phase 115-volt distributing line if each wire can carry 15 amperes?

**Prob. 22–7.** A three-wire three-phase 230-volt unity-power-factor motor requires 1 kw to operate it. How much current must each lead to the motor carry?

**68. Power in a Balanced Three-wire Three-phase System at any Power Factor.** Load the line of Fig. 140 with three induction motors each operating at 80 per cent power factor and drawing 2 kw from the line as shown in

Fig. 146. We know that the power carried by the line must equal $3 \times 2$, or 6 kw, since there are three motors each taking 2 kw. If we wish to compute the power carried over the line by means of the current in the line we must proceed as follows:

$$\text{Apparent power taken by each motor} = \frac{2000}{0.80}$$
$$= 2500 \text{ volt-amperes.}$$
$$\text{Current taken by each motor} = \frac{2500}{110}$$
$$= 22.7 \text{ amperes.}$$
$$\text{Current in each main} = 22.7 \times 1.73$$
$$= 39.4 \text{ amperes.}$$

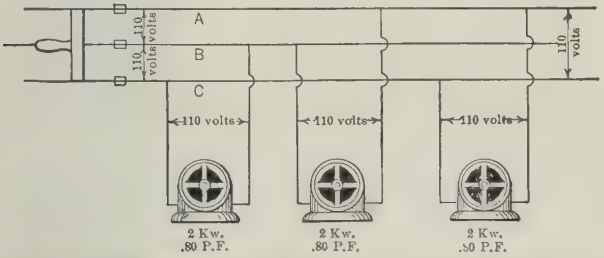



Fig. 146. Balanced delta-connected load of 80 per cent power factor on three-phase three-wire mains.

Knowing the current in the line to be 39.4 amperes, if the power factor of the load were unity, we could say that the power carried by the line would be $1.73 \times 39.4 \times 110 = 7500$ watts. But we have seen that the true power carried by the line is $2 \times 3$, or 6 kw, or 6000 watts, which is only $\frac{6000}{7500}$, or 0.80 of the product.

Note that 0.80 is also the power factor of the motors. Thus, in order to obtain the total power carried by the line it is necessary to multiply the product of the line current (39.4 amperes) times the line voltage (110 volts) not only by 1.73 but also by the power factor of the load (0.80). In

other words, the product (1.73 × line current × line voltage) is only the apparent power carried by the line, and it must always be multiplied by the power factor of the load in order to obtain the effective or real power.

**The rule for power in a balanced three-wire three-phase circuit is:**

**Total Power**
**= 1.73 × line current × line voltage × power factor of load.**

**Example.** What power is carried by a balanced three-wire three-phase 110-volt line each wire of which carries 20 amperes? The power factor of the load is 90 per cent.

**Solution.**

$$\begin{aligned}\text{Power} &= 1.73 \times \text{line current} \times \text{line voltage} \times \text{power factor} \\ &= 1.73 \times 20 \times 110 \times 0.90 \\ &= 3425 \text{ watts.}\end{aligned}$$

**Prob. 23–7.** How much current flows in a balanced three-wire three-phase system delivering 35 kw at 92 per cent power factor and 230 volts?

**Prob. 24–7.** A three-wire three-phase line can carry 33 amperes in each wire. At what voltage must it operate in order to carry 22 kw at 87 per cent power factor if the load is balanced?

**Prob. 25–7.** A balanced three-wire three-phase line carries 12.5 amperes in each line wire at 550 volts. If the power delivered is 10 kw, what is the power factor of the load?

**Prob. 26–7.** A balanced three-wire three-phase line delivers power to nine three-phase motors, each taking 1250 volt-amperes at 230 volts and 78 per cent power factor.

(*a*) What current flows in the line?
(*b*) What power does the line carry?

**Prob. 27–7.** If the motors of Prob. 26 were single-phase motors and operated on a single-phase line:

(*a*) How much current would each line wire carry?
(*b*) How much power would the line carry?

**Prob. 28–7.** Solve Prob. 26 on the assumption that the motors are two-phase four-wire motors operating on a four-wire two-phase line.

**69. Voltage Relations in Three-phase Four-wire Systems.** The four-wire three-phase system is sometimes used in order to obtain two voltages. The voltages produced by this system can be seen clearly from Fig. 147 which is the same as Fig. 133, except that a " neutral " wire is brought out from the point in the armature where the three windings are joined to make the star-connection.

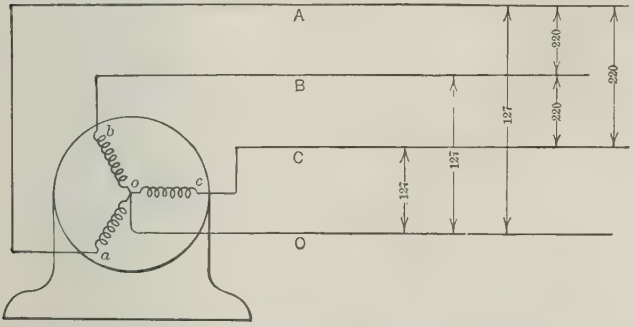



FIG. 147. Four-wire system gives two distinct sets of three-phase voltages, one set being 1.73 times the other set and 30° out of phase with it. Compare Fig. 133 to 136.

Thus, if there are 127 volts across each armature phase, the voltage from the neutral to either line wire $A$, $B$ or $C$ would be 127 volts, because each armature winding is placed between one main wire and the neutral. The voltage between the mains $A$, $B$ and $C$, as we have seen, would be $1.73 \times 127$, or 220 volts. Thus we have available not only the three-phase pressure of 220 volts for motors, but also 127 volts for lamps.

**70. Current in the Mains of a Balanced Four-wire Three-phase Line.**

($a$) **Loads having the same power factor.** Consider the four-wire three-phase line of Fig. 148, in which the voltage between the mains $A$, $B$ and $C$ is 208 volts, and between the

neutral $O$ and any one of the Mains $A$, $B$ and $C$ is $\frac{208}{1.73}$, or 120 volts.

We shall first take up the case in which the three-phase motor $M$ as well as the lamps has unity power factor. The current in each motor coil is 10 amperes as marked. Each main must, therefore, carry $10 \times 1.73$, or 17.3 amperes to feed the motor.

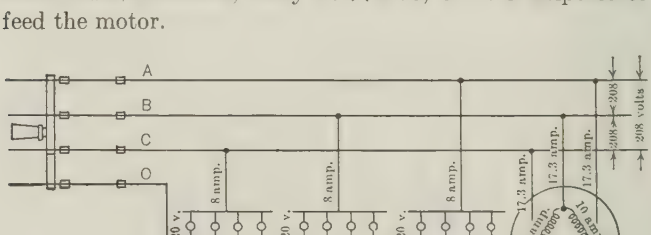



FIG. 148. Four-wire three-phase system. Each line wire carries vector resultant of the currents in all load taps connected to it. See Fig. 149 and 150.

Each group of lamps takes a current of 8 amperes. This current in each case is in phase with the main line current and has merely to be added to the 17.3 amperes carried by the line for the motor supply, since the power factor of the motor load and lamp loads is exactly the same.

The main line current thus equals $17.3 + 8$, or 25.3 amperes.

Since the system is balanced, the neutral carries no current, just as the neutral in a balanced single-phase or direct-current three-wire system carries no current.

This is explained as follows:

Since the current in the neutral wire $O$ is the resultant of the currents flowing between $O$ and $A$, $O$ and $B$, and $O$ and $C$, each current differing in phase with either of the other two

by 120°, we may draw the vector diagram, Fig. 149. Vector $OA$ represents the 8 amperes flowing in Group III, lagging 120° behind vector $OB$ representing the 8 amperes flowing in Group II, which in turn is 120° behind the vector $OC$ representing the 8 amperes flowing in Group I.

We have seen from Fig. 131 that when two equal voltages ($A_1A_2$ and $B_1B_2$) have an angle of 120° between them, their sum or resultant voltage ($A_1B_2$) is equal in value to either of

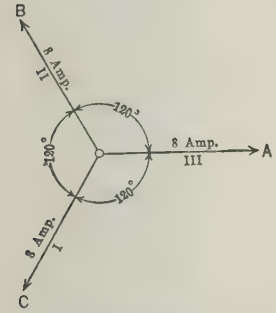



FIG. 149. Represents currents in balanced star-connected loads I, II, III of Fig. 148.

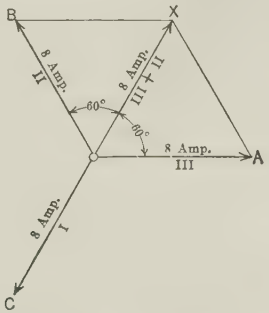



FIG. 150. Neutral Main $O$ of Fig. 148 carries current represented by vector $OA$ between III and II, $OX$ between II and I, zero current between I and generator.

the two voltages and has a phase difference of 60° with each of the two voltages. Similarly, the resultant of any two currents added to each other at an angle of 120° is equal in value to either of the two currents and has a phase difference of 60° to each of the two component currents.

Thus the resultant of the currents $OA$ and $OB$ can be represented by the vector $OX$, Fig. 150. This vector will equal 8 amperes and be at an angle of 60° to both $OA$ and $OB$.

But note that this vector $OX$ representing combined

currents in III and II is equal to the vector *OC* representing current in I and is in the direction exactly opposite to it. Thus the combined currents fed to the neutral for any two groups exactly neutralizes the current fed to the neutral from the third group and no current flows over the neutral to the generator when the system is balanced. Of course if the load consists of single-phase groups separated from one another as in Fig. 148, the current *OA* (Fig. 150) will flow in the neutral wire between III and II, current *OX* between II and I, and zero current between I and the generator.

**Prob. 29–7.** If the motor in Fig. 148 were a unity-power-factor motor requiring 1.5 kw and there were 60 lamps of 150 watts each, so connected to the neutral as to balance the system, what current would flow in the neutral? Construct diagram similar to Fig. 148 to show arrangement of lamps and neutral using one lamp to represent a group. State number of lamps this representative lamp stands for.

**Prob. 30–7.** A four-wire three-phase system is to be used to operate eight three-phase 230-volt motors operated at unity power factor, and six hundred 75-watt lamps. Each motor takes 4.2 kw.

(*a*) For what voltage should the lamps be ordered, if connected between lines and neutral?

(*b*) What current flows in each main if system is balanced?

(*c*) Show by diagram similar to Fig. 148 how the load should be distributed.

(*b*) **Loads of different power factor.** Let us replace the unity-power-factor three-phase motor in Fig. 148 with a three-phase motor carrying the same current of 10 amperes in the motor coils but at a power factor of 80 per cent lagging, and leave the same lamp load connected to the line. When the motor takes a balanced load of unity power factor, the 10 amperes in each coil of the motor is in phase with the voltage across that coil or across the line wires between which it is connected. Also the 17.3 amperes in the lead connecting each motor terminal to a line wire is in phase with the voltage

between that line wire and the neutral wire, just as the current in the lead from each line wire to one of the star-connected loads I, II, III is in phase with the voltage from that line wire to the neutral $O$, when the loads I, II, III are non-inductive. When the power factor of $M$ is 80 per cent lagging, the 10 amperes in each coil lags 37° behind the voltage across that coil or between line wires, and the 17.3 amperes in each motor lead lags 37° behind the voltage between that lead or line wire and neutral. Therefore, if the loads I, II, III are non-inductive, and $M$ operates at 80 per cent power factor

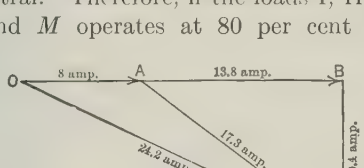



FIG. 151. Currents in all taps to same main in Fig. 148 are represented in relation to same voltage (in this case, from main to neutral), and then added vectorially.

lagging, it follows that the 17.3 amperes in each motor lead lags 37° behind the 8 amperes in each lead to I, II, III. The resulting current to the mains would, therefore, be the combination of a current of 8 amperes with one of 17.3 amperes lagging 37°. To find the value of this resulting current, construct Fig. 151. Vector $OA$ represents the 8 amperes carried by one main to supply the lamps. Vector $AC$ drawn lagging 37° behind $OA$ represents the 17.3 amperes carried by the same main to supply the motor. Vector $AB$ represents the in-phase component of $AC$, that is, in phase with $OA$. Vector $BC$ represents the quadrature component of $AC$ (that is, in quadrature with $OA$) and lags 90° behind the in-phase component $AB$. The vector $OC$ represents the vector sum or resultant of the in-phase components $OA$ and

$AB$, and the quadrature component $BC$. $OA$, being used as a basis of reference, has no quadrature component.

$$\begin{aligned} OC &= \sqrt{OB^2 + BC^2} \\ &= \sqrt{21.8^2 + 10.4^2} \\ &= 24.2 \text{ amp.} \end{aligned}$$

The current flowing in each main, therefore, equals 24.2 amperes when the power factor of the motor is 80 per cent.

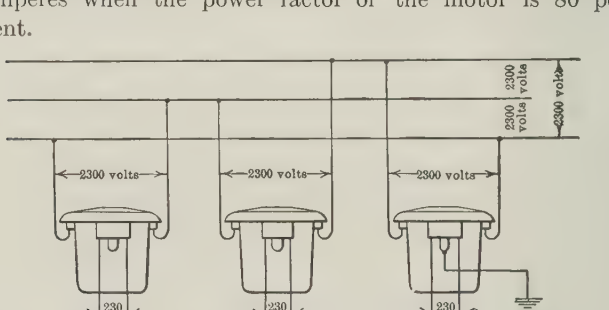



FIG. 152. Delta-delta connection of single-phase transformers on three-phase system.

**Prob. 31–7.** If the power factor of the motor in Fig. 148 were 70 per cent lagging, what current would flow in the mains?

**Prob. 32–7.** What is the power factor of the total load supplied in Prob. 31?

**Prob. 33–7.** Two three-wire three-phase 220-volt motors are attached to a four-wire three-phase line. The first motor takes 10 kw at 70 per cent lagging power factor; the second, 15 kw at 85 per cent lagging power factor. What current flows in each main?

**Prob. 34–7.** If ninety 150-watt 127-volt lamps are attached to the line of Prob. 33 in such a way as to balance the system, what current would flow in the mains?

**71. Transformer Connections in Three-phase Systems.** In connecting distributing transformers to a high-voltage three-wire three-phase line, a single three-phase transformer may be used or three single-phase transformers. In either case several schemes of connection are possible. Fig. 152 and 153 show the primary windings of three single-phase transformers connected in delta to the 2300-volt line and the secondary windings connected in delta to a 230-volt three-wire three-phase distributing system. This is called a **delta-delta** connection, since both the primary and secondary

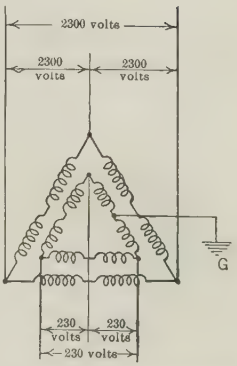



FIG. 153. Electrical connections of Fig. 152. Primary and secondary of each transformer consists of two halves in series.

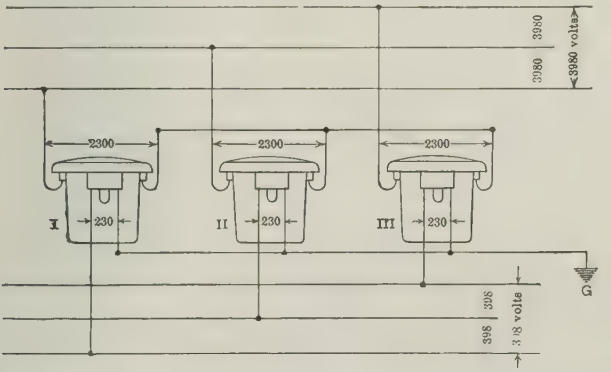



FIG. 154. Star-star connection of single-phase transformers on three-phase system.

sides of the transformers are connected in delta to their respective circuits.

Note that the neutral point of one of the secondary windings is generally grounded. The transformer cases should also be grounded.

Fig. 154 and 155 show a **star-star** connection for the same transformers, both the primaries and secondaries being star-connected to their respective circuits. Note that the neutral point of the low-tension side is generally grounded, and also that the voltage of both circuits will be 1.73 times what it was for the same transformers delta-connected.

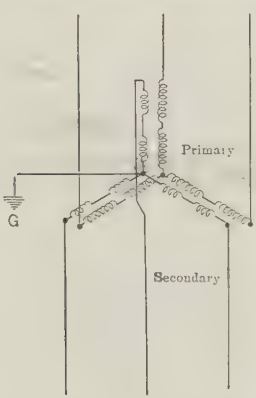



FIG. 155. Conventional diagram of electrical connections for Fig. 154.

**Prob. 35–7.** Construct diagrams similar to Fig. 152, 153, 154 and 155 for these transformers delta-star connected. Connect the primaries in delta, marking voltages between lines and putting in proper ground connection.

**Prob. 36–7.** Construct diagrams similar to Fig. 152, 153, 154 and 155 for these transformers star-delta connected.

**72. Measurement of Power and Power Factor in a Balanced Three-wire Three-phase Circuit.** The power in a balanced three-wire three-phase circuit can be measured by attaching two wattmeters as in Fig. 156. Note that the wattmeter $W_1$ to read the power taken by the motor $M$ has its current coil connected in one lead, in this case, $C$, and the voltage coil is between lead $C$ and lead $B$.

The current coil of wattmeter $W_2$ must be placed in line $A$, so that its voltage terminals may be attached to line $A$ and line $B$. The algebraic sum of these two wattmeter readings is the effective three-phase power taken by the motor.*

* The two wattmeters should both be connected in the same manner to each line; that is, if the + or right-hand side of the current and the

The apparent power is found by connecting the voltmeter $V$ and the ammeter $A$ as in Fig. 156 and multiplying the product of their indications by 1.73. If the system is only

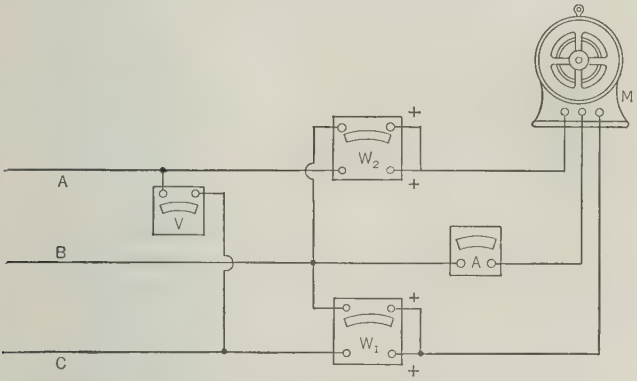



FIG. 156. Total power carried by any three-phase system, balanced or unbalanced, at any power factor, may be measured correctly by two wattmeter readings as here indicated.

slightly out of balance the average readings of an ammeter placed in each lead successively may be taken as the current, and the average voltage between leads as the pressure. The power factor may then be found from the equation

$$\text{Power factor} = \frac{\text{Effective power}}{\text{Apparent power}}$$

$$= \frac{\text{Sum of wattmeter readings}}{1.73 \times \text{line volts} \times \text{line amperes}}.$$

voltage terminals of $W_1$ are connected to the motor side of the line, then both the current and voltage + or right-hand terminals of $W_2$ should be connected to the motor side of the line. If both instruments now indicate properly, add the two indications. If one wattmeter tends to indicate negative value, reverse either the current or the voltage connections and subtract the reading of one wattmeter from the reading of the other.

**Prob. 37–7.** Two wattmeters connected like $W_1$ and $W_2$ in Fig. 156 indicate 3250 and 1620 watts, respectively, the voltmeter, 440 volts, and the ammeter, 7.3 amperes.

(*a*) What power does the motor take?

(*b*) What is the power factor of the motor?

**Prob. 38–7.** A wattmeter connected like $W_1$ in Fig. 156 indicated 1.2 kw. Another wattmeter connected as $W_2$ indicated 2 kw. The average reading of an ammeter when placed in the three leads was 8.6 amperes. The average voltage between leads was 230.

(*a*) What power was taken by the motor?

(*b*) What was the power factor of the motor?

## SUMMARY OF CHAPTER VII

In a SINGLE-PHASE SYSTEM the alternating voltage reaches its maximum value or its zero value at the same instant in all parts of the system. A single-phase system may be two-wire or three-wire, like the corresponding direct-current systems.

In a POLYPHASE SYSTEM the alternating voltage reaches its maximum value or its zero value at different instants in various parts of the system. The polyphase systems most commonly used are THREE-PHASE and TWO-PHASE.

A THREE-PHASE SYSTEM usually has three wires, say A, B, C. Loads are connected from A to B, from B to C, and from C to A (DELTA CONNECTION, represented by the symbol $\Delta$); or, one end of each of the three loads is connected to one of the wires A or B or C, and the other three load terminals are connected to a common point known as the "neutral" (STAR CONNECTION, represented by the symbol Y). In a four-wire three-phase system the neutral points (N, or O) of all the three-phase loads are connected to each other and to the neutral point of the three-phase winding of the generator, by the fourth wire. The three-wire system is most usual.

In any three-phase system there is a phase difference of 120°, representing the time required to pass through one-third of a cycle, between the voltages A to B, B to C, and C to A, each to each; or between the voltages N to A, N to B, and N to C, each to each. The former are called the DELTA VOLTAGES, and the latter are called the WYE VOLTAGES. In a balanced three-phase system, the voltage between any pair of lines is equal to 1.73 times the voltage between any line and neutral.

A TWO-PHASE SYSTEM usually has four wires, say $A_1$ and $A_2$, $B_1$ and $B_2$. Loads are connected from $A_1$ to $A_2$, and from $B_1$ to $B_2$. The voltage from $A_1$ to $A_2$ is 90° out of phase with the voltage from $B_1$ to $B_2$. These two voltages should have equal numerical value. Sometimes, but rarely, two of the wires are combined into one, forming a three-wire two-phase system.

A polyphase system is BALANCED when all three wires of the three-phase, or all four wires of the two-phase system, carry equal amounts of current.

Generators for single-phase, two-phase or three-phase do not differ from one another essentially, except as to the manner in which the coils in the armature winding are connected together and to the terminals.

Any given three-phase generator may be easily changed from $\Delta$ to Y connection without disturbing the coils; in $\Delta$ it can deliver 1.73 times as much current per terminal but at only $\frac{1}{1.73}$ times as much voltage between terminals as in Y. The power capacity is the same in either case.

TOTAL OR RESULTANT CURRENT flowing in any given main of a three-phase system, to supply several loads connected to this main is found as follows:

1°. From power factor of each load, find phase angle between its current and the voltage which produces it.

2°. From known phase relations between the $\Delta$ and Y voltages, find phase angle between each individual load current and some one voltage, say the voltage from that main to neutral. Thus, by comparison, we arrive at the phase relations of all individual load currents to one another.

3°. Draw vector diagram showing individual load currents that flow from same main, in their proper phase relation to one another.

4°. Select one vector as base and, knowing all the angles, resolve each vector into two components, one in phase with the base vector and one in quadrature (at right angles) with it. By algebraic additions find in-phase component and quadrature component of total or resultant current in the main.

5°. Amperes in main is equal to square root of sum of squares of total in-phase component and total quadrature component.

The CURRENT IN EACH WIRE of a balanced three-phase three-wire system is equal to the current in each phase of the load if the loads are Y-connected, or is equal to $\sqrt{3}$ (or 1.73)

times the current in each phase of the load if the loads are Δ-connected. In a balanced, three-phase four-wire system, the neutral wire carries no current between the generator and the load, but may carry current between the loads if they are not all located at the same point.

TOTAL POWER IN ANY POLYPHASE SYSTEM, two-phase or three-phase, balanced or unbalanced, is equal to the sum of products obtained by multiplying amperes in each load by volts across that load by power factor of that load.

If a three-phase system is BALANCED, the total current and the total power factor in each phase are the same, and the calculation of total power for the entire system becomes simplified as follows:

Total power in balanced three-phase system

$= 1.73 \times$ amperes per line wire $\times$ volts between line wires $\times$ power factor of load.

POWER FACTOR OF A THREE-PHASE SYSTEM is equal to the ratio of the sum of watts in all loads of all phases, to the sum of indicated volt-amperes in all loads of all phases. This power factor has no significance unless the system is balanced, when it is equal to the power factor of each of the three phases considered as a single-phase circuit.

TOTAL POWER in a three-phase system is usually MEASURED by means of TWO WATTMETERS (or by a " polyphase wattmeter " which is really two wattmeters in the same instrument). One of these wattmeters has its current coil connected in series with line wire A and its voltage coil connected from line wire A to line wire B. The other wattmeter must be connected in EXACTLY similar or symmetrical fashion, with its current coil in wire C, and its voltage coil from C to B. The algebraic sum of simultaneous readings on these two wattmeters is equal to the total power, for all conditions of load. Unless the load is balanced and has unity power factor, the two readings will be unequal; in fact, one of them may be negative, necessitating reversal of connections for one of the wattmeter coils in order to make the reading, in which case the algebraic sum of readings becomes an arithmetical difference.

## PROBLEMS ON CHAPTER VII

**Prob. 39 7.** If the induction motors of Prob. 16 were added to the lamp load of Fig. 140 (one motor to each lamp group and each

lamp group taking 10 amperes), how much current would each of the mains receive from the station?

**Prob. 40–7.** A 2300-volt three-wire three-phase transmission line has 48 single-phase distributing transformers supplying 1 kw each at 88 per cent power factor, and 18 single-phase transformers each supplying 2 kw at 95 per cent power factor. If the line is balanced, what current must the power station supply to each main?

**Prob. 41–7.** If the load of Prob. 39 were all on a single-phase line at the same voltage between wires, how much current would be supplied to each line wire?

**Prob. 42–7.** If the total load of Prob. 39 were supplied by a four-wire two-phase line, how much current would be carried by each line wire? Assume the load to be balanced.

**Prob. 43–7.** What current would have to be supplied to each wire of a single-phase line to supply the transformers of Prob. 40?

**Prob. 44–7.** What current would have to be supplied to each wire of a four-wire two-phase line to supply the transformers of Prob. 40, if the load were balanced?

**Prob. 45–7.** Each main of a three-wire three-phase 115-volt distributing system can carry 30 amperes. How many 60-watt 115-volt lamps can be attached to this system if properly balanced?

**Prob. 46–7.** If the system of Prob. 45 were a four-wire two-phase 115-volt system, how many 60-watt lamps could be attached, assuming the system properly balanced?

**Prob. 47–7.** How many 60-watt 115-volt lamps could be attached to a single-phase 115-volt system, using the same size wires as in Prob. 45?

**Prob. 48–7.** How many 60-watt lamps could be used on the distributing system of Prob. 45, if it were used as a three-wire single-phase line and the load were properly balanced?

**Prob. 49–7.** In Fig. 157 each lamp takes 1.2 amperes. The three-phase motor takes 12.4 amperes at 78 per cent power factor lagging. Find:

(*a*) Current delivered to each line wire at switch.

(*b*) Power factor of total load on line.

(*c*) Total power delivered at switch.

**Prob. 50–7.** If the three lamp groups of Fig. 157 were replaced by one three-phase motor taking 4.8 amperes per lead and operating

at 90 per cent leading power factor, what would be the answers to the three parts of Prob. 49?

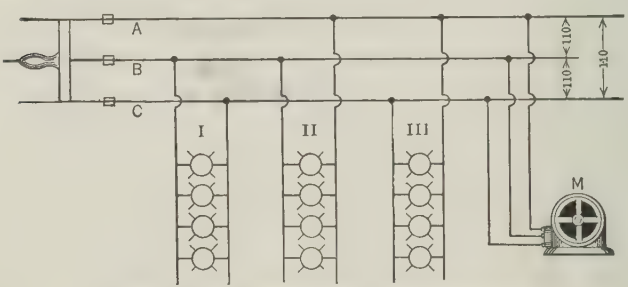



FIG. 157. Balanced delta load on three-phase line.

**Prob. 51–7.** How much power does the motor in Fig. 148 draw from the line?

**Prob. 52–7.** What total power does the line in Fig. 148 deliver?

**Prob. 53–7.** If the main switch in Prob. 24 is fed by three single-phase transformers, what rating must each transformer have?

**Prob. 54–7.** Connect the three transformers of Fig. 152 or 154 in star-star to a three-phase three-wire 2300-volt line. (*a*) What will be the voltage on the low-tension side of each transformer? (*b*) What will be the voltage between wires of the low-tension distributing line?

**Prob. 55–7.** If each transformer in Fig. 152 is rated at 10 kv-a, what current can be supplied to the secondary line without exceeding the rating?

**Prob. 56–7.** If each transformer of Fig. 154 is rated at 10 kv-a, what current can be supplied to the secondary line without exceeding the rating?

**Prob. 57–7.** What power is carried by a balanced three-phase three-wire line which carries 52 amperes at 550 volts if the power factor of the load is 75 per cent?

**Prob. 58–7.** By aid of vector diagrams prove that the voltages $A$ to $B$, $B$ to $C$, and $C$ to $A$ must be 120° apart in phase if each of these voltages has a value of 110 volts, or in general if the three line voltages are equal.

**Prob. 59–7.** Three single-phase transformers, each with its two low-tension coils connected in series, are connected in delta to a 230-volt low-tension three-phase three-wire distributing system. If another three-wire system be connected to the mid-points of the three transformer windings, will this also be a three-phase system? If so, what will be the voltage between any two line wires?

**Prob. 60–7.** What must be the voltage ratio of each of three single-phase transformers connected in star-delta in order that they may take power from a 13,200-volt three-phase transmission line and deliver it to a three-phase three-ring rotary converter with 384 volts between rings or wires?

**Prob. 61–7.** If the transformers of Prob. 60 are to deliver altogether 1600 kv-a at 82.5 per cent power factor, how many amperes are taken from each high-tension line wire and how many amperes are delivered by each lead to a ring on the converter?

**Prob. 62–7.** In attempting to make the star-star connections shown in Fig. 154 and 155, the wireman got the low-tension connections of transformer II reversed. The three voltages between secondary mains are no longer equal; what are they?

**Prob. 63–7.** In attempting to make the delta-delta connections shown in Fig. 152 and 153, the wireman got the low-tension terminals of one of the transformers reversed. What effect would this have when the connections are completed? Illustrate by a vector diagram, after the manner of Fig. 132.

**Prob. 64–7.** A three-phase three-wire transmission line with 66,000 volts between wires, supplies a three-phase synchronous motor through step-down transformers with star-connected primaries and delta secondaries. The motor delivers 1500 horse power at 92 per cent efficiency, and takes power at 94 per cent power factor. The transformer efficiency is 99.1 per cent. (*a*) What ratio is required in each transformer, if the motor requires 4600 volts between terminals? (*b*) What current is delivered to each motor terminal? (*c*) How many amperes are taken from each high-tension line wire?

**Prob. 65–7.** A water-cooled rheostat consisting of three similar coils of iron wire connected in delta is used as an artificial load for testing a three-phase generator. When so connected the rheostat takes altogether 180 kw. If the three sections were reconnected in star, how much power would the rheostat take from the generator at the same voltage between line wires?

**Prob. 66–7.** A balanced load of incandescent lights and three-phase induction motors is carried at the end of a three-wire feeder which takes 76 kw from the station switchboard with 25 amperes per wire and 2300 volts between wires. What is the power factor of this feeder?

**Prob. 67–7.** The distributing transformers on the feeder of Prob. 66 and the feeder itself require for excitation, altogether, 5.1 kv-a at 60 per cent power factor. How much power is consumed by the lamps and motors together, and at what power factor?

**Prob. 68 7.** The lamp load of Prob. 66 and 67 consists of one hundred 150-watt 115-volt lamps on each phase connected in a three-wire single-phase system, the transformers being connected in delta on the high-tension side. Draw a sketch of the connections. Calculate how much power must pass on along the feeder to the induction motors, and at what power factor.

**Prob. 69–7.** The motors of Prob. 66, 67, 68 are connected directly to the feeder and operate at 2300 volts. How many amperes must be delivered to them over each wire of the feeder?

**Prob. 70–7.** If 2.1 kw out of the 5.1 kv-a specified in Prob. 67 are consumed in the line wires themselves at unity power factor as heat on account of their resistance, how many kilovars are taken by the line and the transformers? How many kilowatts are lost in the transformers?

## CHAPTER VIII

## CALCULATION OF WIRE SIZES FOR VARIOUS DISTRIBUTING SYSTEMS

In calculating the size of wire to be used in alternating-current distributing systems we must first determine the current that the wire is to carry, and select a wire which will carry this current without over-heating. This selection can usually be made from the Underwriters' "Table of Allowable Current-carrying Capacity of Wires." See Table II in Appendix.

The wire chosen should then be checked up to see that the voltage used in overcoming the impedance of the line does not exceed the allowable amount, — usually between 3 per cent and 5 per cent of the voltage at the load when lamps form part of the load and 10 per cent when the entire load consists of motors, heating appliances, and the like.

**73. Single-phase Two-wire System with Lamp Load. Neglecting Voltage Drop.** The most common system of wiring for an average lamp load is the two-wire single-phase. Fig. 158 represents such a system for a two-story building.

**Example 1.** Panel B feeds four circuits each having twelve 40-watt lamps. Each branch therefore normally carries 480 watts. However, as there is always a likelihood that lamps of greater wattage may be used in some sockets, it is best to assume that each branch may be loaded to the maximum wattage ordinarily permitted for any branch circuit, which is 660 watts if the usual No. 14 wire (the smallest size permitted) is run for the branch circuits. This means that Main No. 2 must carry $4 \times 660$, or 2640 watts. For a line voltage of 110 volts, each wire must carry $\frac{2640}{110}$, or 24 amperes.

Referring to Table II in the Appendix, we see that for Main No. 2 we should use No. 10 rubber-covered wire, which will safely carry 25

amperes. If each of the branches from panel board A is rated to carry 660 watts, then Main No. 1 must carry $6 \times 660$, or 3960 watts. Each wire of Main No. 1 will therefore have to be large enough to carry $\frac{3960}{110}$, or 36 amperes. This will require a No. 6 rubber-covered wire, according to Table II in the Appendix.

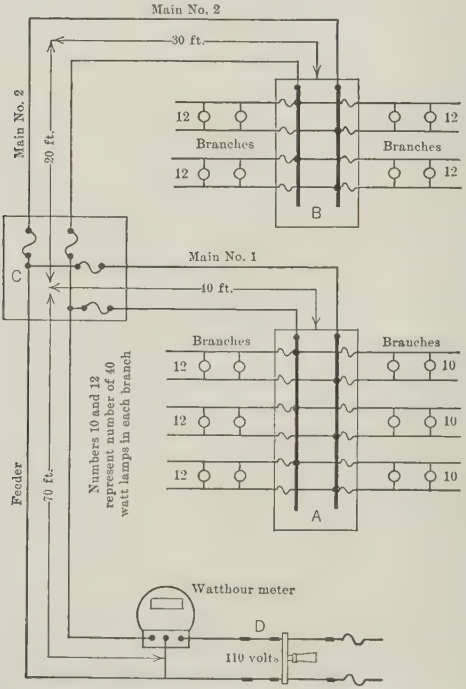



FIG. 158. Diagram of single-phase two-wire system for a two-story building; showing feeder, mains, and branches. Cut-outs or fuses of proper capacity must be installed in smaller wire wherever it joins a larger one.

The feeder (from the meter to the first floor cut-outs) must carry the current for Main No. 1, 36 amperes, and for Main No. 2, 24 amperes, or 60 amperes. This feeder must therefore be No. 4 rubber-covered wire, according to Table II of the Appendix.

**74. Voltage Drop in Single-phase Two-wire System.** The above determination of wire size for the different parts of the system does not take into consideration the drop in voltage along the line.

As we have seen, this drop should not exceed 5 per cent of the voltage at the load in the case of lamp loads; and this drop may be distributed to best advantage according to the following table adapted from Cook's "Interior Wiring."

TABLE V

VALUES OF MAXIMUM VOLTAGE-DROP ALLOWANCE FOR LOADS WHICH INCLUDE LAMPS.

| | In per cent | In voltage between wires for | | | |
|---|---|---|---|---|---|
| | | 110 volts. | 115 volts. | 120 volts. | 240 volts. |
| Branches | 1.5 | 1.65 | 1.72 | 1.8 | 3.6 |
| Mains | 1.0 | 1.10 | 1.15 | 1.2 | 2.4 |
| Feeders | 2.5 | 2.75 | 2.88 | 3.0 | 6.0 |
| Total | 5.0 | 5.50 | 5.75 | 6.0 | 12.0 |

From this table we see that we should not have more than 5.5 volts total drop from the feeder switch to the most remote lamp, in a 110-volt lighting system.

Let us first calculate the drop to a lamp in Group B, which is situated farthest from the service point D.

**Drop in branch.** — We will assume that the distance to the load center* of the longest branch in Group B does not exceed $54\frac{1}{2}$ ft, since this length produces the maximum allow-

* The load center is the point on the branch at which the lamps may be considered to be concentrated, for convenience in calculating. The drop to the farthest lamp will be practically the same as the drop to the load center. For method of determining the location of this point see Cook's "Interior Wiring" or Croft's "American Electricians' Handbook."

able voltage drop. We thus have 109 ft of line wire in the branch, outgoing and return.

The resistance of 1000 ft of No. 14 from Table III, Appendix, is 2.521 ohms.

$$\text{Resistance of 109 ft} = \tfrac{109}{1000} \times 2.521 = 0.275 \text{ ohm.}$$

The largest permissible current in each branch is $\frac{660}{110}$, or 6 amperes.

$$\text{Voltage drop to overcome resistance} = 6 \times 0.275 = 1.65 \text{ volts.}$$

This value, 1.65 volts, is allowed by Table V for the drop in branches on a 110-volt line.

**Drop in main.** Main No. 2 extends to the cutout C on the first floor, a distance of 30 ft + 20 ft, or 50 ft. This part of the circuit requires a No. 10 wire, 2 × 50, or 100 ft long.

By Table III in Appendix, 1000 ft of No. 10 copper wire has a resistance of 0.9972 ohm.

$$\text{Resistance of 100 ft} = \tfrac{100}{1000} \times 0.9972 = 0.0997 \text{ ohm.}$$

The current carried by Main No. 2 is the current of the four branches from the cutout box B, each of which may carry 6 amperes. The greatest current in Main No. 2, therefore, equals 4 × 6, or 24 amperes.

Voltage to overcome resistance of Main No. 2 equals

$$24 \times 0.0997 = 2.39 \text{ volts.}$$

This is more than double the 1.10 volts of Table V for the allowable drop in mains. But before deciding the wire is too small, it is well to see if the feeder drop will not be enough smaller than the 2.75 volts allowed, to make up the difference.

**Drop in feeder.** The feeder consists of $70 \times 2$, or 140 ft of No. 4 copper wire having a resistance according to Table III of 0.248 ohm per 1000 ft.

$$\begin{aligned} \text{Resistance of 140 ft} &= \tfrac{140}{1000} \text{ of } 0.248 \\ &= 0.0347 \text{ ohm.} \end{aligned}$$

As this may have to carry 60 amperes,

$$\begin{aligned} \text{Drop due to resistance} &= 60 \times 0.0347 \\ &= 2.08 \text{ volts.} \end{aligned}$$

This is somewhat under the 2.75 volts allowable.

The total voltage drop due to resistance in feeders, mains and branches out to the farthest lamp from box B, when all the lights are on, would be

$$\begin{aligned} \text{Drop in branch} &= 1.65 \\ \text{Drop in main} &= 2.39 \\ \text{Drop in feeders} &= 2.08 \\ \text{Total} &= 6.12 \text{ volts} \end{aligned}$$

This is somewhat over the amount (5.50 volts) allowed by Table V.

Moreover, we have not yet considered the drop due to the reactance of the line. Let us assume that the wires are installed "open" at the standard distance of 2.5 inches between centers of conductors (see National Electrical Code, Section 501 j).

By Table IV of the Appendix, we see that the 60-cycle reactance of the No. 14 wire spaced $2\frac{1}{2}$ inches as used in the branches is 0.1079 ohm per 1000 ft.

Consider average branch to be $54\frac{1}{2}$ ft long; that is, having 109 ft of wire.

Reactance of each branch is $\frac{109}{1000} \times 0.1079 = 0.01176$ ohm.

Maximum current in each branch is $\frac{660}{110} = 6$ amperes.

Maximum reactance drop is $6 \times 0.01176 = 0.0706$ volt.

This drop is too small to be considered and is usually neglected in practical computations.

The reactance per 1000 feet of No. 10 conductor spaced 2.5 inches is 0.0953 ohm, at the frequency of 60 cycles per second commonly used for lighting circuits.

$$\begin{aligned}\text{Reactance of Main No. 2} &= \tfrac{100}{1000} \text{ of } 0.0953 \\ &= 0.00953.\end{aligned}$$

As Main No. 2 carries 24 amperes, the voltage to overcome reactance

$$\begin{aligned}&= 24 \times 0.00953 \\ &= 0.229 \text{ volt.}\end{aligned}$$

The 60-cycle reactance of 1000 ft of No. 4 wire (stranded) with $2\frac{1}{2}$-inch spacing is 0.0764.

$$\begin{aligned}\text{Reactance of 140 ft} &= \tfrac{140}{1000} \times 0.0764 \\ &= 0.0107 \text{ ohm.} \\ \text{Reactance drop in feeders} &= 0.0107 \times 60 \\ &= 0.642 \text{ volt.} \\ \text{Total reactance drop} &= 0.64 + 0.23 \\ &= 0.87 \text{ volt.}\end{aligned}$$

In order to check up the total line drop and be certain that it does not exceed the limits given in Table V, it is generally more satisfactory to assume that the line has the maximum drop allowed by the table. This determines what the lowest possible voltage at the farthest lamp can be. Thus, if we assume that this line has the full 5.5 volts line drop allowed in Table V, then the voltage at the farthest lamp would be 110 − 5.5, or 104.5 volts. We now determine what voltage must be impressed on the main switch D in order to give 104.5 volts at the farthest lamp and still allow 6.12 volts resistance drop and 0.87 volt reactance drop in the line. If the switch voltage necessary is less than 110 volts then the line drop must be within the limits allowed. If more than 110 volts are required at the main switch to maintain

the lowest allowable voltage at the farthest lamp, then the line drop exceeds the allowable amount.

We can see at a glance that in this case we have somewhat more line drop than is allowed, since the resistance drop alone of 6.12 volts is 6.12 − 5.5 or 0.62 volt more than the total drop of 5.5 volts of the table. In addition to this we also have a reactance drop of 0.87 volt.

But this 0.87 volt reactance drop in the line is not so serious as at first appears, since it must be remembered that the voltage to overcome reactance is 90° out of phase with the voltage to overcome resistance. It is, therefore, well to apply the following check to the work.

Construct Fig. 159, vector $AB$ to represent the voltage of 104.5 volts across the farthest lamp.

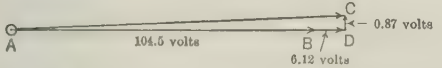



FIG. 159. To get 104.5 volts at the farthest lamp requires $AC$ volts at the service switch $D$, Fig. 158.

Since the power factor of the lamps is unity, the resistance drop in the line must be in phase with the voltage across the lamps. Vector $BD$ drawn along the same line as $AB$ may, therefore, represent the 6.12 volts used to overcome the line resistance.

The voltage to overcome the line reactance must lead the voltage to overcome the line resistance by 90° because the current through reactance alone lags 90° behind the voltage, while the voltage to send the current through resistance is in phase with the current. Thus vector $DC$ leading by 90° can represent the 0.87 volt used to overcome the line reactance. The vector $AC$ will then represent the feeder switch voltage necessary to overcome 6.12 volts due to line resistance and the 0.87 volt due to line reactance and supply 104.5 volts to the lamp.

The value of the feeder-switch voltage may then be found as follows:

$$\begin{aligned} AC &= \sqrt{(AB + BD)^2 + DC^2} \\ &= \sqrt{(104.5 + 6.12)^2 + 0.87^2} \\ &= 110.6 \text{ volts (practically).} \end{aligned}$$

If we consider the resistance drop alone (6.12 volts) the feeder switch voltage would still have to be 104.5 + 6.12 or 110.6 volts (practically).

The reactive drop of 0.87 volt, therefore, does not appreciably affect the line drop. But since the feeder-switch voltage is only 110 volts it is, therefore, unable to supply the lamps with 104.5 volts. The line drop is thus greater than 5 per cent. It is necessary, therefore, to make Main No. 2 of the next larger stock size of wire, which is No. 8.

**Prob. 1–8.** Calculate the voltage at the farthest lamp in Group B of the above example, using No. 8 wire for Main No. 2, and considering both resistance and reactance drop.

**Prob. 2–8.** What will be the voltage across the farthest lamp of Group A in the above example.

**Prob. 3–8.** Using Fig. 158 as a basis, but changing the dimensions to the following, determine the size of the feeders and mains to meet requirements of Fire Underwriters, and check for voltage drop to farthest lamp in Panel B.

| | |
|---|---|
| Voltage at feeder switch | 115 volts |
| Length of feeder, to Main No. 1 tap | 38 ft |
| Length of Main No. 1 | 45 ft |
| Length of Main No. 2 | 75 ft |

Panel B has six 660-watt branches.
Panel A has eight 660-watt branches.
Load of unity power factor.
Cleat construction, 2.5 inches between centers.

**Prob. 4–8.** Check the voltage drop to farthest lamp in Panel A, for the system of Prob. 3 when installed with the wire sizes determined in Prob. 3.

**Prob. 5–8.** If the number of branches on Panel B of Prob. 3 were doubled, determine the wire sizes to be used and check voltage to farthest lamps.

**75. Three-wire Single-phase System. "Code" Wire Size.** When motors are run on the same line with lamps a double advantage over a two-wire system is gained by using three wires. The current necessary to operate a given number of lamps is only half what it is on a two-wire system; and there is available for the larger motors a voltage which is double that of the lamp voltage, thus halving the current necessary for a given horse-power load. The following example serves to illustrate these advantages.

**Example 2.** In the wiring layout of Fig. 160, each of the eight lamp branches is to be wired to carry the maximum 660 watts. Each panel has two small-motor circuits, for fans, vacuum cleaners, etc., at an average lagging power factor of 60 per cent. Each of these circuits is to be wired to carry the maximum 660 volt-amperes. In addition Panel A has a 3-horsepower 220-volt motor of 81 per cent lagging power factor and Panel B, a 220-volt 2-horsepower motor of 79 per cent lagging power factor.

1. Calculate sizes of wire needed for each part of the system with all loads on full.

2. Calculate the voltage of farthest lamp under ordinary running conditions.

**Main No. 1.** When a three-wire single-phase system is properly installed, the loads are so distributed and balanced that, under full-load conditions, no current flows in the neutral wire, regardless of what the power factor of the loads may be. It is necessary, therefore, to find the current in the outside wires only.

Since the main-switch voltage is 115 — 230 volts, lamps and small motors must be supplied to run on $\frac{115}{1.05}$, or 109.5 volts. This will require 110-volt lamps. The large motors must be rated at $\frac{230}{1.05} = 219$ volts, or 220 volts practically.

**Panel A.** Each outside wire to Panel A feeds two lamp branches of 660 watts each. Each outside wire, therefore,

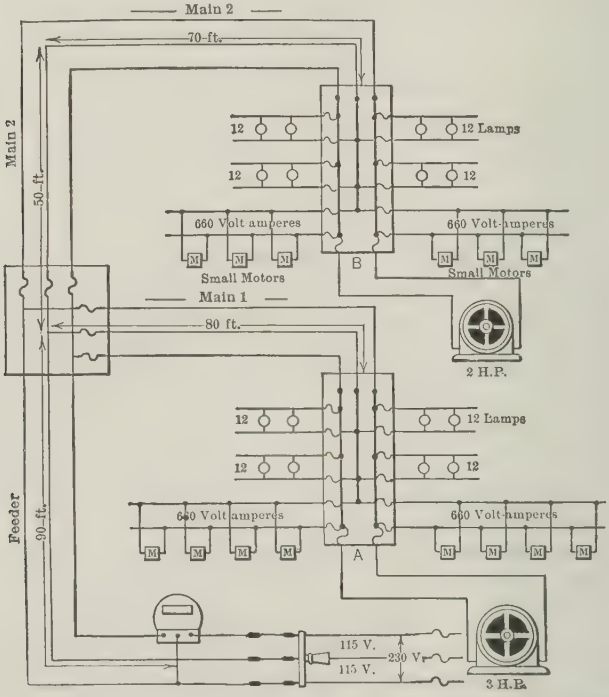



FIG. 160. Three-wire single-phase system for distribution in a building. Note the two-wire branches balanced on the three-wire mains at the cut-out panels *A*, *B*. Small single-phase motors are on separate branches connected to the outer mains; large motors would have separate mains and feeder. Fan motors, *M*, taking less than 660 volt-amperes are plugged into outlets on ordinary branches.

carries practically 6 amperes to each lamp branch or 12 amperes to the two. This current is at unity power factor. Each outside wire also carries 660 volt-amperes or 6

amperes to a small-motor branch. This current has probably a 60 per cent power factor lagging.

In addition each outside wire carries current to a 3-hp 220-volt motor. By Table VI (Appendix), we see that a 3-hp 220-volt two-phase induction motor takes 8.0 amperes at full load. A 3-hp 220-volt single-phase motor would take double this current per wire, since all the current must be brought to the motor by one circuit instead of by two circuits. The current per motor lead would, therefore, be 16.0 amperes. In order to allow for short-time overloads, and also to provide a reasonable amount of additional current-carrying capacity, it is customary, for a single motor, to consider its normal load equal to $1\frac{1}{4}$ times full load. Thus the above motor would be considered to draw $1\frac{1}{4} \times 16$, or 20 amperes, and from Table VII it is found that No. 10 wire is required.* This current has probably a power factor of 81 per cent lagging (see Table VIII, Appendix).

We thus have flowing in the outside wires of Main No. 1, a lamp current of 12 amperes, unity power factor, a motor current of 6 amperes, 60 per cent power factor, and a motor current of 20 amperes, 81 per cent power factor.

The resulting current in Main No. 1 may be found as follows:

| Appliances. | Indicated current. | Power factor. | Reactive factor = $\sqrt{1-\text{Power factor}^2}$ | Active component = Indicated current × Power factor | Reactive component = Indicated current × Reactive factor |
|---|---|---|---|---|---|
| | amp | | | amp | amp |
| Lamps | 12 | 1.00 | 0.00 | 12.0 | 0.0 |
| Small motors | 6 | 0.60 | 0.80 | 3.6 | 4.8 |
| Large motor | 20 | 0.81 | 0.59 | 16.2 | 11.8 |
| Line current | | | | 31.8 | 16.6 |

* The factor $1\frac{1}{4}$ thus used is called the "demand factor" of a single motor. See paragraph 76.

The vector diagram representing this current condition of Main No. 1 is shown in Fig. 161, from which the following equation can be taken to solve for the line current.

$$\text{Total line current in Main No. 1} = \sqrt{31.8^2 + 16.6^2}$$
$$= \sqrt{1011 + 276}$$
$$= 35.9 \text{ amp.}$$

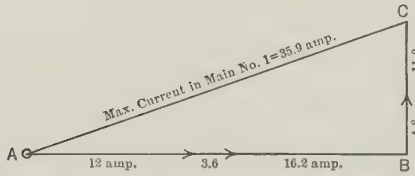



FIG. 161. Vector sum of all currents flowing from Main No. 1 to branches at *A*, Fig. 160, is *AC* amperes.

This, according to Table II, requires No. 6 wire for the outside wires of Main No. 1. It is necessary to make the neutral wire the same size as the outer wires unless the current in each outer wire exceeds 200 amperes. This enables one side of the system to be kept in operation even though the other side is opened by some mishap. Thus Main No. 1 would consist of three No. 6 wires.

**Main No. 2.** As in Main No. 1, each of the outside wires of Main No. 2 must carry 12 amperes at unity power factor to supply two lamp branches of 660 watts each, and 6 amperes at 60 per cent power factor to supply one branch of 660 volt-amperes for fractional horse-power motors or appliances.

In addition Main No. 2 must supply a 220-volt 2-hp induction motor of 79 per cent power factor (Table VIII). According to Table VI a two-phase 220-volt 2-hp motor requires 5 amperes at full load. A single-phase motor of same horsepower would require $2 \times 5$, or 10 amperes. The motor leads must be heavy enough to carry $1\frac{1}{4} \times 10$, or 12.5 amperes to allow for possible continued overload, and, therefore, must be of No. 12 wire according to Table VII.

Mains No. 2 must, therefore, carry:

Lamp current.....12 amperes at unity power factor.
Small-motor current..6 amperes at 60 per cent power factor.
Large-motor current..12.5 amperes at 79 per cent power factor.

The resulting current in Main No. 2 may be found as follows:

| Appliances. | Indicated current. | Power factor. | Reactive factor = $\sqrt{1-\text{Power factor}^2}$ | Active component = Indicated current × Power factor | Reactive component = Indicated current × Reactive factor |
|---|---|---|---|---|---|
| | amp | | | amp | amp |
| Lamps..... | 12 | 1.00 | 0.00 | 12.0 | 0.0 |
| Small motors... | 6 | 0.60 | 0.80 | 3.6 | 4.8 |
| Large motor... | 12.5 | 0.79 | 0.62 | 9.9 | 7.8 |
| Line current ..... | | | | 25.5 | 12.6 |

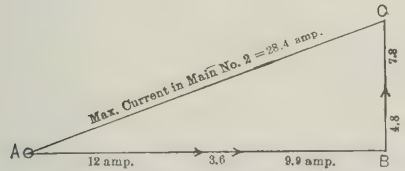



FIG. 162. Vector sum of all currents flowing from Main No. 2 to branches at *B*, Fig. 160, is *AC* amperes.

From Fig. 162, representing the above active and reactive components of the line-current, we may write the equation:

$$\text{Line-current in Main No. 2} = \sqrt{25.5^2 + 12.6^2}$$
$$= \sqrt{650 + 158.8}$$
$$= 28.4 \text{ amp.}$$

The three wires of Main No. 2, according to Table II, should be size No. 8.

**Feeders.** The feeders must carry the current necessary to supply the appliances connected to both Panel A and Panel B.

Panel A requires 31.8 amperes (active) and 16.6 amperes (reactive).

Panel B requires 25.5 amperes (active) and 12.6 amperes (reactive).

Active current in feeders = 31.8 + 25.5 = 57.3 amperes.

Reactive current in feeders = 16.6 + 12.6 = 29.2 amperes.

From Fig. 163, we can see that the feeder current can be obtained by the equation:

$$\begin{aligned}\text{Total current in feeders} &= \sqrt{57.3^2 + 29.2^2} \\ &= \sqrt{3283 + 853} \\ &= 64.3 \text{ amp.}\end{aligned}$$

According to Table II, the three wires of the feeder must be No. 4 gauge.

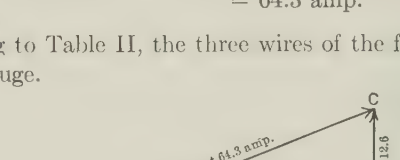



FIG. 163. Vector sum of all currents flowing from feeder to mains of Fig. 160 is represented by $AC$, equal to 64.3 amperes.

Thus the outside wires of the feeder in this case must carry not more than 64.3 amperes which, according to Table II, would require the (rubber-covered) conductors to be of No. 4 gauge. This presumes, however, that every motor will be running at the same time, with all lamps burning, a most improbable state of affairs. However, as the load is much more likely to increase than to diminish in the course of time, it is the practice of many engineers not to load any branch circuit initially to more than about 80 per cent or 90 per cent

of its permitted capacity (660 volt-amperes), and not to make any feeder or main of smaller capacity than the sum of the maximum capacities of all branches connected to it. In view of usual experience with wiring systems, it is considered that the cost of excess investment in copper, which for a while may not be used to its full capacity, is less than the probable cost of changing the size of copper as the load grows.

**76. Voltage Drop to Farthest Lamp Due to Resistance.** Consider a lamp on branch from Panel B.

**Drop due to resistance of branch wires.** Inasmuch as we do not know the length of the branch from Panel B, we will assume that we have the maximum branch drop allowed in Table V, for a 110-volt branch, or 1.65 volts. It is then necessary to limit the length of a branch, assumed to carry the maximum allowable current of 6 amperes, to such value as will consume this voltage.

**Drop due to resistance of Main No. 2.** The length of Main No. 2 is 70 + 50, or 120 ft. There are thus $2 \times 120$, or 240 ft of No. 8 wire in the outer wires.

Resistance of 1000 ft of No. 8 copper wire (Table III) = 0.6271 ohm.

Resistance of 240 ft of No. 8 copper wire = $\frac{240}{1000}$ of 0.6271 = 0.1505 ohm.

**Demand factor.** In calculating what **carrying capacity** a wire must have, it is good practice to use the **full-load* currents** of all the motors, assuming them all to be running at the same time. But in computing the **voltage drop** on the line, we wish to know what this drop will be under usual running conditions. Since it is not the usual condition to have all the motors running at full load at the same time, we need not use all of the full-load currents of all the motors

* When a few large motors are installed in combination with small motors or lamps it is good practice to use $1\frac{1}{4}$ full-load current for the large motors.

attached to the line. We accordingly use only that fraction of the arithmetical sum of full-load currents which it has been found in general practice is the fraction of the full-load currents usually flowing in the line. **This fraction, which the usual line-current is of the sum total of full-load currents, is called the "demand factor" of the line,** and can be found tabulated for various combinations of motors in Table IX of the Appendix. Note that the more motors there are on the line, the less likelihood there is of their all running at full-load and consequently the lower the demand factor.

Assume that there are five small motors to each branch.

Full-load current for small motor branch = 6 amperes.

Average demand factor for 5 motors = 0.65 (Table IX).

Usual small-motor current = $0.65 \times 6 = 3.90$ amperes.

Power factor for small motors = 0.60.

Reactive factor (Paragraph 75) = 0.80.

Usual active current (small motors) = $0.60 \times 3.90 = 2.34$ amperes.

Usual reactive current (small motors) = $0.80 \times 3.9 = 3.12$ amperes.

We have already considered the demand factor in calculating the maximum current of the 2-hp motor attached to Panel B, which was found to consist of 9.9 amperes active current and 7.8 amperes reactive current.

We are now able to compute the total usual current in Main No. 2, as follows:

| | Active component | Reactive component |
|---|---|---|
| | amp | amp |
| Lamps | 12 | 0 |
| Small motors | 2.3 | 3.1 |
| Large motors | 9.9 | 7.8 |
| Total in Main No. 2 | 24.2 | 10.9 |

From Fig. 164 we find:

Resultant usual current in Main No. 2 = $\sqrt{24.2^2 + 10.9^2}$
= 26.6 amperes.

Resistance drop in Main No. 2 = 26.6 × 0.1505 = 4.00 volts.

**Drop in feeders due to resistance.**

Length of outer feeder wire = 2 × 90 = 180 ft

Resistance of 180 ft of No. 4 copper wire = $\frac{180}{1000}$ × 0.248 = 0.04464 ohm.

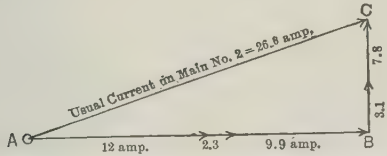



FIG. 164. Usual current in Main 2, less than maximum, on account of demand factor. Compare Fig. 162.

The usual total current in feeder is the vector sum of the usual total currents in Main No. 1 and Main No. 2.

| | Active component | Reactive component |
|---|---|---|
| | amp | amp |
| Usual current for Main No. 2 | 24.2 | 10.9 |
| Usual current for Main No. 1 | | |
| Lamps | 12.0 | 0.0 |
| Small motors (same as for Panel B) | 2.3 | 3.1 |
| Large motor (Paragraph 75) | 16.2 | 11.8 |
| Total components usual current in feeder | 54.7 | 25.8 |

Total usual current in feeder (from Fig. 165)

= $\sqrt{54.7^2 + 25.8^2}$
= 60.5 amperes.

Drop in feeders due to resistance = 60.5 × 0.04464
= 2.70 volts.

The total resistance drop out to the farthest lamp equals

| | |
|---|---|
| Resistance drop in branch ..... | 3.3 volts |
| Resistance drop in Main No. 2.... | 4.0 " |
| Resistance drop in feeder..... | 2.7 " |
| Total resistance drop ..... | 10.0 volts |

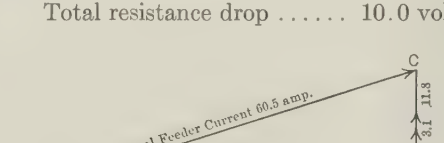



FIG. 165. Usual current in feeder is less than maximum, on account of demand factor. Compare Fig. 163.

**77. Total Pressure Drop to Farthest Lamps.** The total drop in pressure at the farthest lamps is the combined drop in branch, main, and feeder due to both the resistance and the reactance of the line. Of this, the reactance drop of the branch is usually negligible, due to the small current and comparatively short length of wire.

**Reactance drop of Main No. 2.** Assuming the outer wires to be installed 6 in. apart, the 60-cycle reactance of the 240 ft of No. 8 wire can be found by reference to Table IV.

Reactance of 1000 ft of No. 8 spaced 6 in. = 0.1103 ohm.
Reactance of 240 ft = $\frac{240}{1000}$ of 0.1103 = 0.02647 ohm.
Reactance drop in Main No. 2 = $26.6 \times 0.02647$
= 0.70 volt.

**Reactance drop of feeders.** The reactance drop of the 180 ft of No. 4 wire in the feeders carrying 60.5 amperes is found in the same way.

Reactance of 1000 ft of No. 4 stranded wire spaced 6 in.
= 0.0963 ohm (Table IV).
Reactance of 180 ft = $\frac{180}{1000} \times 0.0963 = 0.01733$ ohm.

Reactance drop in feeders $= 60.5 \times 0.01733$
$= 1.048$ volts

Total reactance drop out to farthest lamp equals

Reactance drop in Main No. 2 = 0.70
Reactance drop in feeders = 1.05
Total reactance drop = 1.75 volts.

**Combined resistance and reactance drop.** The resistance drop to the farthest lamps was found to be about 10.0 volts, and the reactance drop 1.75 volts. The resistance drop is made up of 3.3 volts drop in the branch and 6.7 volts in the main and feeder. But since in a three-wire single-phase system the drop along the outer wires of the main and feeder is distributed across the lamps on both sides of the circuit, only half of the main and feeder drop affect any one 110-volt lamp. Thus the resistance drop along main and feeder for each lamp equals $\frac{6.7}{2}$, or 3.35 volts. The resistance drop in the branch wires was found to be 1.65 volts which affects each lamp. Thus, total resistance drop to each lamp equals 3.35 + 1.65 = 5.0 volts. The reactance drop equals $\frac{1.75}{2}$, or 0.88 volts.

**Check on percentage line drop.** It is now necessary to find how the actual voltage at the lamps will be affected by this 5.0 volt resistance drop and 0.88-volt reactance drop in the line. As we have seen, the easiest method is to assume that the lamps have the 109.5 volts computed on page 235 from Table V and to compute what feeder switch voltage must be maintained to produce this voltage. If the feeder switch voltage required does not exceed the given 115 volts, the line drop is not excessive.

The power factor of the current delivered to Panel B can be found from the usual indicated current 26.6 amperes, active component 24.2 amperes, and lagging reactive component 10.9 amperes, as calculated in Par. 76.

$$\text{Power factor} = \frac{\text{Active component}}{\text{Indicated current}}$$
$$= \frac{24.2}{26.6}$$
$$= 91.0 \text{ per cent.}$$

$$\text{Reactive factor} = \frac{\text{Reactive component}}{\text{Indicated current}}$$
$$= \frac{10.9}{26.6}$$
$$= 41.0 \text{ per cent.}$$

The angle of lag corresponding to these power and reactive factors as given in Table I is approximately 25°.

Construct Fig. 166 as follows:

Draw $AX$ of indefinite length to represent the direction of the vector of the current and the active component of voltage at Panel B.

Draw vector $AC$ to represent the least permissible indicated voltage at the farthest lamps, namely 109.5.

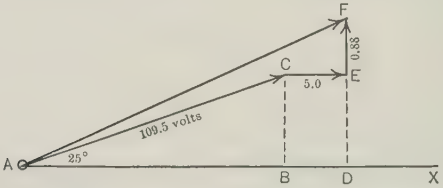



FIG. 166. Finding the voltage ($AF$) required at one end of a line, in order to deliver voltage $AC$ at the other end.

Since the power factor is lagging, the active component of voltage must **lag 25° behind** the indicated voltage. Thus vector $AC$ representing the indicated voltage should **lead** the active component line by 25°. The main switch voltage must equal the vector sum of this lamp voltage of 109.5, the resistance line drop of 5.0 volts and the reactance line drop of 0.88 volt.

Add the resistance line drop of 5.0 volts to the lamp voltage of 109.5, by drawing the vector $CE$. This must be drawn in the same direction as $AX$ since this drop is in phase with the current.

Add the reactance line drop of 0.88 volt by drawing the vector $EF$. This must be drawn leading upward at 90° to the direction of $AX$, since the voltage to overcome reactance must lead the current by 90°. Both vector $CE$ and vector $EF$ are drawn much longer than they really are, in order to make the diagram clear. This does not affect the computation, however.

The vector $AF$ now represents the vector sum of the lamp voltage 109.5, the resistance line drop 5.0 and the reactance line drop 0.88, and is, therefore, the voltage which the main switch must deliver in order to maintain a full-load voltage of 109.5 at the lamps.

The value of $AF$ can be found as follows:

Draw dotted lines $CB$ and $ED$, perpendicular to $AX$.

$$AF = \sqrt{AD^2 + DF^2}.$$
$$AD = AB + BD.$$
$$AB = 109.5 \times \text{power factor}$$
$$= 109.5 \times 0.91$$
$$= 99.65 \text{ volts.}$$
$$BD = CE = 5.0$$
$$AD = 99.65 + 5.0$$
$$= 104.65.$$
$$DF = DE + EF.$$
$$DE = CB = 109.5 \times \text{reactive factor}$$
$$= 109.5 \times 0.41$$
$$= 44.9.$$
$$EF = 0.88.$$
$$DF = 44.9 + 0.88$$
$$= 45.78.$$
$$AF = \sqrt{104.65^2 + 45.78^2}$$
$$= 114.3 \text{ volts.}$$

The main switch voltage must, therefore, be 114.3 volts in order to overcome the line reactance and line resistance and still leave 109.5 volts at the lamps. As we have 115 volts at the main switch, the sizes of wires planned are sufficiently large to insure satisfactory service.

**Prob. 6–8.** Calculate the voltage drop to the farthest lamp in Example 2 with the lamps only from both panels turned on.

**Prob. 7–8.** If, in Fig. 158, a balanced three-wire single-phase system were used in which all three wires are of the same size, how many pounds of copper wire would be saved? The weight per thousand feet of copper wire can be found in Table III.

This problem demonstrates the advantage of using a three-wire single-phase system instead of the two-wire layout, even for a lamp load only.

**Prob. 8–8.** Check the line drop to the lamps wired from Panel A in Fig. 160.

**Prob. 9–8.** Calculate the size of wire to be used for feeders and mains in a system like Fig. 160, except that Panels B and A are interchanged.

**Prob. 10–8.** Calculate Example 2 for a 25-cycle line.

**Prob. 11–8.** What size branch wires should be run to a 5-horsepower single-phase 440-volt induction motor? Assume a demand factor of 1.25.

**Prob. 12–8.** How many 1-horsepower 110-volt single-phase induction motors can be fed by a No. 6 conductor? Assume a demand factor of 1.25.

**Prob. 13–8.** Compute the line drop (including reactance and resistance) in the branch wires of Prob. 8–8, if the distance to the main is 150 ft.

**78. Two-phase Circuits. Size of Wire.** In calculating the size of wire to be used in a two-phase four-wire system, treat each phase as a single-phase system carrying one-half the power delivered to the two-phase appliances.

**Prob. 14–8.** Calculate the size of wire for feeder, mains and branches of the same length as in Fig. 158 if the panels were each

fed by a four-wire two-phase 110-volt system. Total load as in Example 1.

**Prob. 15–8.** Calculate the size of wire for feeders, mains and branches of Fig. 160 if each of the two panels is fed by a four-wire two-phase system. Use loads and distances of Fig. 160 and Example 2. Lamps and motors 110 volts.

**Prob. 16–8.** Calculate wire sizes for system of Prob. 14, using 25 cycles instead of 60 cycles.

**79. Size of Wire for Three-phase Three-wire Systems.** Since polyphase induction motors are self-starting, polyphase systems are desirable, where many motors or large motors are to be operated. The most economical polyphase system in the matter of wiring and in generator and motor equipment is the three-wire three-phase.

**Example 3. Size of Main No. 2.** Consider the layout in Fig. 167. The wire sizes are determined as before, by computing the amount of current each part of the circuit has to carry. The lamp load on Main No. 2 is balanced and consists of three branches, each wired to carry 660 watts. With main-switch voltage 115, lamps must be used of $\frac{115}{1.05}$, or 109.5 volts. The nearest standard lamp has the 110-volt rating. At 110 volts 6 amperes are used per branch. The lamp current in each Main No. 2 would, therefore, be $1.73 \times 6$, or 10.4 amperes.

The two motors are three-phase motors and each is supposed to take a balanced load from the three phases.

The 1-hp motor may take 6.6 (full-load) amperes (Table VI), or $1\frac{1}{4} \times 6.6 = 8.25$ amperes possible running current, at 70 per cent power factor. (Table VIII).

The 2-hp motor may take 12.0 (full-load) amperes, or $1\frac{1}{4} \times 12.0 = 15.0$ amperes possible running current, at 79 per cent power factor.

Each wire of Main No. 2 must, therefore, carry the com-

bination of 10.4 amperes at unity power factor, 8.25 amperes at 70 per cent power factor, and 15.0 amperes at 79 per cent power factor.

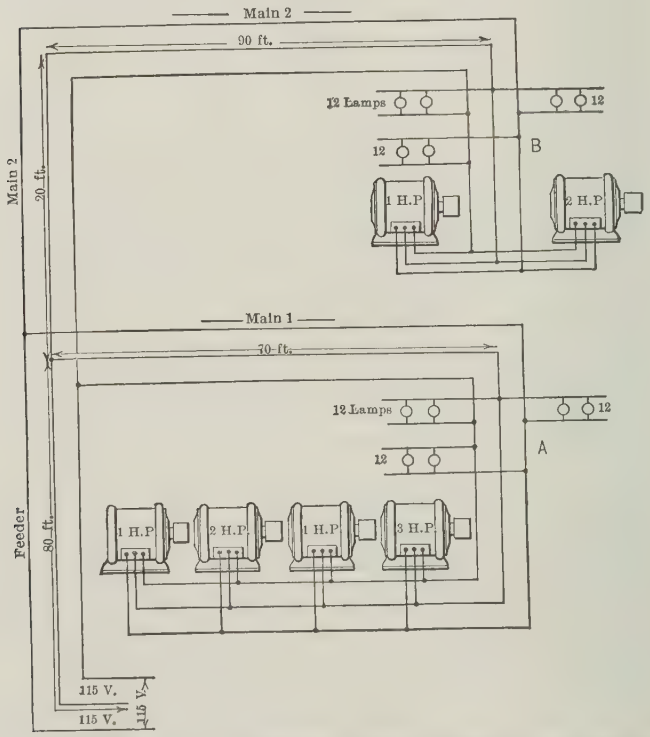



FIG. 167. Three-phase distributing system. Fuses or cut-outs are not shown, but should be inserted in a smaller wire wherever it joins a larger one. Often the motor system is entirely separate from the lighting system.

This current can be resolved into its power and reactive components as follows:

| Appliances | Running current | Power factor | Reactive factor | Power component | Reactive component |
|---|---|---|---|---|---|
| | amp | | | amp | amp |
| One 1-hp motor..... | 8.25 | 0.70 | 0.714 | 5.77 | 5.89 |
| One 2-hp motor..... | 15.0 | 0.79 | 0.613 | 11.85 | 9.20 |
| Lamps..... | 10.4 | 1.00 | 0.000 | 10.4 | 0.00 |
| Total components ..... | | | | 28.02 | 15.09 |

The current in Main No. 2 thus consists of a power component of 28.02 amperes and a reactive component of 15.09 amperes.

To determine resulting current, construct Fig. 168.

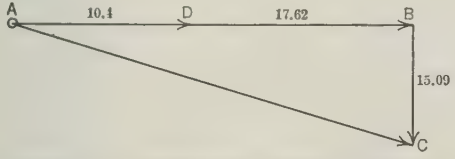



FIG. 168. Vector $AC$ represents the current in Main No. 2. $AD$ represents current of lamps; $DB$, power current of motor; $BC$, reactive current of motors.

$AD$ = 10.4 amperes power current of lamps.
$DB$ = 17.62 amperes power current of motors.
$BC$ = 15.09 amperes reactive current of motors.
$AC$ = resulting current in Main No. 2

$$= \sqrt{\overline{AB}^2 + \overline{BC}^2}$$
$$= \sqrt{28.02^2 + 15.09^2}$$
$$= 31.8 \text{ amperes.}$$

Power factor of Main No. 2 $= \dfrac{28.02}{31.8} = 88$ per cent.

Angle of lag = 28° (approx.).

To carry 31.8 amperes, No. 8 wire should be installed for Main No. 2. By Table VII, the leads to the 1-hp motor should be No. 14, and to the 2-hp motor, No. 12.

**Size of Main No. 1.** The lamp current in each conductor of Main No. 1 will be the same as in each of Main No. 2, namely, 10.4 amperes, according to Fig. 167.

The maximum running loads of induction motors may equal $1\frac{1}{4}$ times the full load for a considerable length of time.

| Motors | Full-load current | Running current = $1\frac{1}{4}$ full-load current | Power factor | Reactive factor | Power component | Reactive component |
|---|---|---|---|---|---|---|
| | amp | amp | | | amp | amp |
| Two 1-hp. . . | $2 \times 6.6 = 13.2$ | 16.5 | 0.70 | 0.714 | 11.54 | 11.78 |
| One 2-hp. . . . | 12.0 | 15.0 | 0.79 | 0.613 | 11.85 | 9.20 |
| One 3-hp. . . . | 18.0 | 22.5 | 0.81 | 0.587 | 18.22 | 13.20 |
| Total motor current (components) . . . . . | | | | | 41.61 | 34.18 |

The total current in Main No. 1 is, therefore, made up of the 10.4 amperes power current of lamps, 41.61 amperes power component of the motors, and 34.18 reactive components of the motors.

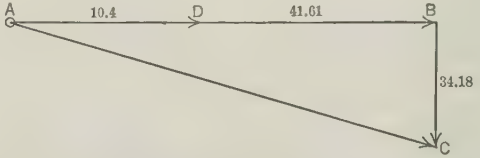



FIG. 169. Vector $AC$ represents the resultant current flowing in Main No. 1. It is made up of the lamp current $AD$, the motor power component, $DB$, and the motor reactive component, $BC$.

Combine these components as in Fig. 169.

$AD$ = 10.4 amperes power current of lamps.

$DB$ = 41.61 amperes power component of motors.

$BC$ = 34.18 amperes reactive component of motors.

$AC$ = combined current of lamps and motors

$$= \sqrt{\overline{AB}^2 + \overline{BC}^2}$$

$$= \sqrt{52.01^2 + 34.18^2}$$

= 62.2 amperes in Main No. 1.

$$\text{Power factor} = \frac{52.01}{62.2} = 83.6 \text{ per cent.}$$

Angle of lag = 33° (approximately).

Each wire of Main No. 1 must be of No. 4 gauge in order to carry 62.2 amperes. By Table VII leads to the 3-hp motor must be No. 10.

**Size of feeders.** The current in the feeders is found by adding the power components of the currents in Mains No. 1 and No. 2, adding the reactive components of Mains No. 1 and No. 2 and taking the square root of the sum of the squares of these values. Fig. 170 represents the addition.

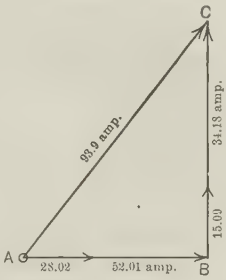



FIG. 170. Total current $AC$, in feeder of Fig. 167 combination of $AC$, Fig. 168, and $AC$, Fig. 169.

$AB$ (sum of power components) = 28.02 + 52.01 = 80.03 amperes.

$BC$ (sum of reactive components) = 15.09 + 34.18 = 49.27.

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{80.03^2 + 49.27^2}$$
$$= 93.9 \text{ amperes.}$$

To carry this current a No. 1 conductor will be required (Table II).

**80. Resistance Voltage Drop in Three-phase Systems.**

**Drop in branch.** The lengths of the lamp branches being various and unknown, we will assume a drop of 1.65 volts, the maximum allowable for a 110-volt 660-watt branch. (Table V.)

**Drop in Main No. 2.** Current taken from each line wire by balanced lamp load = 1.73 × 6 amperes = 10.38 amperes at unity power factor.

The usual motor current is found as follows:

| Motor. | Full-load power factor. | Reactive factor = $\sqrt{1 - \text{Power factor}^2}$ | Full-load current, Table VI. | Active component of current. | Reactive comp. of current. |
|---|---|---|---|---|---|
| | | | amp | amp | amp |
| 1-hp.... | 0.70 | 0.714 | 6.6 | 4.62 | 4.72 |
| 2-hp.... | 0.79 | 0.613 | 12.0 | 9.48 | 7.36 |
| Total motor components ..... | | | | 14.10 | 12.08 |

Total active component of current in Main No. 2 consists of 14.10 motor current and 10.38 amperes lamp current, or $14.10 + 10.38 = 24.48$ amperes.

Reactive component = 12.08 amperes.

Total line current in Main No. 2 $= \sqrt{24.48^2 + 12.08^2}$

$= 27.3$ amperes.

The resistance of one wire of Main No. 2 consisting of 110 ft of No. 8 copper wire equals $\frac{110}{1000} \times 0.6271 = 0.06898$ ohm.

Usual resistance drop **along one wire** equals

$$27.3 \times 0.06898 = 1.88 \text{ volts.}$$

**Drop in feeders.** The current taken from Main No. 1 by the balanced lamp load equals $1.73 \times 6 = 10.38$ amperes.

The usual current taken by the motors can be found as follows:

| Motors. | | Full-load power factor Table VIII. | Full-load reactive factor, $\sqrt{1-\text{Power factor}^2}$ | Full-load current, Table VI. | Full-load active component | Full-load reactive component |
|---|---|---|---|---|---|---|
| No. | Size. | | | amp | amp | amp |
| 2 | 1 | 0.70 | 0.714 | $2 \times 6.6 = 13.2$ | 9.24 | 9.42 |
| 1 | 2 | 0.79 | 0.613 | 12.0 | 9.48 | 7.36 |
| 1 | 3 | 0.81 | 0.587 | 18.0 | 14.58 | 10.55 |
| Full-load components of currents ..... | | | | | 33.30 | 27.33 |

Demand factor for four motors = 70 per cent (Table IX).
Usual active component of motor current = $0.70 \times 33.30$
= 23.31 amp.
Usual reactive component of motor current = $0.70 \times 27.33$
= 19.13 amp.

Usual active components for total current in Main No. 1 consists of 10.38 amperes for lamps and 23.31 amperes for motors, or $10.38 + 23.31 = 33.69$ amperes.

Components of usual current in feeder:

| | Active. | Reactive. |
|---|---|---|
| | amp | amp |
| For Main No. 1 | 33.69 | 19.13 |
| For Main No. 2 | 24.48 | 12.08 |
| Total | 58.17 | 31.21 |

Total current in feeder = $\sqrt{58.17^2 + 31.21^2}$
= 65.6 amperes.

The resistance of each conductor of feeder is the resistance of 80 ft of No. 1 copper wire or $\frac{80}{1000} \times 0.1237 = 0.009896$ ohm.

The resistance drop in each feeder conductor equals $65.6 \times 0.009896$, or 0.65 volt.

Total resistance drop to lamps fed through Panel B:

Resistance drop in both conductors of branch
= 1.65 volts.
Resistance drop in one conductor of Main
No. 2 = 1.88 volts.
Resistance drop in one conductor of feeder = 0.65 volt.
Total resistance drop per leg = 4.18 volts.

**81. Equivalent Distance in Wire Spacing.** The three wires of a three-wire system usually are not equidistant from

each other; with open wiring there may be two and one-half inches between the neutral wire and either of the two outside wires, and five inches between the two outside wires. Before we may find the reactance from Table IV, it is therefore necessary to find the "equivalent distance" between the wires. This is done by means of the equation:

$$\text{Equivalent distance} = \sqrt[3]{\text{product of the three distances}}$$
$$= \sqrt[3]{2.5 \times 2.5 \times 5}$$
$$= 3.15 \text{ inches.}$$

The equivalent distances for the usual spacings will be found in Table X of the Appendix. The reactances for exactly these equivalent distances are not given in Table IV but may be obtained approximately from that table.

**82. Reactance Drop in Three-phase Systems.**

**Reactance drop in Main No. 2.** Reactance of 110 ft of No. 8 wire with the equivalent spacing of 3.15 in. $= \frac{110}{1000} \times 0.095 = 0.0105$ ohm.

Reactance drop in one wire of Main No. 2

$$= 0.0105 \times 27.3$$
$$= 0.287 \text{ volt.}$$

**Reactance drop in feeder.** Reactance of 80 ft of No. 1 stranded wire with equivalent spacing of 3.15 in. $= \frac{80}{1000} \times 0.0735 = 0.00588$ ohm.

$$\text{Reactance drop of one wire of feeder} = 65.6 \times 0.00588$$
$$= 0.386 \text{ volt.}$$

Total reactance drop $= 0.287 + 0.386 = 0.673$ volt.

Total resistance drop = 4.18 volts (page 255).

We have assumed the drop in the branch to be maximum and to be due entirely to resistance.

**83. Percentage of Line Drop. Voltage to Neutral.** In the above example, it will be noted that we have computed the resistance drop and the reactance drop of one conductor

only. It is not convenient to use the drop along two conductors because each conductor acts either as the line or the return for the current in two phases. In order to find what percentage the line drop is of the voltage across the switch, we have heretofore compared the drop along two conductors (line and return) with the usual or full-load voltage between the same two conductors. Obviously it is not proper in determining the percentage line drop in a three-wire three-phase system to compare the voltage drop along one conductor with the voltage between two conductors.

It is, therefore, customary to use as a standard not the voltage between two conductors, but the voltage between one conductor and the neutral point of the load.

If the load is star-connected, we have a definite point as the neutral and can actually measure the voltage between a conductor and the neutral. Thus, in Fig. 171 when we have three lamps star-connected to a three-wire three-phase system, each lamp is connected between the neutral point $N$ and one of the three line wires $A$, $B$ or $C$. As the voltage between any two of the line wires is 115 volts, the voltage across any one of the lamps must be the voltage from a line wire to the neutral or $\frac{115}{1.73} = 66.4$ volts. Thus the voltage across lamp 1 is 66.4 volts and across lamp 2 is 66.4 volts, although the two lamps are in series between the mains $A$ and $B$. It will be remembered that the reason why the voltage between $A$ and $B$ does not have to equal the arithmetical sum of 66.4 + 66.4 volts, is because the two voltages are not in phase, but at an angle of 60° with each other.

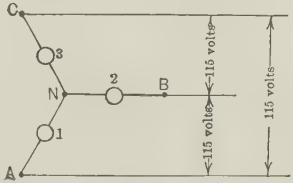



FIG. 171. "Neutral point" of three-phase line is $N$ if loads 1, 2, 3 are exactly alike. Then, voltage $AN = BN = CN = 0.577$ times line voltage.

Therefore, the resultant of the voltage across the two lamps equals 66.4 × 1.73, or 115 volts. In finding the percentage line drop out to any one of these lamps we must compare the drop in volts with the voltage to neutral, that is, the 66.4 volts across the lamp.

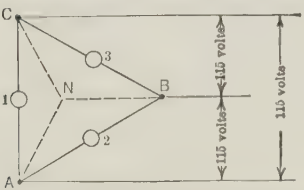



Fig. 172. With delta-connected load, neutral point is imaginary, but same voltage relations hold as in Fig. 171.

But most lamp loads, as in Example III, are delta-connected, and the voltage across each lamp is the voltage between two line wires, as the 115 volts in Fig. 172. But as we cannot compare the drop in one line wire due to two phases with the voltage between wires which constitute one phase, we, even in the case of a delta-connected load, use the voltage to an imaginary neutral point. In the case of Fig. 172, we would find the voltage drop along one wire, and, as in Fig. 171, compare this drop with the voltage to neutral, $\frac{115}{1.73}$ or 66.4 volts.

Thus in Example III, with a switch voltage of 115 volts, the voltage to neutral would be 66.4 volts. Since the maximum allowable drop along one line is 5 per cent, the voltage to neutral at lamps must be at least $\frac{66.4}{1.05} = 63.2$ volts.

The percentage line drop to the farthest lamp can now be found as in paragraph 77. The power factor and reactive factor of current delivered to Panel B are found from the following data of paragraph 80, page 254.

Indicated line-current = 27.3 amperes.
Active component = 24.48 amperes.
Reactive component = 12.08 amperes.

$$\text{Power factor} = \frac{\text{Active component}}{\text{Indicated current}}$$
$$= \frac{24.48}{27.3}$$
$$= 89.7 \text{ per cent.}$$

$$\text{Reactive factor} = \frac{\text{Reactive component}}{\text{Indicated current}}$$
$$= \frac{12.08}{27.3}$$
$$= 44.3 \text{ per cent.}$$

These factors correspond to an angle of practically 26° (Table I). Construct Fig. 173, similar to Fig. 166.

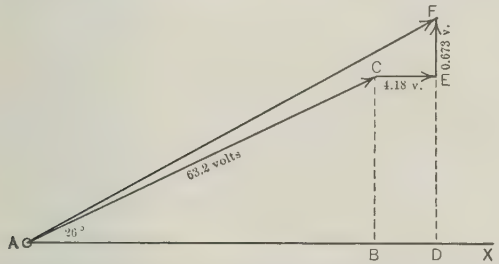



FIG. 173. Finding the voltage ($AF$) required at one end of a line in order to deliver voltage $AC$ at the other end. Compare Fig. 166 for single-phase line.

Draw $AX$ of indefinite length to represent the direction of the active component of voltage-to-neutral at the lamps, or the direction of the vector for current in a line wire.

Draw vector $AC$ to represent the voltage-to-neutral of 63.2 volts at the lamps, leading the active component of voltage-to-neutral by 26°.

Draw vector $CE$ to represent the 4.18 volts drop (page 255) due to the resistance of one line wire. This vector is drawn in

the same direction as the active component of voltage-to-neutral, because the drop is in phase with the active component of voltage-to-neutral, or in phase with current in the line wire.

Draw vector $EF$ to represent the 0.673-volt drop (page 256) due to the reactance of the line. This vector should be drawn leading the active component of voltage, or the line current, by 90° because the drop due to reactance leads the line-current by 90°.

The voltage-to-neutral at the switch must equal the vector sum of the voltage-to-neutral at the lamps, the line resistance drop in one wire and the line reactance drop in one wire. This vector sum is represented by the vector $AF$, the value of which may be found as follows:

$$AF = \sqrt{AD^2 + DF^2}.$$

$$AD = AB + BD.$$

$$\begin{aligned} AB &= 63.2 \times \text{power factor} \\ &= 63.2 \times 0.897 \\ &= 56.7. \end{aligned}$$

$$BD = CE = 4.18.$$

$$\begin{aligned} AD &= 56.7 + 4.18 \\ &= 60.88. \end{aligned}$$

$$DF = DE + EF.$$

$$\begin{aligned} DE &= 63.2 \times \text{reactive factor} \\ &= 63.2 \times 0.443 \\ &= 28.0. \end{aligned}$$

$$EF = 0.673.$$

$$\begin{aligned} DF &= 28.0 + 0.673 \\ &= 28.67. \end{aligned}$$

$$\begin{aligned} AF &= \sqrt{60.88^2 + 28.67^2} \\ &= 67.3 \text{ volts.} \end{aligned}$$

The voltage-to-neutral at the main switch must, therefore, be 67.3 volts in order to have not more than 5 per cent line drop.*

Since the voltage to neutral at the switch is $\frac{115}{1.73}$, or only 66.4 volts, the sizes chosen for the feeders and for Main No. 2 may not give quite satisfactory service. Main No. 2 should be increased to No. 6 B & S.

It is generally the case when motors form a part of the load that the sizes calculated to carry safely the running loads, do not cause excessive line drops for the usual loads on the circuits.

**Prob. 17–8.** Calculate the per cent line drop to the farthest lamp in Example 3, using No. 6 wire for Main No. 2.

**Prob. 18–8.** What is the voltage drop to the lamps on Panel A in Example 3?

**Prob. 19–8.** If in Fig. 167 an additional 10-horsepower motor is connected to panel B and a 5-horsepower motor is connected at panel A, what size conductors will be required for branches, mains, and feeders? Assume each motor to be on a separate branch.

**Prob. 20–8.** What will be the per cent line drop to the farthest lamp in Prob. 19?

**84. Skin Effect of Large Conductors.** When conductors of 300,000 circular mils, or larger, are run, it is found that the resistance to the flow of an alternating current is somewhat greater than that given in Table III. This increase in resistance is produced by the crowding of the current to the outer part or "skin" of the conductor and has the effect of actually decreasing the conductor area through which the current passes, producing a corresponding apparent increase

* For highest accuracy, we should take into account that the current in feeder is not exactly in phase with current in Main No. 1 and Main No. 2, or that the angle is not 26° as in Fig. 173 for all parts of the system, which fact has been neglected here. No error of practical importance results, and the work is much simplified.

in resistance. The effect increases with the frequency of the alternations, and results in a slightly greater voltage drop in the conductor. The amount of this drop may be determined from any good electrical handbook.

## SUMMARY OF CHAPTER VIII

The SIZE OF WIRE that should be used in each part of a distributing system is usually determined, for interior wiring and relatively short distances, by two considerations only: HEATING OF THE WIRE on account of its resistance, and REDUCTION OF VOLTAGE due to the impedance of the wire.

Heating the wire beyond a certain temperature damages the insulation on it, tending to produce short-circuits and grounds with attendant risk of fires. Insurance cannot be had against damage from fire unless the current in each size of conductor is automatically limited to a certain value by means of a cut-out (fuse or circuit breaker) in series with that conductor. These values are specified in a table of "ALLOWABLE CARRYING CAPACITIES OF WIRES" to be found in the "NATIONAL ELECTRICAL CODE" prescribed by the fire insurance companies.

Reduction of voltage, due to impedance of the conductor, causes flicker of lights or change of motor speed as the amount of load on the system changes. Experience indicates that for GOOD SERVICE the CHANGE OF VOLTAGE AT THE LOAD should not exceed 3 PER CENT TO 5 PER CENT FOR LIGHTS and 10 PER CENT FOR MOTORS.

Less voltage drop for a given current or load is obtained by reducing the impedance of the circuit; larger size of conductors means less resistance, and closer spacing means less inductance and reactance. Minimum spacings for usual circuit voltages are specified in the National Electrical Code.

In SELECTING THE SIZE OF WIRE, the following procedure is convenient:

1. Determine how many amperes each part of the load will require, from known or estimated values of the power, voltage and power factor.

2. By vector addition of these currents for all loads connected thereto calculate the actual total current which must flow in each branch circuit, main and feeder, and the corresponding phase angle.

3. From the Table of Allowable Carrying Capacities (Table II) select the smallest size wire that will safely carry this value of total current.

4. Decide what (minimum) spacing of wires shall be used, and if necessary calculate the EQUIVALENT DISTANCE. Then from suitable tables determine the total reactance and total resistance, in ohms, for each conductor of branch, main and feeder.

5. From the service voltage known to be available at feeder switch, and the maximum allowable per cent voltage drop, calculate minimum allowable voltage at the load (lamps or motors, at end of branches).

6. Knowing the amount of current in each conductor and its phase relation to the load voltage, ADD BY VECTOR DIAGRAM the total resistance volts and the total reactance volts to the minimum load voltages and determine thereby what voltage at feeder switch is necessary to deliver this minimum voltage at load.

7. If this calculated feeder switch voltage is less than the voltage actually available there, the sizes of conductors selected (according to N. E. C.) are correct; if not, larger sizes of wire must be tried.

Conductors smaller than No. 14 B & S gauge may not be used anywhere in accordance with the N. E. C. except in lighting fixtures between the outlet box and the lamp socket.

Induction motors can safely take more than their normal full-load current for a considerable length of time, and the conductors should be selected for a current 1.25 times full-load current of large motors. Where mains or feeders supply several motors, conductor material may be saved by taking into account the demand factor.

DEMAND FACTOR is the ratio of the maximum demand of any system, or part of a system, to the total connected load of the system or of the part of system under consideration. Values determined from measurements on various classes of load are to be found in engineering handbooks. In general, an increase in either the number or variety of consuming devices served causes a decrease in the demand factor. Typical values for motor loads are given in Table IX of the Appendix. Demand factors for lighting loads may vary from 100 per cent for store lighting to only 20 per cent for some classes of residence; in the modern trend of development it is safest to assume 100 per cent as has been done in this book.

The size of wire is generally controlled by the allowable carrying capacity (N. E. C.) when the circuit is short, and by the allowable voltage drop when the circuit is long. When the drop controls, the sizes of feeder, mains and branches should be adjusted to one another so that the total drop is distributed approximately as indicated in Table V, in order to give best service.

In SINGLE PHASE INSTALLATIONS, the THREE-WIRE SYSTEM is almost always used for mains and feeder, and two-wire for the branches, which are evenly divided or arranged so that the same load is drawn between neutral and either outer wire. If the system is so BALANCED, there is no current and no voltage drop in neutral. However, neutral should be of same size wire as each outer wire.

The advantage of the three-wire system of single-phase over the two-wire system is a considerable saving of conductor material, amounting to at least 25 per cent. Two-phase offers no saving of copper over single-phase, for the same voltage. Three-phase three-wire is the most economical system of distribution. In any system, the amount of conductor material and consequently the cost of system is greatly reduced by using higher voltage, since thereby the amount of current to be carried is less.

In three-phase three-wire systems, resistance volts in phase with the line current and reactance volts in quadrature with the line current are added by vectors to the load voltage between line wire and neutral in order to find service voltage between line wire and neutral. This latter multiplied by $\sqrt{3}$, or 1.73, gives the voltage between line wires at feeder switch or service point.

## PROBLEMS ON CHAPTER VIII

**Prob. 21–8.** Find the size of wire to be used in feeder and mains to supply the lights from Panels A and B, Fig. 158, if three-wire three-phase system is used instead of two-wire single-phase. Use the same distances and loads as in Fig. 158, balancing the phases. Check the voltage drop to farthest lamp.

**Prob. 22–8.** Compare the amounts of copper used to supply loads on Fig. 158, under the different systems.

(*a*) Two-wire single-phase (Example I).
(*b*) Three-wire single-phase (Prob. 7–8).
(*c*) Three-wire three-phase (Prob. 21–8).

**Prob. 23–8.** What per cent of copper could be saved by installing the system of Prob. 3 8 as a three-wire three-phase system? Check voltage drop to farthest lamp under this system.

**Prob. 24–8.** Calculate the voltage drop to farthest lamp for the correct size of conductors to supply load of Prob. 5–8 if a three-wire three-phase system is installed.

**Prob. 25–8.** If the motors in Fig. 160 are changed to 115-volt three-wire three-phase motors of the same horse power, and a three-wire system is installed, calculate sizes of wire for the installation. Divide lamps into 3 branches of sixteen 40-watt lamps each.

**Prob. 26–8.** Check voltage drop to farthest lamp of Panel A in Prob. 25.

**Prob. 27–8.** Compare the weight of copper necessary for installation to the load of Prob. 25, under the following different systems:

(*a*) Two-wire single-phase (115-volt motors).
(*b*) Three-wire single-phase (230-volt motors).
(*c*) Three-wire three-phase (115-volt motors).
(*d*) Four-wire two-phase (115-volt motors).

**Prob. 28–8.** Compute Prob. 24, using 25 cycles instead of 60.

**Prob. 29–8.** If single-phase motors of same horse power and voltage were installed in Fig. 167, and the system were changed to a two-wire 115-volt single-phase. calculate the sizes of wire to be used for the different parts.

**Prob. 30–8.** Change the motors in Fig. 167 to 230-volt single-phase motors of the same horse power. What size conductor must be installed in feeder and mains if the system were changed to three-wire 115 : 230-volt single-phase?

**Prob. 31–8.** If motors in Fig. 167 are changed to four-wire two-phase motors of the same horse power, what size conductor must be installed using the four-wire 115-volt two-phase system?

**Prob. 32–8.** Compare the amounts of copper used in the different systems represented by Example III, Prob. 29, 30, 31.

**Prob. 33 8.** Show how two indicating wattmeters would be connected in at the feeder switch of Fig. 167 to measure the total load.

**Prob. 34–8.** What is the demand factor of a traffic signal containing 3 lamps, for the red, yellow, and green lights? The yellow light comes on with the green just before the signal changes to red.

**Prob. 35–8.** A group of one hundred twenty 100-watt lamps is to be supplied by a 115-volt, three-wire, three-phase system. With lamps arranged as a balanced load, what size conductors must be used? Will this size conductor be adequate to carry the current if the fuse in one line burns out while all the lamps are turned on?

**Prob. 36–8.** If all the lamps fed from one of the outer wires on Panel B of Fig. 160 were burned out or turned off, but all other loads remained unchanged, what then would be the amount of current in each of the (three) conductors in Main 2? What would be the total current in each feeder conductor?

**Prob. 37–8.** If half of the lamps on one side of the neutral of Panel A, Fig. 160, were replaced by fan motors consuming the same amount of volt-amperes, at 0.60 power factor, calculate how many amperes flow in each conductor of Main 1 and of the feeder, all other loads remaining unchanged.

**Prob. 38–8.** A three-wire single-phase circuit delivers 15 kv-a at 120 volts to a load having 100 per cent power factor, and also 6 kv-a to another load of 50 per cent power factor at the same location, the loads being on opposite sides of neutral. How many amperes flow in each conductor of the circuit, and what size does the N. E. C. require it to be? Rubber-covered wire is used.

**Prob. 39–8.** The circuit of Prob. 38 is 100 feet long and the wires are spaced 3 inches apart in the same plane. Calculate: (*a*) Equivalent distance; (*b*) Total resistance and reactance drop in each conductor. Frequency 60 cycles; (*c*) Volts required between neutral and each outer wire at the input end of main, in order to deliver the stated voltages at the output end. Is the total voltage drop on either side too much for good lighting?

**Prob. 40–8.** Solve Prob. 39 on a basis of 500 feet length for the main. Adjust size of conductors if necessary to give proper amount of drop. Compare results with those of Prob. 39, and draw conclusions.

**Prob. 41–8.** A single-phase two-wire feeder of 600,000 circular-mil cable delivers 450 amperes at 440 volts 60 cycles and 85 per cent power factor to a distance of 800 feet. The conductors are spaced 6 inches apart. If the voltage at the input end of feeder is maintained constant, by what percentage will the voltage at the output end rise as the load is reduced to zero (that is, what will be the "voltage drop" expressed as per cent)?

**Prob. 42–8.** Solve Prob. 41 on the assumption that power factor of load is 50 per cent instead of 85 per cent. Compare the results, and draw conclusions.

**Prob. 43–8.** What must be the size of two (equal) smaller two-wire feeders in parallel, to deliver the same load as the single large feeder of Prob. 41? Rubber-insulated wire in both cases; same spacing. Assuming the cost of conductor to be in direct proportion to weight of metal, what per cent is saved by this substitution?

**Prob. 44–8.** Calculate for the parallel feeders of Prob. 43 the per cent rise of voltage at output end of feeders from full load to zero load, voltage at input end being maintained constant. Compare this result with the corresponding figure for Prob. 41, and draw conclusions.

**Prob. 45–8.** Try to explain why the heavier conductors in Table II cannot be permitted to carry as large a current in relation to their size, as the smaller conductors.

**Prob. 46–8.** A No. 0 single-phase two-wire feeder was originally installed to carry a load of 120 amperes, but the load grew to 220 amperes and the station operator attempted to meet the situation by paralleling another two-wire feeder of No. 1 gauge. Both feeders have the same spacing, 6 inches, between conductors, and both have rubber insulation. Do you think that together they are sufficient?

**Prob. 47–8.** If the No. 1 feeder of Prob. 46 were carrying the largest current permitted for it by N. E. C. (Table II), what would be the total voltage drop along the wires due to impedance? The feeders are 500 ft long. How many amperes must the No. 0 feeder in parallel then be carrying, since the impedance drop must be identically the same along both feeders? Are these two currents in phase with each other? How much is the total current? What ratio between current in each feeder and total current? How do these results bear on Prob. 46?

**Prob. 48–8.** By definition, one ampere of alternating current generates heat at the same rate in any given resistance as one ampere of direct current in the same resistance. How many watts are lost in the feeder of Prob. 41? What per cent is this of the power put into the feeder?

**Prob. 49–8.** If in Prob. 41 the same amount of power were delivered at the same power factor but at double voltage (880 volts), to what fraction of its former value would the heat loss in conductors due to their resistance, be reduced?

**Prob. 50–8.** If the resistance loss in Prob. 49 at doubled voltage were permitted to be the same percentage of power input as in Prob. 41, what per cent reduction could be made in weight of conductors?

# CHAPTER IX

## MOTORS, STARTERS AND CONTROLLERS

During the past fifty years alternating-current power systems have grown to the point of almost entirely displacing direct-current systems for general power and lighting purposes. Much of this growth was due to the greater flexibility of the a-c system, but the development of a wide variety of efficient a-c motors and control devices has also been of great importance.

**85. Torque.** The measure of the tendency which a motor has to turn is called the **torque** of the motor. It is measured by the product of the pounds pull at the rim of the pulley times the radius of the pulley in feet. Thus, a motor, which will exert 6 pounds pull at the rim of a pulley of $1\frac{1}{2}$ ft radius, is said to have a torque of $6 \times 1\frac{1}{2}$, or 9 pound-feet torque.

If the pulley is at rest when the torque is measured, the value thus obtained is called the **starting torque** of the motor. The torque of the motor while running is called the **running torque,** or merely the torque, and its value is usually quite different from that of starting torque.

The running torque of motors up to about 200 horsepower may be measured by a "Prony brake." A simple form of this device is shown in Fig. 174. A loop of rope, fastened between the two spring balances $A$ and $B$, is wound around the pulley and the tension is adjusted until the motor is loaded by friction to the desired amount. The **difference** between the readings of the two balances multiplied by the radius $R$ of the pulley is the value of torque exerted by the motor. Thus, if balance $A$ reads 20 pounds, balance $B$

reads 5 pounds, and the radius of the pulley is 6 inches ($\frac{1}{2}$ foot), the torque delivered is $(20 - 5) \times \frac{1}{2}$, or $7\frac{1}{2}$ pound-feet.

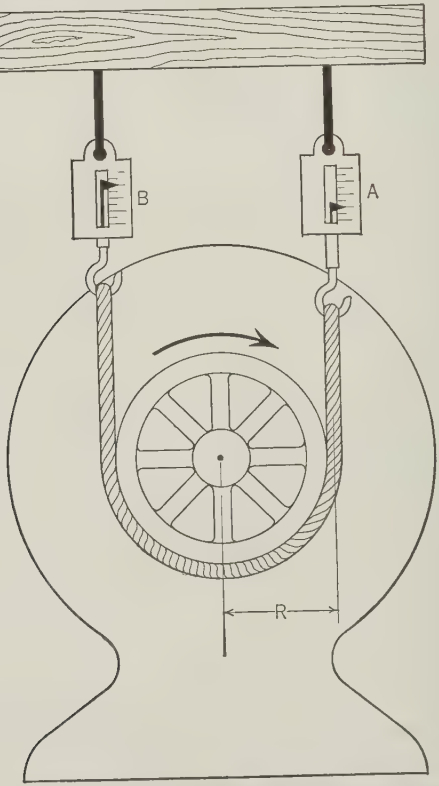



FIG. 174. A Prony brake, used to measure torque and horsepower of a motor.

If the speed of this motor is also measured, the horsepower output can be calculated from the formula:

$$\text{Horsepower} = \frac{\text{Torque (pound-feet)} \times \text{Speed (rpm)}}{5255}.$$

In the above example, suppose the speed is 1720 rpm. The horsepower is then

$$\frac{7.5 \times 1720}{5255} = 2.45 \text{ horsepower.}$$

**86. Efficiency.** The efficiency of a motor always means the relation between output and input:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}.$$

Of course, it is necessary to express both output and input in the same units. Frequently the output of a motor is measured in horsepower and the input is measured by a wattmeter in watts. Usually it is better to convert the output from horsepower to watts by multiplying by 746, which is the number of watts which is equivalent to one horsepower. Thus, in the example above, the output of the motor might have been expressed as $2.45 \times 746$, or 1830 watts. If the input to this motor was 2200 watts, the efficiency would be $\frac{1830}{2200} = 0.832$, or 83.2 per cent.

**Prob. 1–9.** A motor fitted with a Prony brake, as shown in Fig. 174, runs at a speed of 1150 rpm. The balance $A$ reads 55 pounds, and balance $B$ reads 15 pounds. If the pulley is $2\frac{1}{2}$ feet in diameter, what is the torque developed?

**Prob. 2–9.** What is the output of the motor of Prob. 1 in horsepower? In watts?

**Prob. 3–9.** If the motor of Prob. 1 draws 10 kilowatts from the line, what is its efficiency?

**Prob. 4–9.** What torque is exerted by a 100-horsepower, 600-rpm motor, when delivering full load?

**Prob. 5–9.** The efficiency of the motor of Prob. 4 is 89 per cent at full load. What power does it take from the line?

**87. Synchronous Motors.** It is a general fact concerning all electric generators that under proper conditions they will operate as motors. Similarly, most electric motors will operate as generators. Of course, some modifications or

adjustments are often necessary to cause a generator to operate successfully as a motor, or vice versa, but the general statement is universally true.

It follows, then, that alternating-current generators of the type shown in Fig. 137, 138 and 139 may operate as alternating-current motors. When so used they are called **synchronous motors,** because the rotor revolves in synchronism or " in time " with the alternations of the current. This type of motor is generally constructed with the permanent magnetic poles on the rotating part. These poles are excited by direct current from some outside source. The armature is on the stationary frame as in Fig. 137 and 139. Since the current in the armature is continually alternating the magnetic poles formed by the armature windings are continually changing from north to south polarity and from south to north, and the revolving field poles have to move around in time with these changes so as always to keep a pole of the proper (opposite) polarity close to each changing pole of the armature.

Thus the rotor of Fig. 138 has 30 poles and the armature has 30 poles. Consider one phase only. If the armature is connected to a 60-cycle circuit, any given north pole produced by the armature windings will change to a south pole during the next $\frac{1}{120}$ of a second, and the south pole of the rotor which was opposite this north pole of the armature must move on and be succeeded by a north pole. There being 30 poles on the rotor, the rotor must turn through $\frac{1}{30}$ of a revolution in order to present its opposite (next) pole to the south pole now produced in this position by the armature winding. Since each armature pole changes every $\frac{1}{120}$ of a second, the rotor must move through $\frac{1}{30}$ of a revolution in each $\frac{1}{120}$ of a second to keep up with the changes of the armature poles. This is what is meant by the rotor revolving **in synchronism** with the alternations of the current.

The **rated speed** of this motor is its synchronous speed,

which can be found as follows: If the motor makes $\frac{1}{30}$ of a revolution in $\frac{1}{120}$ of a second, in a whole second it would make $120 \times \frac{1}{30}$, or 4 revolutions; thus, the speed would be $60 \times 4$, or 240 revolutions per minute.

Note that the speed of a synchronous motor may be found by the equation

$$\text{Speed (rev. per min.)} = \frac{\text{Frequency} \times 60}{\text{Number of pairs of poles}}$$

$$= \frac{60 \times 60}{15}$$

$$= 240 \text{ rpm.}$$

The advantages of these motors are:

(*a*) They run at constant speed for all loads up to the limit of their capacity, when supplied with power at constant frequency. If too great a load is put upon them, however, they fall "out of step" and stop.

(*b*) The field windings can be "over-excited" and the current taken by the armature will then have a leading power factor. This tends to correct whatever lagging power factor other devices may put upon the line as seen in the preceding chapters. When so used they are called **synchronous condensers.**

Until comparatively recent years, synchronous motors were subject to two disadvantages which seriously limited their use. They were not self-starting under load, and in smaller sizes they were unstable and likely to fall out of step due to surges. Modern designs have overcome these disadvantages, particularly in the larger sizes.

**Prob. 6–9.** At what speed will the 30-pole generator of Fig. 138 and 139 run as a synchronous motor on a 25-cycle line?

**Prob. 7–9.** At what speed will a 24-pole 25-cycle synchronous motor operate?

**Prob. 8–9.** How many poles must a synchronous motor have in order to operate at 720 rpm on a 60-cycle system?

**Prob. 9–9.** At what speed will the motor in Prob. 8 operate on a 25-cycle line?

**88. Induction Motors. Polyphase.** The polyphase induction motor as illustrated in Fig. 175, 176, and 177 is the simplest and most common form of alternating-current motor for industrial use. Note that the stator or armature, shown

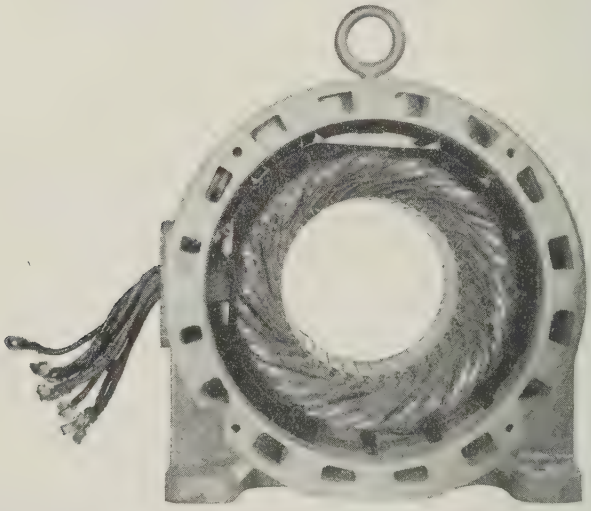


FIG. 175. Stator of polyphase induction motor. *Westinghouse Elec. & Mfg. Co.*

in Fig. 175, consists of a laminated steel frame, having slots in which formed coils are laid. The rotor, shown in Fig. 176, is of the **squirrel-cage** type; its " winding " is made by casting aluminum into holes punched in the laminations which form the core. The polyphase current from the supply line is led into the stator windings only. The rotor has no electrical connection to the line, thus doing away with all brushes and slip rings.

This motor is self-starting, even when connected to its normal full load, and has the characteristics of a direct-current shunt motor. That is, when unloaded, it has a certain definite speed, depending upon the number of poles

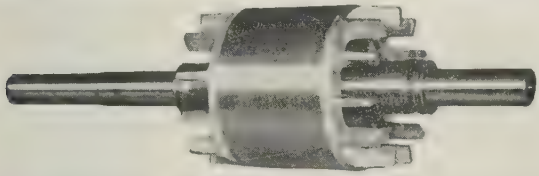


FIG. 176. Rotor of polyphase induction motor. Squirrel-cage type. *Westinghouse Elec. & Mfg. Co.*

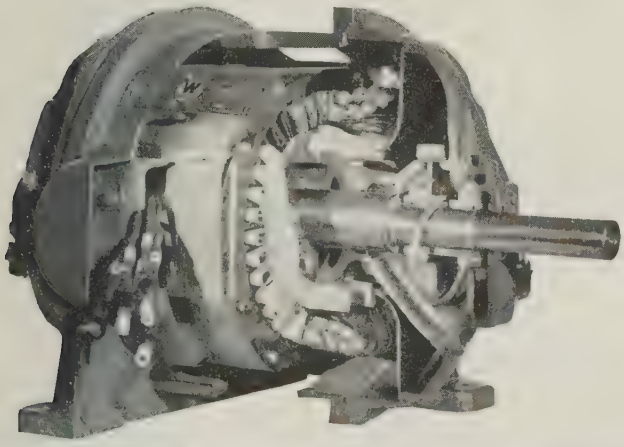


FIG. 177. Assembled squirrel-cage induction motor with frame cut away to show internal appearance. *Westinghouse Elec. & Mfg. Co.*

on the stator and the frequency of the power supply. As the load increases, the speed falls off slightly at first then more and more until, at a considerable overload, the motor stops. Due to the very simple and rugged construction, it demands no attention except occasional oiling.

**89. Starting Torque of a Polyphase Motor. Rotating Field.** The action of a polyphase induction motor may be seen from a study of Fig. 178, 179 and 180. Fig. 178 shows

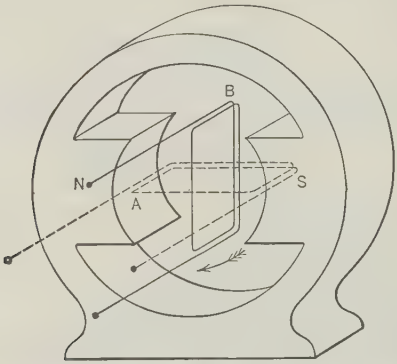



FIG. 178. Skeleton view of a two-phase generator. Coil *A* generates maximum voltage at same instant coil *B* generates no voltage.

the arrangement of the two armature coils of a two-phase generator. Fig. 179 is a diagrammatic representation of the two-phase generator of Fig. 178 connected to a two-phase

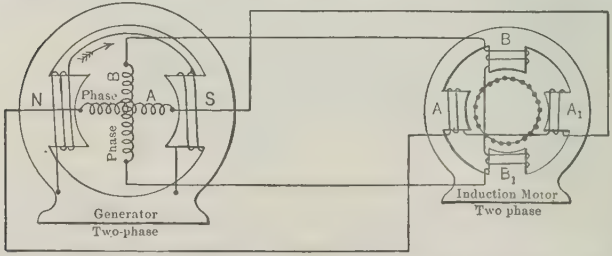



FIG. 179. Two-phase two-pole induction motor supplied by the generator of Fig. 178.

induction motor. Phase *A* of the generator is connected to two Poles *A* on the stator of the motor, and Phase *B* of the generator is connected to the two Poles *B* of the motor. The

Poles $A$ and $B$ of the motor are represented as though they were salient or distinct like the poles of a direct-current motor. It will be seen from the illustration of the stator $S$ of an actual induction motor in Fig. 175, that the poles are not distinct, but are merely regions of the frame surrounded by coils. It requires careful tracing out of the windings of the stator of an actual motor to determine how many poles it has. The effect, however, is the same as though the poles stood out from the frame as shown in Fig. 178 and 179.

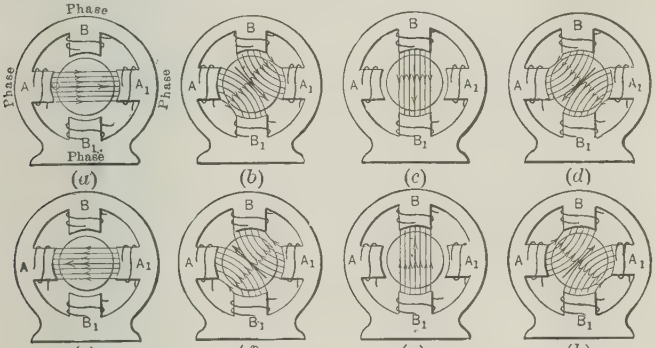



FIG. 180. Two-phase two-pole induction motor. Diagrams show how the direction of the resultant magnetic flux due to combined action of two phases makes one complete revolution during each cycle of line voltage.

It will be seen that in Fig. 178 and 179, at the instant shown, Phase $A$ of the generator is cutting across the magnetic field at the fastest rate, while Phase $B$ is in the neutral position and therefore not cutting the magnetic field at all. Therefore, at this instant a maximum voltage is set up in Phase $A$ of the generator and is sending current through the coils of Phase $A$ in the motor, while there is no voltage in Phase $B$ of the generator and thus no current through the coils of Phase $B$ of the motor. Thus we have a strong mag-

netic field from Pole $A$ to Pole $A_1$ of the motor and no field from Pole $B$ to Pole $B_1$, as is seen from Fig. 180*a* which represents the magnetic conditions in the motor at the instant shown in Fig. 178. The arrow in the center of Fig. 180*a* represents the general direction of the magnetic field set up; note that it is from $A$ to $A_1$.

When the armature of the generator has turned 45° from its present position, Phase $A$ will not be cutting magnetic lines so rapidly and Phase $B$ will have begun to cut magnetic lines. Thus the current in the coils of Phase $A$ of the motor will have decreased and a current will have started in Phase $B$. The magnetic field would then assume some shape like Fig. 180*b* in which the general direction of the field has been turned around so that it goes from Poles $A$ and $B$ to Poles $A_1$ and $B_1$. Comparing the direction of the arrow in Fig. 180*b* with that in Fig. 180*a*, we see that the magnetic field has rotated through practically 45°.

When the armature of Fig. 178 has turned through another 45°, Phase $A$ will be in the neutral position and generating no voltage while Phase $B$ will be generating its maximum voltage. Thus the magnetic field between poles $A$ and $A_1$ of the motor will have died out and the field from Pole $B$ to Pole $B_1$ will be at its maximum as shown in Fig. 180*c*. Note that the arrow showing the general direction of the magnetic field has advanced 90° from the direction of the field in Fig. 180*a*.

Fig. 180*d* shows the motor field condition at an instant 45° later, when Phase $A$ of the generator has begun to cut lines of force in the opposite direction and so has begun to magnetize Poles $A$ and $A_1$ in the opposite direction by reason of a reversed current in this phase. Poles $B$ and $B_1$ have grown weaker, because Phase $B$ of the generator has passed the point of maximum cutting and is now sending less current to magnetize the $B$ poles. The resultant field of the motor at this instant has turned around another 45°.

In Fig. 180*e*, Phase *A* of the generator will be cutting lines again at the maximum rate, only in the negative direction, and Phase *B* will be again in the neutral position. The result is a magnetization of Poles *A* and $A_1$ in the direction opposite to that of Fig. 180*a*, and no magnetization of Poles *B* and $B_1$.

As the current in Phase *A* again grows less and that in Phase *B* acquires strength in its negative direction the magnetic field shifts around to the position shown in Fig. 180*f*.

Passing successively through the positions shown in Fig. 180*g* and 180*h*, the direction of the motor field finally swings around into the position shown at the starting of the cycle in Fig. 180*a*.

Note that as the current in the phases passes through one complete cycle the magnetic field of the motor has swung around through one complete revolution. In other words, we have here a motor with a **rotating magnetic field, although it is the magnetic lines which rotate and not the pole structure.**

Note that although the representations of the two-phase motor in Fig. 179 and 180 have the appearance of showing a **four-pole** motor, they really represent a **two-pole** motor. If Fig. 180*b* represented a four-pole motor, Pole *A* being north, Pole *B* would have to be south. But Pole *B* like Pole *A* is north, and is, therefore, a part of the north pole area. In other words, all adjacent poles of the same polarity are counted as one pole area, or simply as one pole. Therefore, in Fig. 180*b*, Poles *A* and *B* constitute one north pole, and Poles $A_1$ and $B_1$ constitute one south pole. Similarly, in Fig. 180*d*, poles *B* and $A_1$ constitute one north pole and $B_1$ and *A* together form one south pole.

**90. Why the Rotor Tends to Revolve.** The fundamental reason why the rotor of a polyphase induction motor revolves is because the polyphase currents in the stator windings

produce a rotating field. The action of this rotating field as it cuts the copper rods of the rotor sets up voltages and currents in the rods. The stator magnetism then pushes the current-carrying rods around in the direction of rotation of the field.

This is seen more clearly if we consider the action on one rod of the rotor shown under the face of Pole *A* in Fig. 180*a*. If this rod is standing still, the field moving **up** across it would cause a voltage to be set up in it tending to cause a current to flow **out** of the face of the paper toward the reader.

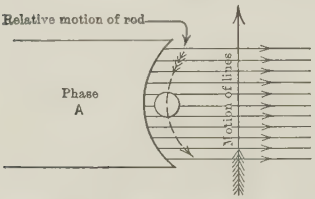



FIG. 181. Compare Fig. 180*a*.

To test this by the right-hand rule, consider the field to stand still and the bar to move down as in Fig. 181. The relative motion is the same as if the rod were standing still and the field moving up. Placing the thumb of the right hand in the direction of the motion of the rod, the forefinger in the direction of the magnetic lines, the middle finger shows the direction of the voltage induced in the bar to be **out** as shown in Fig. 182.

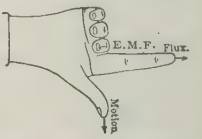



FIG. 182. Directions of magnetic flux, motion and induced voltage bear a fixed relation to one another.

Since all the bars of the squirrel-cage rotor are soldered to rings at both ends, a path of low resistance is offered to any current tending to flow in the rods. Thus we have the elements of a motor, conductors carrying a current in a magnetic field. The rod shown in Fig. 180*a* with an induced current flowing **out** would be pushed up across the pole and the drum to which it was attached would tend to rotate clockwise in the same direction that the field is rotating.

This can be shown as follows:

Fig. 183 represents the rod carrying a current outward, placed in the magnetic field as in Fig. 180*a*. Note that the field about the wire due to the current in the wire is circular in a counter-clockwise direction, so that above the bar the circular field is in the direction opposite to the parallel field of the stator pole. But below the bar, the circular field is in the same direction as the parallel field of the stator pole.

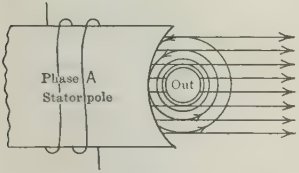



Fig. 183. Showing direction of voltage induced in rod of Fig. 181.

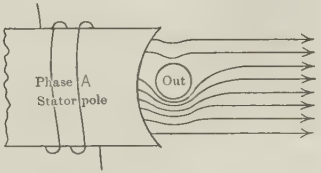



Fig. 184. Current produced by induced voltage of Fig. 181, 183 distorts the magnetic field and produces a force acting upward.

This results, as shown in Fig. 184, in a thinning of the magnetic lines above the rod and in a strengthening of the field below the rod. When we remember that magnetic lines of force act like stretched rubber bands, we can see that the rod will be forced **upward.**

As there is a large number of these rods on the rotor acted upon at all instants by the magnetic field, a large total force is exerted tending to turn the rotor in a clockwise direction, which is also the direction in which the field is rotating.

**91. Unloaded Induction Motor. Synchronous Speed.** If the frequency of the generator in Fig. 178 is 60 cycles per second, then the field of the two-pole two-phase induction motor must rotate 60 times a second or $60 \times 60$, or 3600 times a minute. The speed at which the field of an induction motor rotates is called the **synchronous speed** of the motor

and can be found by the same equation that indicated the speed of a synchronous motor.

$$\text{Synchronous speed} = \frac{\text{frequency} \times 60}{\text{pairs of poles}}.$$

In this case

$$\text{Synchronous speed} = \frac{60 \times 60}{1}$$
$$= 3600 \text{ rpm.}$$

We have seen that the rotor of an induction motor when standing still tends to rotate in the same direction as the field of the stator. If the rotor is unloaded, it will rotate faster and faster until it has a speed almost equal to the speed of the field or the synchronous speed. The rotor speed, however, can never quite equal the speed of the field, because if it did, the field would no longer be cutting the bars of the rotor, and thus no current would be induced in the bars. Since it is the reaction of the circular field produced by the current in the bars upon the surrounding field of the stator windings which causes the force on the rotor bars, as soon as this current stops flowing in the bars, the turning force on the rotor stops and the rotor slows down.

When the rotor is unloaded very little force is required to turn it, therefore the current in the bars need not be great. Accordingly, the rotor revolves just enough slower than the field to allow the field to cut the bars a little and generate sufficient current in the bars to produce force to overcome what little friction or other opposition may be offered to the motion. It is customary to regard the no-load speed as practically synchronous speed.

**Prob. 10–9.** What would be the synchronous speed of the induction motor in Fig. 180, on a 25-cycle line?

**Prob. 11–9.** An induction motor has 12 poles. At approximately what speed will it rotate when unloaded on a 60-cycle system?

**Prob. 12–9.** The zero-load speed of an induction motor is 1795 rpm when connected to a 60-cycle system. How many poles must the stator have?

**92. Effect of Load upon Speed of Induction Motor. Slip.** When we place the rated load upon the rotor of an induction motor, of course more magnetic force is required to turn it. The rotor merely slows down and the field, continuing to rotate at the same speed, cuts the bars of the rotor at a greater rate. This increases the current in the bars until enough force is produced to keep the rotor turning even with the load attached.

The amount by which the rotor falls off from synchronous speed is called the **slip** of the motor for this load. The slip in revolutions per minute is found by subtracting the speed at any given load from the synchronous speed. The slip is generally stated, however, as percentage of the synchronous speed.

**Example 1.** The synchronous speed of a certain induction motor is 1200 rpm. The full-load speed is 1140 rpm. Find:

(*a*) The slip in rpm.
(*b*) The percentage slip.

**Solution.**

(*a*) Slip = synchronous speed − full-load speed.
$= 1200 - 1140$
$= 60$ rpm.

(*b*) Percentage slip $= \dfrac{\text{slip}}{\text{synchronous speed}}$
$= \frac{60}{1200}$
$= 5$ per cent.

**Prob. 13–9.** The full-load speed for the motor of Prob. 12 is 1725 rpm.

(*a*) What is the slip at full load in rpm?
(*b*) What is the percentage slip at full load?
(*c*) What is the slip at zero load?

**Prob. 14–9.** A certain induction motor on a 60-cycle system has a full-load speed of 696 rpm and a zero-load speed of 718 rpm. Calculate:

(*a*) How many poles stator must have.
(*b*) The synchronous speed.
(*c*) The per cent slip at full load.
(*d*) The per cent slip at zero load.

**Prob. 15–9.** On what frequency must an 8-pole induction motor be operated in order to have a synchronous speed of 375 rpm?

**Prob. 16–9.** What full-load speed will a 12-pole induction motor have when operating on a 60-cycle system with a 5.5 per cent slip?

**Prob. 17 9.** A 6-pole induction motor has a 5 per cent slip at full load. What is its speed at full load on a 60-cycle system?

**93. Current and Full-load Power Factor of Induction Motor.** An induction motor is like a transformer with a rotating secondary. The stator is the primary and the rotor is the secondary. The current taken by the primary coil of any transformer is almost directly proportional to the current in the secondary, and the power factor of the primary current becomes practically equal to whatever the power factor of the secondary happens to be, especially when the motor is loaded. Thus the current and power factor of the stator windings depend upon the current and power factor of the rotor, which in turn depend upon the resistance and reactance of the rotor circuit.

The resistance of the rotor is made very low by brazing or soldering the bars into an end-ring or, as in Fig. 176, by actually casting the rotor "winding" into the laminations. Under these conditions the resistance is practically that of a short circuit. The reactance of the rotor at or near full load is usually greater than the resistance, because the bars are surrounded by a good magnetic path of soft steel. The impedance of the rotor, therefore, consists largely of the reactance. The reactance depends upon the frequency of the induced currents in the rotor circuit, and, therefore, it

varies with the load and is always in direct proportion to the slip (see paragraph 95).

We have seen that the power factor of a circuit is equal to the $\frac{\text{resistance}}{\text{impedance}}$, and when the reactance is relatively large, the power factor is correspondingly low. This is true of the induced rotor current. The voltage set up in the bars by the rotating field causes a current to flow against the impedance. The power factor of this current must be low for the heavier loads because the reactance of the rotor circuit corresponding to the larger values of slip is much greater than the resistance. Thus the power factor of the current taken by the stator must be low for heavy loads.

The power factor of an induction motor, therefore, changes with the change in load. With no load on the rotor, the greater part of the current in the stator windings is that part necessary to magnetize the field. This current is called the magnetizing current and, on account of the large reactance of the stator coil, has a low power factor. As more and more load is put on the rotor, a larger and larger power component of current is taken by the stator windings. The magnetizing current, although practically constant, therefore becomes a smaller and smaller part of the total stator current. Thus the power factor rises as the load increases until it reaches a maximum at about full load. At overloads, due to the increased rotor reactance with excessive slip, the power factor again decreases. The values of the current taken by the more common sizes of two-phase induction motors at full load are given in Table VI. The values of the usual power factors of these full-load currents are given in Table VIII. Note that the larger the motor the higher the power factor.

**94. Starting Current.** We have seen that the starting current of small induction motors is usually taken as about twice the full-load current. This is easily accounted for by

the fact that when the bars of the rotor are standing still, the rotating field sweeps across them at a faster rate than when they are rotating in the same direction as the field. Thus a higher voltage is induced in the bars at starting and a greater current flows than after the rotor has attained its speed.

Although it is customary to use twice the full-load current as the starting current, many squirrel-cage motors would take five or six times the full-load current if thrown directly on the line. Accordingly, starting devices are used to enable motors of this type to get up a certain speed before the full voltage is applied. These devices are explained later in this chapter.

**95. Power Factor of Starting Current.** The power factor of the starting current is low, usually not higher than 50 per cent for motors up to 5 horsepower. The main cause for the power factor of the starting current being lower than the power factor of the full-load current is the fact that the reactance of the rotor is greater when the rotor is at rest than when it is rotating. We have seen that the larger the reactance the lower the power factor, other conditions remaining unchanged.

The greater starting reactance is caused as follows: The reactance depends upon the frequency of the current, the higher the frequency the greater the reactance against it. (See paragraph 57.) When the rotor is standing still, each bar is cut by the magnetic flux from two poles of the rotating field during each cycle, since the field rotates in step with the alternations of the voltage and current in the stator or the supply line. The frequency of the voltage induced in the rotor bars is, therefore, the same as the frequency of the current in the stator. In other words, the rotor, when stationary, is just like the secondary of any transformer, and the frequency of the induced current must equal the frequency of the primary current.

Now if, on the other hand, the rotor is revolving at the same speed as the field, then the bars are not cut at all by the magnetic lines and the frequency of rotor voltage and current is zero. But when the rotor of Fig. 180 falls behind and makes just one fewer revolutions per second than the field makes, then the bars are cut by the magnetic field of a stator pole twice each second and a voltage of a frequency of one cycle per second is induced in the bars. At full load the slip of most motors is about 5 per cent, so the frequency of the induced voltage in the rotor would be about 5 per cent of 60, or 3 cycles per second on a 60-cycle system, with the motor running at full-load speed. Thus the induced voltage in the rotor at rest has the same frequency as the line or has 100 per cent of line frequency, while the induced voltage in the rotor at full-load speed is only about 5 per cent of the line frequency. The reactance of the rotor circuit is, therefore, $\frac{100 \text{ per cent}}{5 \text{ per cent}}$ or 20 times as great, at starting as at full-load speed, and the power factor is correspondingly lower.

**96. Selection of Motors. Pull-out Load.** From the foregoing it is seen that the power factor is lower at the start than at full-load speed. But it is also true that the smaller the load on an induction motor the lower the power factor; that is, the power factor is bad for either very light loads or heavy overloads. Furthermore, the efficiency is also low at light load and at heavy overload, being greatest usually at or near rated full load. In selecting an induction motor, therefore, for a given duty, care should be taken to get one having the proper rated horsepower.

Do not get a motor of too great horsepower for the job, because it will not be running at full load and will, therefore, have low power factor and low efficiency. This increases the cost of running the motor and increases all the effects of bad power factor on the line, such as lowering the line voltage and increasing the line losses.

Do not get a motor of too small horsepower, because if the motor is overloaded for a long time it will heat up the stator windings to such an extent that the insulation becomes brittle, increasing the chance for short circuits, and shortening the useful life of the motor.

Furthermore, the load which an induction motor will carry cannot be increased indefinitely even for a short time. As we increase the load on such a motor the rotor gradually slows down in order to acquire a greater torque by means of the increased rotor currents. But when we reach between two and three times the full-load torque, the rotor, instead of merely slowing down a little more, suddenly stops altogether, and unless the power is thrown off the motor will soon be burned out. The torque at which an induction motor stops is called the **pull-out torque** and varies from 2.5 to 3.5 times the full-load torque.

It is, therefore, well to obtain from the manufacturers of the machines to be motor-driven the exact horsepower and speed of the motor required, and to install a motor which meets closely the requirements. In general, it is better to determine the requirements by tests on the machines, or by finding the results of such tests.

**97. Reversing Direction of Rotation.** In order to reverse the direction of rotation of a two-phase induction motor it is necessary to reverse the connections of one phase only, leaving the connections of the other phase as before.

The fact that the direction of the rotation of the field is changed by the reversal of one phase can be seen by again considering Fig. 179 to 180*h*.

Suppose that we reverse the connections of Phase *B* to the motor in Fig. 179. Then Fig. 180*a* still represents the magnetic field condition of the motor when the armature coils of the generator are in the position shown in Fig. 178 and 179. But when the armature coils have proceeded through 45°, the motor field conditions will no longer be

represented by Fig. 180*b*, because, if the connections to Poles $B$ and $B_1$ have been reversed the direction of their magnetic fields must have been reversed. Thus Pole $B$ would become south and Pole $B_1$ north, and combining with the field of $A$ and $A_1$ they would produce the field direction shown in Fig. 185. Note that the arrow showing general field direction has turned counter clockwise instead of clockwise as in Fig. 180*b*, showing that the direction of field rotation has been reversed.

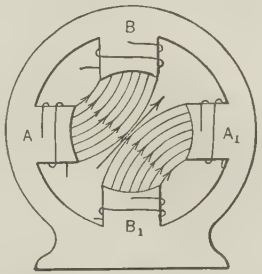



FIG. 185. Phase $B$ of Fig. 180 has been reversed, and the magnetic flux now revolves oppositely.

**Prob. 18-9.** Draw diagrams similar to Fig. 180*a* to 180*h*, showing the field rotation throughout one cycle when the connections of Phase $B$ of the motor are the reverse of those shown in Fig. 179.

**98. Three-phase Induction Motors.** Fig. 186 shows in skeleton the arrangement of the three armature coils of a three-phase generator. In Fig. 187, the three-phase star-connected generator of Fig. 186 supplies three-phase power over a three-wire line to the three-phase induction motor. As the two-phase induction motor in Fig. 179 was represented with distinct poles, so the three-phase motor in this figure is represented with distinct poles. Phase $B$ is connected to Poles $B$ and $B_1$, Phase $C$ is connected to Poles $C$ and $C_1$, and Phase $A$ is connected to poles $A$ and $A_1$. The three phases are star-connected within the motor, so that three leads only are brought out from the frame. The armature of the generator is shown in Fig. 187 at the instant at which Phase $C$ is cutting no lines, and, therefore, no voltage is set up in Phase $C$. Phase $B$ is cutting across the north pole of the generator. Phase $A$ is cutting across the south pole at the same rate that $B$ is cutting across the

north pole. Thus the same voltage is induced in Phase $A$ as in Phase $B$, only in the opposite direction with respect to

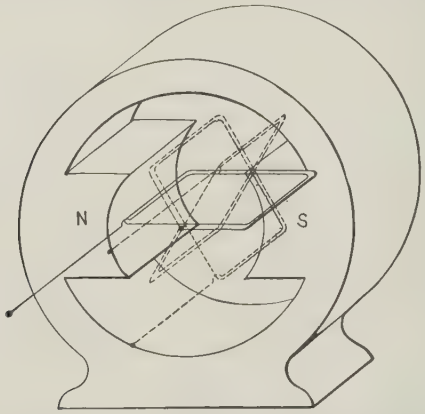



FIG. 186. Skeleton view of a three-phase generator, star-connected. Voltages in three coils reach their respective maximum values one-third cycle apart.

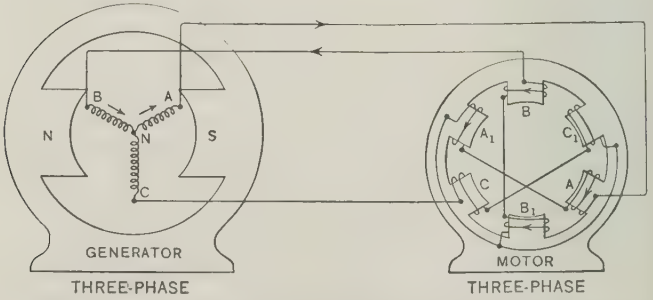



FIG. 187. Three-phase two-pole induction motor driven by generator of Fig. 186.

line terminals, one being toward terminal $A$ and the other being away from terminal $B$, as indicated by the arrows.

Fig. 188*a* represents the generator with the armature in the position at the instant shown in Fig. 187. Fig. 188*b* represents the magnetic field of the motor at this instant. Note that no field is produced by poles $C$ and $C_1$, since there is no current in Phase $C$. The fields produced by Pole $A$ and Pole $B$ are opposite in direction because the currents in Phases $A$ and $B$ are opposite in direction. This causes Pole $A_1$ to have the same north polarity as Pole $B$, and Pole $A$ to have the same south polarity as $B_1$. Thus the magnetic lines pass from the two north poles $A_1$ and $B$ to the two south poles $B_1$ and $A$.

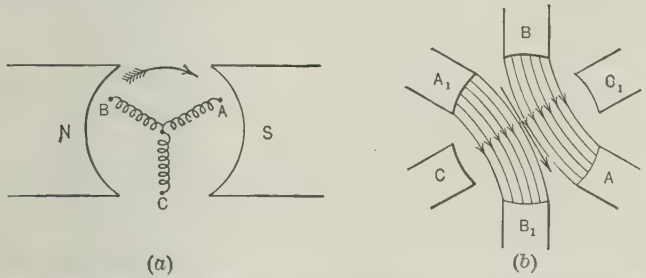



FIG. 188. Fig. 188, 189, 190 illustrate how the magnetic field of the induction motor of Fig. 187 rotates in synchronism with the generator.

Fig. 189*a* represents the position of the armature 60° later. Phase $B$ is now cutting no lines and has no induced voltage. Phase $A$ is still cutting across the south pole, having passed through its maximum value and again decreased until it now has the same value it had in Fig. 188. Phase $C$ is now cutting the north pole at the same rate $A$ is cutting the south pole, and thus has the same induced voltage as Phase $A$, only in the opposite direction.

Note that in Fig. 189*b* the $B$ poles have no field, but that the $A$ poles and $C$ poles have fields of equal strength in opposite directions, because the currents in Phase $A$ and $C$ are

in opposite directions. Poles $C$ and $A_1$ being north, send magnetic lines to Poles $A$ and $C_1$ which are south.

In Fig. 190*a* the armature of the generator has passed through still another 60° and Phase $A$ is now cutting no lines

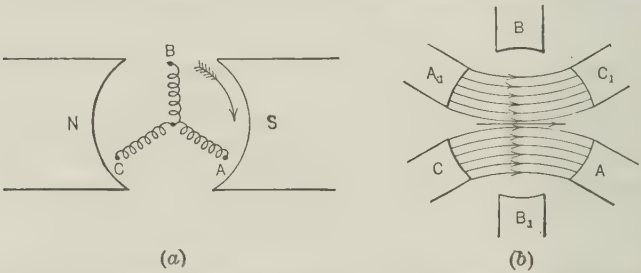



FIG. 189. One-sixth cycle after Fig. 188, the magnetic field of the motor has turned through one-sixth revolution.

and has no voltage induced in it. Phase $C$ is still cutting across the north pole, having passed through its maximum cutting and is now cutting at the same rate as in Fig. 189.

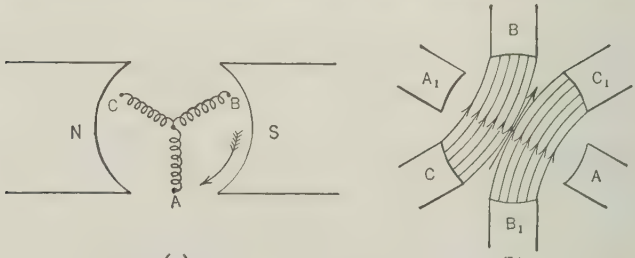



FIG. 190. One-third cycle after Fig. 188, the magnetic field of motor has turned through one-third revolution.

Phase $B$ is cutting the field near the south pole at the same rate that $C$ is cutting the field near the north pole. Thus the voltage induced in Phase $B$ is equal to the induced voltage in Phase $C$, but is in the opposite direction.

Note in Fig. 190*b* that Poles $A$ and $A_1$ have no field. The $B$ poles and $C$ poles have equal fields but in opposite directions, because the voltages of Phases $B$ and $C$ are opposite in direction. Thus Poles $B_1$ and $C$ are north and Poles $B$ and $C_1$ are south.

From the change in direction of the three-phase motor field in Fig. 188, 189, and 190, it is clearly evident that the field is rotating in a counter-clockwise direction in synchronism with the alternations of voltage in the system. Note that although the motor appears to have six poles, it really has but two polar areas and is, therefore, a two-pole motor. Also note that while the rotation of the armature of the generator is clockwise, the field of the motor rotates counter clockwise. This merely happened to be the case because of the manner in which the phases were connected to the motor.

**Prob. 19–9.** Draw diagrams similar to Fig. 190 for three later instants of the cycle, at 60° intervals.

**Prob. 20–9.** Interchange the leads $A$ and $B$ in the motor of Fig. 187, connecting lead $A$ at Pole $B$ and lead $B$ at Pole $A$. Construct three diagrams similar to Fig. 188, 189 and 190. In what direction now is the field rotating?

**Prob. 21–9.** Interchange the leads $A$ and $C$ as placed in Prob. 20–9 and repeat the problem, noting direction of rotation.

**99. To Reverse the Direction of Rotation of a Three-phase Induction Motor.** From Problems 20 and 21 it is seen that **by interchanging any two leads of a three-wire three-phase induction motor, the direction of rotation is reversed.**

**100. Starting Small Polyphase Induction Motors.** The majority of small induction motors of modern design are now started by connecting directly to the line. In doing so, however, protective equipment must be provided which will not burn out due to the relatively large starting current and will still protect the motor from damage due to relatively small overload currents. This is accomplished by means of

the so-called " heater-type " or " thermal-delay " relays and fuses.

The essential protective element of a line-starting device for smaller motors is the thermal overload relay shown schematically in Fig. 191. The line current of the motor passes through a heater coil made of resistance wire. Located within this coil and insulated from it is a strip of " bimetal." This is a strip made by welding two thin pieces of unlike metals together and has the property of changing shape when heated because one metal expands more than the other. When fixed at one end, as shown in Fig. 191, it will deflect from the position shown in solid lines to the position shown in dotted lines when its temperature rises. In its normal position, the bimetal strip acts as a latch holding the contact arm in a closed (solid line) position against the action of a spring. If, however, the strip becomes too hot due to excessive line current, it moves to the trip position (dotted lines) and releases the contact arm which is in turn pulled by a spring to its open (dotted line) position.

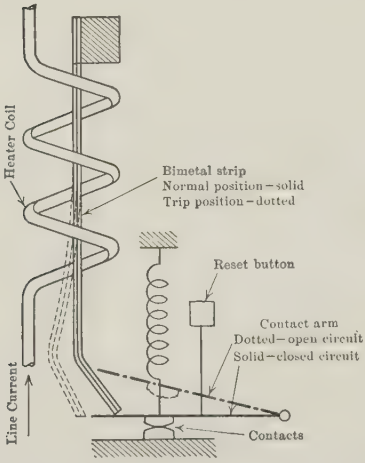



FIG. 191. Schematic diagram of thermal overload relay.

After the overload has been cleared, sufficient time must be allowed for the bimetal strip to cool and return to its normal position. Then, by pushing the " reset " button, shown in Fig. 191, the contact arm can again be latched in the closed position and operation resumed.

Because it requires an appreciable time for the bimetal strip to heat and deflect enough to trip the contact arm, this type of relay will not trip out due to large starting currents which last for only a short time, but will trip on much smaller currents which flow long enough to be harmful to the motor. By proper design of the relay, it is possible to make the bimetal strip heat up at a rate proportional to the rise in motor temperature and thus provide a very sensitive means of protecting the motor even with rapidly changing loads.

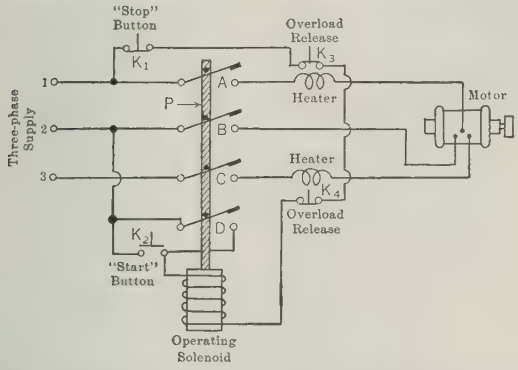



FIG. 192. Connections for push-button control of 3-phase induction motor. Motor is connected directly to line for starting, using thermal overload relays.

Fig. 192 shows the application of the thermal overload relay to a magnetic starter for a three-phase motor. With the motor not operating, the control circuit can be traced starting at line 2. The contacts $K_2$ are open and also the contact $D$ which is connected in parallel with $K_2$. Passing through the operating coil, contacts $K_4$ and $K_3$, which are parts of two thermal overload relays, are both closed. Contact $K_1$ on the stop button, is also closed. If the start button is now depressed, contact $K_2$ closes and the circuit from line 2 to line 1 is completed through the operating coil

and the closed contacts $K_4$, $K_3$ and $K_1$. The energized operating coil now pulls on the operating bar $P$, against either spring force or gravity, and closes the power contacts $A$, $B$ and $C$ to the motor. At the same time, contact $D$ closes and thus "holds" the control circuit closed when the start button is released. Normally the motor is stopped by pushing the stop button, which opens contact $K_1$, releasing the operating bar $P$ and thus allowing contacts $A$, $B$, $C$ and $D$ to open. With contact $D$ open, the stop button can be released.

The contacts $K_3$ and $K_4$, in Fig. 192, are similar to the pair of contacts shown in Fig. 191. They are normally closed when the motor is running but will open if the current in either line 1 or line 3 is excessive. If either contact $K_3$ or $K_4$ opens, the operating solenoid releases bar $P$ and shuts down the motor. The motor cannot again be started until the thermal overload relays are "reset."

Note that thermal overload relays are placed in only two of the three leads to a three-phase motor. Since the line currents to the motor are normally balanced, only one relay is actually required for protection during three-phase operation and it makes no difference in which line it is placed. However, fuses of the ordinary type must always be located ahead of the starter for protection against short circuits,* and if one of these fuses burns out, the motor will try to operate as a single-phase motor (see Par. 106). If a single thermal overload relay is used, and happens to be located in the open line, it will not detect the overload due to single-phase operation, and the motor will overheat. By using two relays, as shown in Fig. 192, the motor is protected against this condition regardless of which fuse burns out.

* The thermal overload element is too slow in operation to act as protection against short circuits. A standard fuse with a current rating about four times the rating of the starter is generally used.

Fig. 193 shows a starter, equipped with thermal overload relays, for starting a 50-horsepower motor directly on the line. Starters of this general type are built for motors as large as 100 horsepower.

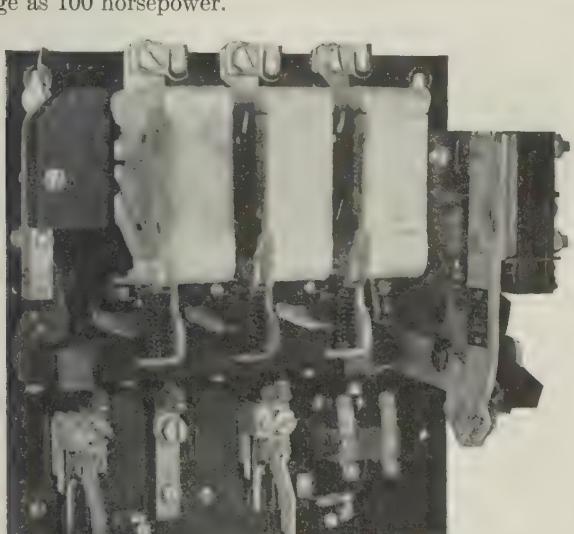


Fig. 193. Line starter with thermal overload relays. *Westinghouse Elec. & Mfg. Co.*

**101. Starting Compensators or Auto-starters.** The most common starter for larger sizes of both two-phase and three-phase induction motors is the compensator or auto-starter. The principle upon which an auto-starter works is shown in Fig. 194, which is a conventional method of representing a two-phase starter. Coils *ab* and *cd* represent two transformer primary coils each wound to operate at 220 volts;

the transformer will have no secondary coils. In the place of secondary coils, tap $x$ is brought out of the middle of coil $ab$ and tap $y$ is brought out of the middle of coil $cd$. The voltage across each half-coil $ax$ and $dy$ will be half the voltage across each whole coil. So if 220 volts from a two-phase system are impressed across each of the coils $ab$ and $cd$, the voltage across $ax$ and across $dy$ will be 110 volts. If Phase $A$ of the induction motor is connected to $a$ and $x$, and Phase $B$ to $d$ and $y$, the voltage across the two phases of the induction motor will be only 110 volts, two-phase. The two coils on

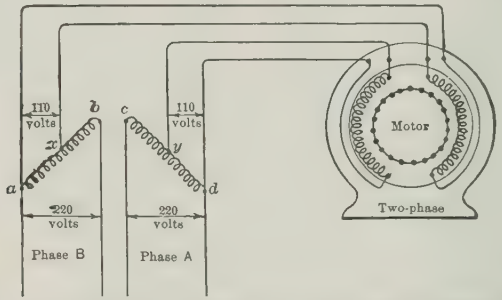



FIG. 194. Auto-transformer arrangement for starting induction motor at reduced voltage; called " auto-starter " or compensator.

the stator of the motor are merely convenient conventional representations of the stator windings and do not at all indicate the actual arrangement of the stator coils.

The motor is thus subjected to only half voltage when starting. As soon as the motor attains the proper speed, a switch disconnects the motor from the compensator and throws it on the line so that it receives full line voltage. The same switch usually also disconnects the compensator from the line. A transformer constructed in this manner, with taps from a single coil, is called an **auto-transformer.** The taps need not be placed midway on the coils but can divide the coils into two parts of any desired voltage. Thus, if a

220-volt coil is tapped at the one-quarter point, the voltage between one end of the coil and the tap will be $\frac{1}{4}$ of 220, or 55 volts, and between the other end and the tap $\frac{3}{4}$ of 220, or 165 volts.

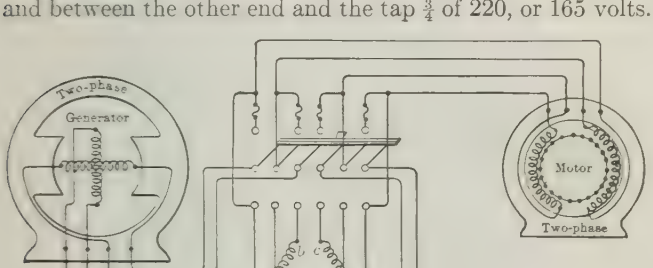



FIG. 195. Wiring diagram of starting compensator and switch, for two-phase induction motor. Starting taps ordinarily at 60 per cent of normal voltage.

Fig. 195 shows the wiring connection for a compensator and switch. When the switch is thrown down, it puts the coils of the transformer across the line, and connects the

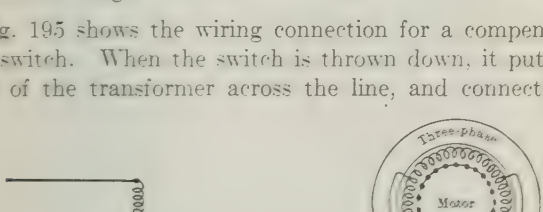



FIG. 196. Wiring diagram of starting compensator for three-phase induction motor, shown without switch.

motor across any part of the coils which is tapped off. When the motor has attained the proper speed, the switch is thrown up and connects the motor directly to the line through the running fuses.

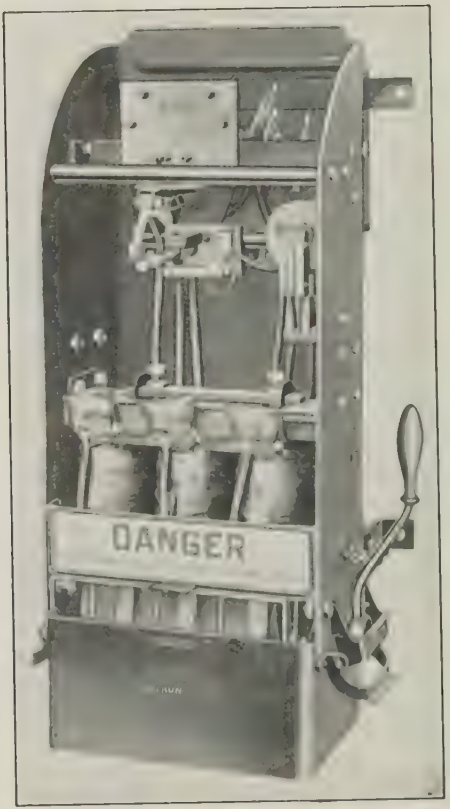



Fig. 197. Three-phase starting compensator for induction motors equipped with no-voltage release (upper right) and overload relay (upper center). The handle on outside is for making starting and running connections. *General Electric Co.*

The same scheme is used for starting three-phase induction motors. Fig. 196 shows the conventional diagram, while the construction of such a three-phase compensator is shown in Fig. 197. Here again the coils may be those of three single-phase transformers star-connected, or they may be the three coils of a three-phase star-connected transformer. The whole coils are connected across the three phases of a three-phase line, while only a certain part of each coil is connected

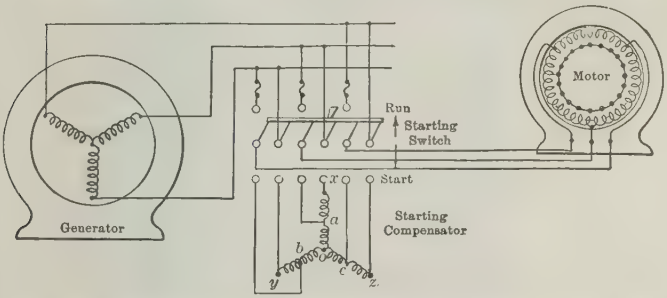



FIG. 198. Starting switch for disconnecting compensator of Fig. 196 from line and motor after attaining full speed.

to a phase of the motor. Each phase of the motor, therefore, is subjected to only part of the line voltage for starting. Fig. 198 gives the wiring diagram for switching the auto-starter to the line and the motor phases to the low-voltage taps, by the down motion of the switch. Throwing the switch up disconnects the auto-starter and throws the motor on the full-line voltage.

When induction motors are operated in inaccessible locations, an automatic starter like that of Fig. 199 may be used. This device is operated by remote control through push buttons and contains a small, motor-driven timer which allows the main motor sufficient time to get up to speed on low-voltage and then automatically switches the motor on the line.

It is not necessary, however, to use three coils in a compensator to operate on a three-phase system. Two coils

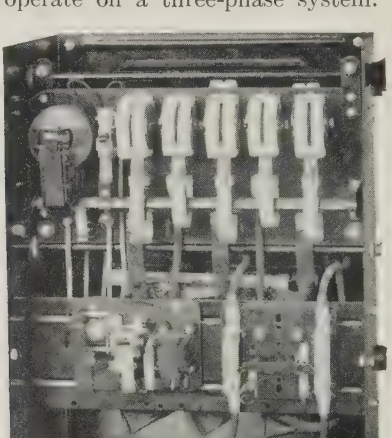


FIG. 199. Automatic starting compensator with thermal overload relays and timing relay. *General Electric Co.*

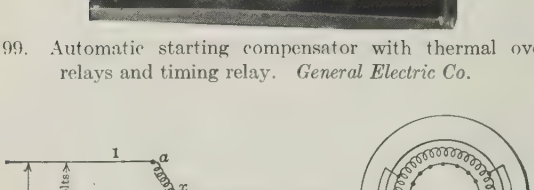



FIG. 200. Starting compensator for three-phase induction motor, simplified to two auto-transformers in open delta.

connected in "**open-delta**" may be used as in Fig. 200. In the open-delta connection, note that coil *ab* is put between lines 1 and 2, coil *bc* between lines 2 and 3, while the third coil

which would naturally go between lines 3 and 1 is omitted. As a matter of fact, the series combination of coils *ab* and *bc* is really between the lines 1 and 3. The point *b* of junction between the coils *ab* and *bc*, and the taps *x* and *y*, are brought to the three-phase motor. This puts the low voltage of *xb*, *by* and *xy*, or the series combination of *xb* and *by*, across the three phases of the motor for starting. The wiring connections for this device are shown in Fig. 201.

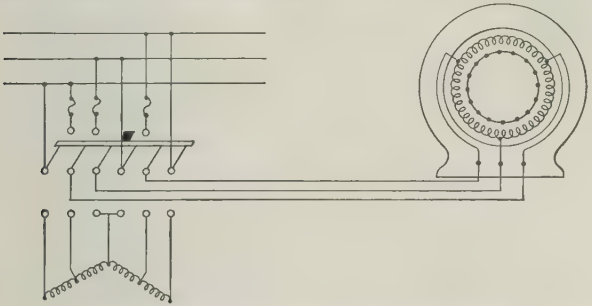


FIG. 201. Starting switch for disconnecting compensator of Fig. 200 from line and motor after attaining full speed.

An induction motor which requires the use of a compensator for starting must also be protected against being connected directly on the line after a power interruption. This is generally taken care of in the design of the compensator by providing a " no-voltage " release in the form of a magnet-operated latch which holds the operating handle in the running position normally, but releases it if power fails. In the automatic compensator of Fig. 199, the operating solenoid will release if power fails and will not pick up until the push button is depressed again.

**Prob. 22-9.** Show by vector diagram that the voltage between *y* and *x* is 110 volts if the voltage from *x* to *b* is 110 volts and from *b* to *y* is 110 volts, when connected as in Fig. 200 to a three-phase line.

**Prob. 23 9.** The voltage across the coil of a single-phase auto-transformer * is 230 volts. The whole coil contains 450 turns. What is the voltage between one end of the coil and a tap, if there are 200 turns between these two points?

**Prob. 24–9.** What is the voltage between the other end of the coil of Prob. 23 and the tap?

**Prob. 25–9.** It is desired to obtain 65 volts from a 115-volt line by means of an auto-transformer wound with 320 turns. Where should the coil be tapped?

**102. Star-delta Connection for Starting Induction Motors.** If the windings of the stator of a three-phase induction motor are star-connected as in Fig. 202, and thrown on to a 220-volt three-wire three-phase line, the voltage across each phase of the winding will be $\frac{220}{1.73}$, or 127 volts, or about 58

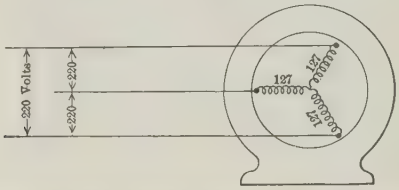



FIG. 202. Each of three phases connected in star gets 58 per cent of line voltage.

per cent of the line voltage. The leads from small three-phase induction motors are sometimes so arranged that the windings can be star-connected in this manner for starting and then be delta-connected by means of a double-throw switch for running. As can be seen from Fig. 203 the windings of the delta-connected motor receive the full-line pres-

* Auto-transformers cannot be used to step down the high voltage of a line in order to bring a low voltage into a building, because one wire of the low-voltage system would then be connected directly to the high-voltage line. Neither can auto-transformers be used for bell ringing, for the same reason; namely, that one wire of the bell circuit would be connected to the lighting system.

sure, in this case, of 220 volts. To use this arrangement, it is necessary to bring out both ends of each winding. This

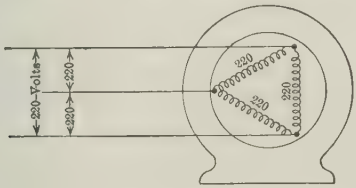



Fig. 203. Each of three phases connected in delta gets full-line voltage.

means that the motor must be supplied with six leads, as shown in Fig. 204.

The switch wiring may be done as in Fig. 204, where throw-

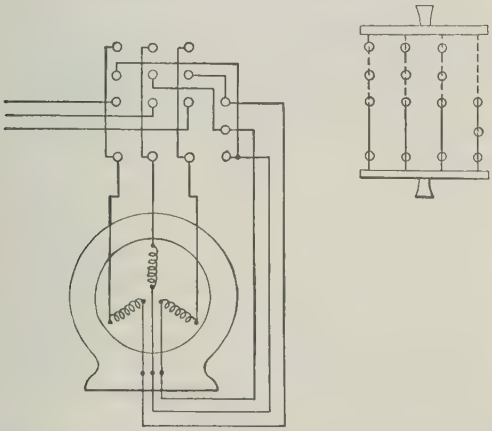


Fig. 204. Starting switch for three-phase induction motor, arranged to connect phases in star for starting and then in delta for running after full speed is attained.

ing the handle down connects the motor coils in star for starting, and throwing it up connects the coils in delta for running.

At the right in Fig. 204 is a switch diagram showing the clips which the knife blades engage when thrown either way. Thermal overload relays and no-voltage release coils are generally used in connection with these switches.

**Prob. 26–9.** An ordinary six-pole double-throw switch can be used as a star-delta switch. Show the wiring connections when such a switch is used.

**Prob. 27–9.** Although not as safe an installation, it is possible to use a three-pole double-throw switch for the star-delta switch. Show the connections for it, when so used.

**103. Starting with Resistance in Series with Stator.** When no other means are available a polyphase induction motor may be started by putting equal resistances in series with each phase, and gradually cutting them out (simultaneously in all phases), as the motor gets up speed. The simplicity of the construction of this type of starter makes the initial cost low, but the resistance grids are bulky, having to be large enough to carry the large starting current while consuming a large part of the line voltage, and they consume a large amount of power. They are, therefore, expensive to operate.

**104. Wound-rotor Polyphase Induction Motors.** On account of the high reactance and low resistance of the squirrel-cage rotor, we have seen that the power factor of the current set up in the rotor bars on starting is very low. This causes the induced rotor currents to lag far behind the induced voltage. The induced voltage in the rotor bars is greatest when the densest parts of the rotating field are sweeping across them. The greatest value of the lagging current in the bars must come later than the greatest value of the voltage. That is what is meant by a lagging current. Thus the current in each rotor bar has its greatest value not at the instants when the densest parts of the rotating field are sweeping across them, but later, after the main part of the field has swept by. Since it is the force between the current

in the rotor bars and the magnetic field which tends to push the rotor around, it is desirable that the greatest value of the current in each bar should occur as nearly as possible at the same time that the strongest part of the magnetic field is passing the bar, in order that the greatest torque may be produced by a given amount of current, or that the least current may be required in order to produce a given torque.

To bring about this result, resistance may be introduced into the rotor circuit to raise the power factor of the rotor currents, because the larger the resistance is in comparison with the reactance, the greater the power factor, as we have

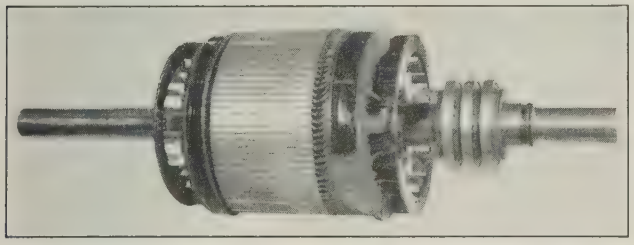


FIG. 205. Wound rotor for three-phase induction motor, used where high torque and adjustable speed are desired. Compare Fig. 176. *Westinghouse Elec. & Mfg. Co.*

seen from Chapter VI. The most successful way of introducing resistance into the rotor circuit is to wind the rotor with insulated wire and bring the terminals out to slip rings as is shown in the rotor of Fig. 205. Brushes bearing on these rings are connected to adjustable resistance grids.

In this way enough resistance can be introduced into the windings of the rotor to produce at starting (zero speed) the greatest force that can be developed for a given amount of current. This occurs when the combined resistance of the rotor windings and external resistance grids equals the reactance of the rotor.

When the rotor gets up speed the slip decreases rapidly,

and we have seen that the voltage induced in the rotor becomes very much less, so in order to keep enough current to maintain the necessary torque, the resistance is cut out. At full load the resistance is dead short-circuited. If the short-circuit is made by means of a switch at the grids, the brushes, of course, are left bearing on the rings. But if, as is sometimes the case, the manufacturer wishes to relieve the motor of the friction of the brushes on the rings, a centrifugal device, attached to the rotor itself, short-circuits the rotor windings, and the brushes are lifted.

There is usually enough external resistance to cut down the starting current to about the full-load current. Therefore no compensator or other special starting switch is needed with a motor having a wound rotor. However, the necessity of slip rings and brushes adds a feature to the motor which decreases its simplicity and ruggedness and adds parts which must be maintained and replaced when worn out.

**105. Speed Control of Polyphase Induction Motors.** The speed of a squirrel-cage induction motor is fixed by the number of poles in the stator, the frequency, and inherent slip of the rotor and cannot readily be changed. Accordingly, when an adjustable-speed alternating-current motor is desired we generally use the wound-rotor type. The full-load speed of this type can be changed through wide ranges, by adjusting the resistance in the rotor circuit. From data published on a 25 hp 60-cycle 8-pole three-phase induction motor of the wound-rotor type, we find that with all external resistance cut out, the full-load speed was 825 rpm. When resistance was introduced into the rotor circuit equal to the reactance at standstill, the starting force was nearly doubled, but the full-load speed fell to 675 rpm. With the maximum safe amount of resistance cut into the rotor circuit, the full-load speed fell to 200 rpm or less than one-quarter of the former speed. Any further increase of resistance would lower the speed so rapidly that it would be likely to " pull-

out," if at any moment a slight increase of load should come on the motor.

In fact the "**speed regulation**" of a wound rotor is very poor, any increase in the load causing a large slowing down and any decrease of load causing a large increase of speed. The efficiency of the motor is also low, on account of the energy consumed in the extra resistance, and the lower speed for the same force.

**Prob. 28–9.** What was the per cent slip of the above motor at full load with the greatest safe resistance in the rotor circuit?

**106. Single-phase Induction Motors.** An ordinary single-phase winding will not produce a rotating field in the stator, because no matter how many poles are made around the stator, the polarity of all the poles changes at the same instant, and thus the magnetic field at all points merely reverses its direction. This produces what is called an **oscillating field.**

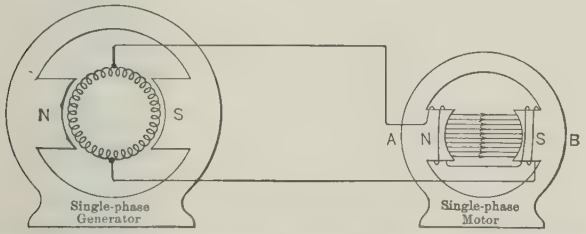



FIG. 206. Single-phase, two-pole motor, represented at the instant when voltage and pole strength of motor are greatest.

This can be seen from Fig. 206, which shows the fields of a single-phase motor magnetized to their greatest strength, because the armature circuit of the single-phase generator is cutting magnetic lines at the fastest rate at this instant and therefore generating the greatest voltage. Note that Pole *A* of the motor is north and Pole *B* is south when the armature coil of the generator is in this position. Fig. 207*a*

shows the motor field when the armature coil of the generator has moved along 45°. Note that the motor field is in the same direction, but is weaker, as the generator coil is not in position to cut lines so fast. In Fig. 207*b* the magnetic field of the motor has died out because the generator coil has now moved 90° from its position in Fig. 206, and is generating voltages which exactly neutralize each other, so that the line

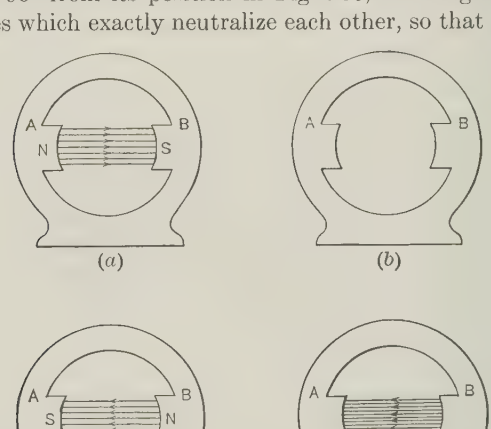



FIG. 207. Illustrating variation of magnetic field of single-phase motor at intervals of one-quarter cycle. Poles change strength and reverse but do not rotate. Such a motor has no starting torque. Compare Fig. 180.

voltage is zero. Fig. 207*c* shows the condition of the motor field when the generator coil has reached the 135° position and is cutting lines in the opposite direction. Thus the motor field has built up in the reverse direction, Pole *B* now being north and Pole *A* south.

In Fig. 207*d*, the generator coil has reached the 180° position and is cutting lines in the opposite direction at the

greatest rate. Therefore the magnetic field of the motor is at its greatest value again, only in the direction opposite to that of Fig. 206.

Note that in all these changes, the field of the motor has merely reversed in direction and no rotating field has been produced. Such currents and poles as are induced in the rotor by the oscillating field of the stator tend to produce equal amounts of torque in opposite directions, and the net starting torque is, therefore, zero.

However, if we can get the rotor of a single-phase induction motor up to such a speed that it rotates nearly in synchronism with the alternations of the current in the stator, then the induced currents in the rotor will continue to occupy such a position in the magnetic field as to produce force or torque tending to keep the rotor revolving even when a load is applied to it. It is necessary, therefore, merely to supply some means of starting the motor and getting its speed up to the point where it develops sufficient torque to keep rotating.

**107. Starting a Single-phase Induction Motor.**

**By hand.** A small (fractional horsepower) single-phase motor can be given enough impulse by hand to make it come up to full speed when the power is thrown on. It will run equally well in either direction when once started.

**By split phase. Reactance start.** Auxiliary coils may be wound on the stator as in Fig. 208 and a so-called split-phase produced. The coils to form Poles $A$ and $A_1$ are the main coils of the single-phase stator and when the main switch is thrown these coils take current directly from the main line. But the coils to form Poles $B$ and $B_1$ are smaller and are called auxiliary coils. They are connected to the line through a reactance, usually of the inductive type, by means of the single-pole switch $S_1$.

The reactance of the auxiliary coils in series with the external reactance is much greater than the reactance of the main coils. Thus the current in the auxiliary coils lags far

enough behind the current in the main coils to cause the field to act somewhat like a two-phase rotating field. Of course

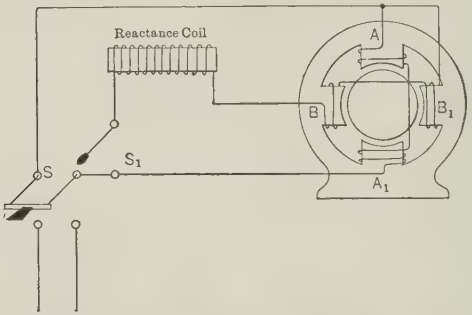



FIG. 208. Split-phase starting of single-phase motor. Switch $S_1$ closed only while starting, making an imperfect two-phase motor.

in a two-phase motor the currents in the two phases are 90° apart, but the current in this split phase does not lag 90° behind the current in the main windings. Therefore the

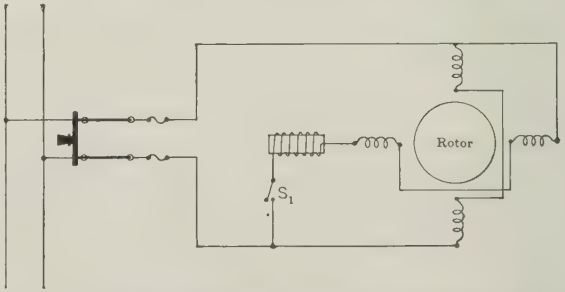



FIG. 209. Conventional electrical diagram to represent Fig. 208. At starting, motor has two circuits with currents less than 90° apart, producing an irregularly rotating field which develops a torque. When up to speed, $S_1$ is opened and we have a single-phase motor.

rotating effect of the field is not as good as in a real two-phase motor, but it serves to start the motor even under considerable load.

After the rotor has attained full speed, the switch $S_1$ is opened and the motor operates as a single-phase motor. Fig. 209 shows more simply the connections of this motor and reactance coil.

**By split phase. Resistance start.** A more compact and cheaper motor can be built if, instead of using an external reactance coil as in Fig. 208, the split-phase effect is obtained by winding the starting coil $B$ with a high-resistance wire. The electrical connections of this type, known as a split-phase resistance-start motor, are shown in Fig. 210. The switch $S$ is usually opened automatically by a centrifugal device when the rotor gets near full speed.

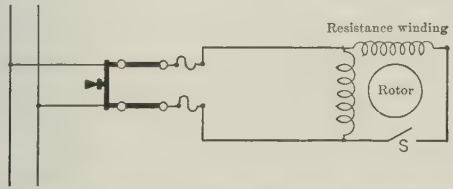



FIG. 210. Circuit diagram of a split-phase, resistance-start motor. Switch $S$ is opened by a centrifugal device when the motor gets up to speed.

**By split phase. Condenser start.** Within recent years there has been an increasing use of single-phase capacitor motors. These are split-phase motors in which a condenser is used in series with the starting winding, as shown in Fig. 211, in order to obtain the necessary out-of-phase field flux for starting. In this case, however, the current in the starting winding leads the current in the main winding instead of lagging. These motors are now frequently made with no provision for disconnecting the starting winding. Some improvement in power factor results from leaving the condenser in the circuit, and with the improvement in quality of condensers which has been made in the last decade, it is found that the condensers will stand such continuous service,

Furthermore, an induction motor running single phase is inherently somewhat noisy because the torque is pulsating at twice the frequency of the voltage supply. With the starting winding and condenser left in the circuit, the magnitude of the torque pulsations is reduced by supplying torque from

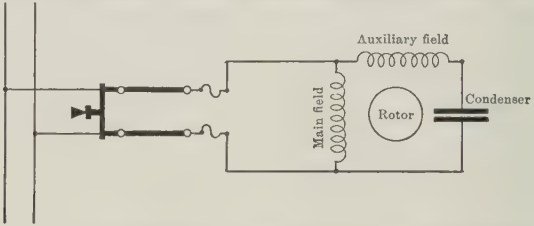



FIG. 211. Circuit diagram of a split-phase, capacitor-start motor.

the starting winding out of phase with the torque due to the main winding. This results in a considerably quieter motor and has led to wide application of the capacitor motor in household devices where noise is objectionable.

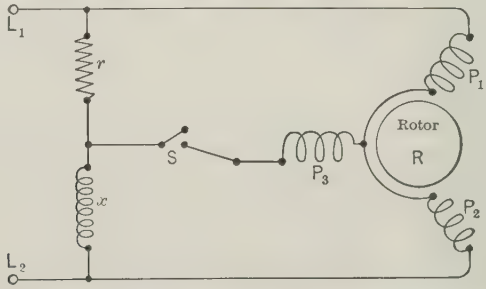



FIG. 212. Phase-splitting arrangement for starting a three-phase motor on single-phase power supply. *S* is opened after attaining full speed.

**By split phase. Three-phase winding.** A three-phase motor after being started will run on a single-phase circuit though it will deliver only somewhat less than half the horsepower. Fig. 212 shows a method of splitting the phases in order to start such a motor.

A resistance $r$ and a reactance $x$ are connected in series across the line and a tap is taken out at the junction of the resistance and the reactance. This tap goes to the third phase winding, the other two phases being in series directly across the line. When the motor has attained sufficient speed the single pole switch $S$ is opened, shutting off the current from coil $P_3$. The resistance and reactance are also disconnected from the line and the motor then operates as a single-phase motor on the coils $P_1$ and $P_2$ in series.

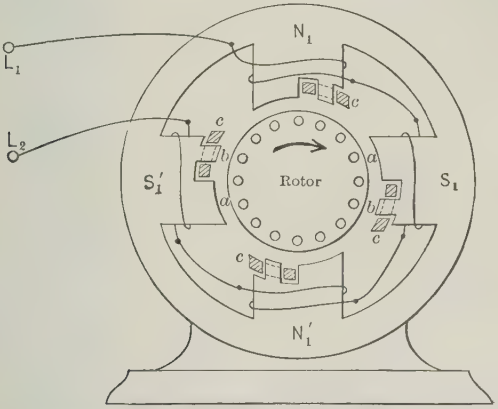



FIG. 213. Single-phase induction motor with shading coils (*c*) in the poles, in order to produce a starting torque. At each reversal of voltage a wave of flux sweeps across the face of each pole from *a* to *b*.

**Shading coils.** The poles of single-phase motors are sometimes equipped with **shading coils.** These are copper rings put around about half of each pole as shown in Fig. 213. The shading coils are labeled *c*. When an alternating current flows in the main coils on the poles, the changing flux cuts the short-circuited shading coil and sets up an opposing current. This opposing current retards the change of the flux in that part of the pole which it surrounds so that

the changes in the field within the shading coil take place later than the changes in the rest of the pole face. This causes a sort of magnetic field wave to sweep across each pole face and produces the effect of a weak rotating field. This effect, however, is enough to start a lightly loaded motor.

**As a repulsion motor.** This method is taken up in the next paragraph.

**108. The Repulsion Motor.** In the "repulsion motor" we have a rotor with a winding quite similar to that employed on the armature of a direct-current machine. At uniform intervals along this winding, taps are connected to bars in a commutator. The brushes which bear upon this commutator are short-circuited together. By shifting these brushes into various positions, we may cause the motor to turn in either direction, or to stand still, when the stator windings are connected to a source of single-phase power. The operating characteristics of this motor are similar to those of a series d-c motor. At zero load, the speed goes indefinitely high, and as the load increases the speed decreases but the torque becomes correspondingly larger. The starting torque is high.

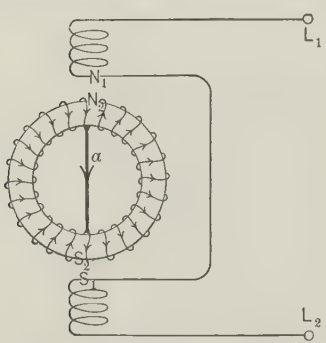



FIG. 214. The currents produced in the short-circuited rotor windings, as marked, cannot produce a torque with the stator field on account of their relative positions.

To understand the operation of the repulsion motor, first consider Fig. 214. The single-phase stator winding connected to line wires $L_1L_2$ produces two poles, let us say, at $N_1$ and $S_1$. Although the rotor is actually drum-wound, a ring winding is shown for simplicity in tracing circuits. A

short-circuit ($a$) is connected between two definite coils which are in line with the stator poles. The flux due to the stator is in fact alternating, and the polarities marked correspond only to a particular part of each cycle. The variation of flux from $N_1$ to $S_1$ induces voltages and currents in the rotor windings short-circuited at $a$, and these currents produce poles on the rotor in line with the short-circuit — or at $N_2S_2$ in Fig. 214. For this position ($a$) of rotor, there can be no torque between $N_1S_1$ and $N_2S_2$, regardless of the strength of stator flux or rotor currents, since the torques developed under each half of any pole are equal and in opposite directions.

If the rotor be turned by hand into the position ($b$) shown in Fig. 215, there will still be zero torque. In this case, the rotor is in the most favorable position to produce torque by interaction between rotor currents and stator flux. But it may easily be seen that the voltages induced in each path of the rotor winding neutralize each other, so that no rotor currents and no rotor poles can be produced.

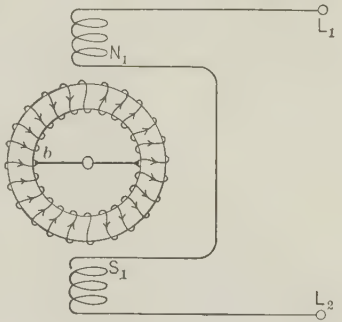



FIG. 215. When the short-circuited rotor of Fig. 214 is in this position, the voltages induced in it by the alternating stator magnetism neutralize one another. Thus there are no rotor currents and no torque, although the rotor is in the most favorable position for developing torque if there were any current.

However, if the rotor be moved to a position somewhere between those shown in Fig. 214 and 215, the resultant emf induced in each rotor path will be greater than zero, and the rotor currents will produce poles on the rotor somewhere between the stator poles, as shown in Fig. 216. Here, if the

rotor is initially in the position (*a*), a clockwise torque will be exerted on $N_2S_2$, and in the position (*b*) a counter-clockwise torque will be produced on $N_2'S_2'$. The torque will not reverse as the current alternates, because both stator and rotor poles reverse simultaneously. In either case, however, this torque will be reduced to zero as soon as the rotor has moved enough to bring the short-circuit into position *cc*, midway between stator poles.

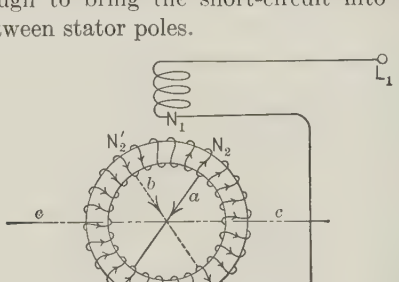



FIG. 216. When the short-circuited rotor of Fig. 214 is in the position *a* a clockwise torque is exerted upon the currents induced in it. When the rotor is in position *b*, a counter-clockwise torque is exerted on it. In either case, the torque lasts only until the rotor has moved into the position *cc*, when the torque becomes zero.

To maintain the torque steadily, it is necessary to adopt means to keep stationary the points on the rotor winding between which the short-circuit is applied. For this purpose, the winding is connected as shown in Fig. 217 to a commutator *CC*, upon which bear the brushes *BB* with a short-circuit between them. The brushes are shifted out of line with the main stator poles $N_1S_1$, whereupon there are induced in the rotor, by transformer action, currents which produce rotor poles at $N_2$ and $S_2$. The stator poles exercise a **repulsive force** upon these rotor poles and produce thereby a torque. By shifting these brushes *BB*, we may have a torque

in either direction, or zero torque. In reality, the internal actions become quite highly complicated by the voltages and currents that arise in the rotor due to speed as soon as the motor begins to turn, but this explanation has been made as simple as possible.

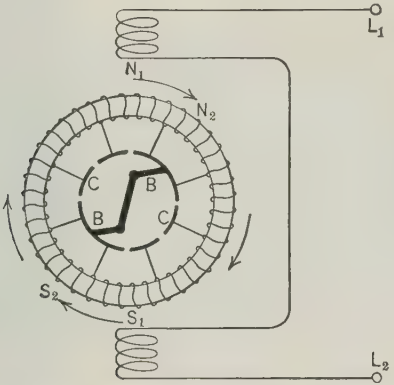



FIG. 217. The repulsion motor. It has a rotor wound like the armature of a direct-current motor, the coils being connected to the commutator $CC$, upon which bear the short-circuited brushes $BB$. If the brushes are set so as to produce the rotor poles in the regions $N_2$, $S_2$, which are neither in line with the stator poles $N_1$ and $S_1$ nor at right angles, a continuous torque will be exerted upon the rotor.

**109. Repulsion Induction Motors.** The straight repulsion motor, which has the characteristics of a series motor, has been applied to various purposes for which the latter would be suitable — such as driving of railroad cars and fans. Its widest application, however, has been as an auxiliary to the single-phase induction motor, to supply the starting torque which the latter inherently lacks. Fig. 218 shows a single-phase induction motor with wound rotor, the rotor winding being tapped to a commutator upon which bear brushes controlled by a centrifugal governor on the shaft. The brushes are short-circuited together and when the motor

is at standstill they bear upon the commutator, being set so as to produce torque by repulsion motor action when the stator is excited. This torque accelerates the motor to nearly synchronous speed, at which point the governor connects all the commutator bars together, and at the same

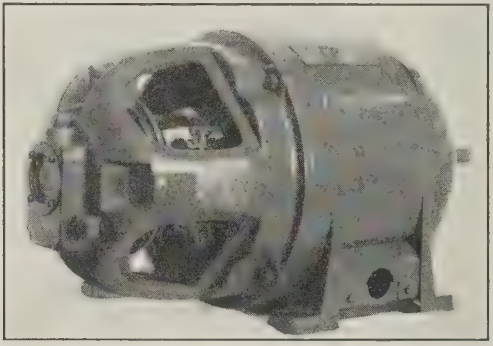


FIG. 218. Single-phase induction motor with commutator and brushes for starting as a repulsion motor. *Westinghouse Elec. & Mfg. Co.*

time throws the brushes out of contact with the commutator, producing practically a squirrel-cage rotor. The motor then operates as a straight single-phase induction motor.

An interesting variation of the repulsion-induction motor is known as the "unity-power-factor motor." The rotor slots contain two distinct windings, a squirrel-cage winding of copper bars at the bottom, and a coil winding at the top connected to a commutator. The electrical connections (with the normal operating characteristics) are shown in Fig. 219. As here indicated, there is also placed in the same slots with the main winding (M.F.) on the stator an auxiliary "compensating winding" (2), the use of which is to improve the power factor of the motor. Two brushes 5 and 6, in line with the stator poles, are short-circuited together, while another pair of brushes (7, 8) fixed midway between the

stator poles is connected in series with the main field. The compensating winding (2) is shunted across the latter brushes (7, 8) and there is included in this circuit a switch operated by centrifugal force which closes the compensating field only after the motor has reached synchronous speed. Between any two brushes there is of course an alternating induced voltage.

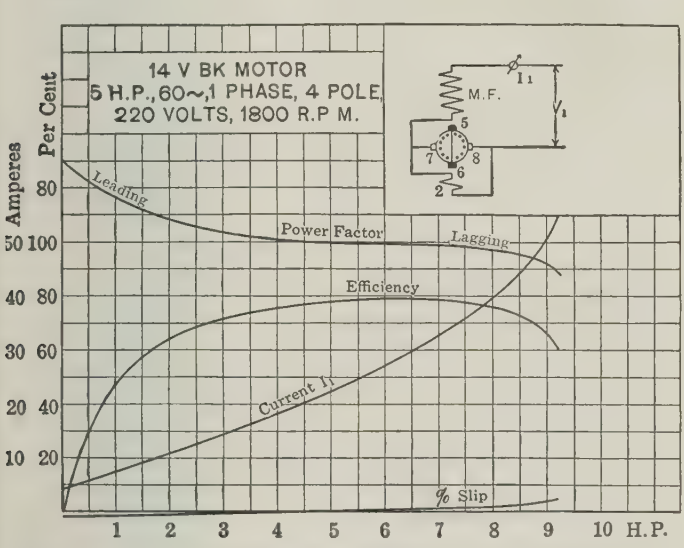



FIG. 219. Curves illustrating the performance, under various loads, of the single-phase, unity-power-factor motor.

In normal operation this motor has at zero load a slip which is negative (speed slightly above synchronism), and the power factor is about 70 per cent **leading.** As the power output increases the speed falls and the power factor rises, the slip being zero and the power-factor unity at about rated load. It should be explained that the power factor may be adjusted by shifting connections on the compensating wind-

ing, and the direction of rotation may be reversed by reversing connections between the main field (M.F.) and the brushes (7, 8). The motor cannot race under any circumstances, because of the squirrel-cage winding in the bottoms of the slots; in this respect it is superior to some other motors which lack the squirrel-cage winding and which will race if some of the brushes become disconnected.

**110. The Series Motor for A-C Circuits.** When the direction of current through a direct-current series motor is reversed without altering the connections between its field and armature windings, the direction of torque and of rotation remain unchanged, because the magnetic poles on both field and armature have their polarity reversed at the same time by the reversed current which flows through both of them. Even if the reversals of current occur rapidly we should expect to find that the torque remains unidirectional; in other words, the series motor should produce a torque tending to turn it in the same direction, when either direct or alternating current is sent through it.

This is in fact the case; but the operation of the motor on alternating-current circuits is decidedly inferior to its performance on direct-current circuits, in the following respects:

*First.* The series motor designed for d-c circuits takes alternating current at a very low power factor, on account of the large amount of inductance in field and armature windings. This is objectionable because with the greatest current which may be carried without overheating, the power developed will be much lower than for the same value of direct current and voltage.

*Second.* There would be excessive heating of the field cores of a d-c series motor operated on an a-c circuit, involving low efficiency and either damage to insulation or reduction of power capacity. This is due to large eddy currents induced in the solid pole-cores. The armature core is laminated even in a d-c machine.

*Third.* The d-c series motor would spark excessively at the brushes if operated on an a-c circuit. This is due principally to alternating voltages and currents induced in the coils that are short-circuited through each brush, by the alternating flux which links with such coils in its path from one field pole to another.

These difficulties are overcome by special windings to such an extent that alternating-current series motors are in successful operation on railway electric locomotives; especially where it is necessary to run the same locomotive on alternating current over part of the system and on direct current over another part. An alternating-current series motor operates even better on direct current than it does on alternating current.

## SUMMARY OF CHAPTER IX

**TORQUE** is the measure of the tendency which a motor has to turn. If the motor exerts one pound force at the rim of a pulley of one foot radius, or two pounds force at six inches radius, it is developing a torque of one POUND-FOOT in either case. The torque, speed and horsepower of any motor are related as indicated in the formula:

$$\text{Horsepower} = \frac{\text{Torque (in pound-feet)} \times \text{Speed (rpm)}.}{5255}$$

**EFFICIENCY** of a motor is always given by the formula:

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}.$$

This result is usually expressed as a per cent value.

**SYNCHRONOUS SPEED** is attained when the rotor moves through the angular distance between centers of adjacent poles during the time required to complete one-half cycle of the line voltage. Synchronous speed is related to the line frequency and the number of poles, as indicated by the formula:

$$\text{Synchronous speed} = \frac{\text{Frequency} \times 60}{\text{No. of pairs of poles}}.$$

SYNCHRONOUS MOTORS have the following distinctive features:

(a) Direct current must be available to excite the field poles; often a small direct-current generator called an "exciter" is coupled to the end of the main shaft.

(b) They run at constant (synchronous) speed up to the limit of their capacity; if too great a load is put upon them, they stop abruptly and become practically a short-circuit on the line.

(c) They cannot "race" under any condition.

(d) They can be made to take lagging, leading or in-phase current from the line, by simply changing the field current from a low value to a high value.

(e) In the smaller sizes they are unstable and likely to "surge" or "hunt," fall "out of step" and stop.

POLYPHASE INDUCTION MOTORS are the most common and useful types of a-c motors. The SQUIRREL-CAGE ROTOR type has the following features:

(a) No electrical connections to the rotor, no slip-rings or brushes; very strong and durable construction; requires little attention.

(b) Is self-starting, even when connected to a heavy load.

(c) Cannot race above synchronous speed under any condition.

(d) Drops below synchronous speed but slightly as load increases up to full load, having the speed-torque characteristics of a d-c shunt motor.

(e) Will carry a large overload before "pull-out torque" is reached; then comes rapidly to standstill.

(f) Has low power factor at starting, also at light loads and at heavy overloads.

(g) Takes large amount of current at starting unless voltage is reduced by auto-transformer or "compensator" for starting.

(h) Speed cannot be controlled except by multiple windings and change of connections, which gives an abrupt and usually a large change of speed.

INDUCTION MOTOR TORQUE is produced by setting up a rotating magnetic field. This field induces currents in the rotor winding and the reaction between the rotor current and the rotating field of the stator produces rotor torque. In order to have currents and torque in rotor, it must rotate slower than stator field, by an amount which is called the "SLIP."

SLIP of squirrel-cage motors is usually less than five per cent at full load; that is, rpm of rotor is 95 per cent of rpm of stator field, or greater. At standstill or starting, slip is 100 per cent and frequency of voltages and currents induced in rotor conductors is same as frequency of line. Rotor frequency and rotor reactance are in direct proportion to the slip; the ratio of reactance to resistance is large at or near zero speed, and consequently the power factor of starting current is low.

WOUND ROTOR may be used instead of squirrel-cage rotor, with the same stator, thus enabling the resistance of rotor circuit to be increased and to be varied at will. Characteristics differ from those of squirrel-cage motor as follows:

(a) Less current drawn from line by same stator at same voltage, when starting.

(b) Higher power factor of starting current.

(c) Greater torque possible for any given value of starting current.

(d) Speed may be controlled over a wide range at any load, by varying the resistance in external circuit connected to terminals of rotor winding.

(e) Speed regulation is relatively poor even with rotor winding short-circuited (highest speed and best regulation), and becomes worse with greater amount of speed control (or of rotor-circuit resistance).

(f) Efficiency usually lower than for squirrel-cage motor under like conditions, becoming rapidly lower with greater amount of speed control.

(g) Slip rings and insulated windings on rotor usually make this motor less rugged and more troublesome than squirrel-cage.

(h) Bulky and expensive rheostats and controllers for rotor circuit.

STARTING polyphase induction motors is accomplished by:

(a) Direct connection to line for small motors, and in some cases even up to 100 horsepower. Thermal overload relays protect the motor against overheating.

(b) Compensators or auto-transformers are used for larger motors.

(c) If both ends of each stator phase are brought out, a three-phase motor can be started with star connection and run with delta connection.

(d) Series rheostats in the stator leads.

(e) With wound rotors, starting resistance may be used in rotor circuit.

PULL-OUT TORQUE is the maximum torque which motor can develop without stopping; it is fixed for any motor by its design, and is usually from two to three times full-load torque, at rated voltage, for polyphase induction motors. For loads which fluctuate widely, this fact may determine the size of motor required.

SIZE OF MOTOR should be quite accurately adjusted to the load; induction motor too small for its load, overheats and develops insulation troubles; too large for its load, operates at low power factor and low efficiency.

STARTING ARRANGEMENTS FOR SINGLE-PHASE INDUCTION MOTORS comprise the following:

(a) Small motors will often come up to full speed in either direction if given a rapid impulse by hand.

(b) SPLIT-PHASE windings will produce a weak rotating field for starting purposes. Such windings may use external inductance or capacitance, or an internal high-resistance winding.

(c) SHADING-COIL may be used on corresponding tip of each stator pole. Rotation will then be in one direction only, from unshaded toward shaded pole-tip.

(d) REPULSION-MOTOR ACTION gives starting torque by use of a commutator and brushes. Change from repulsion motor to induction motor action is caused at or near full speed by centrifugal governor which short-circuits commutator bars together and lifts brushes; or by squirrel-cage winding embedded in bottoms of rotor slots. Such combination is known as "REPULSION INDUCTION MOTOR."

REPULSION MOTOR has rotor like a direct-current armature, turning within a field exactly like induction motor stator. Rotor may turn in either direction or be locked, depending on position of brushes. Speed decreases rapidly with increase of torque, and is very high at zero load, similar to d-c series motor.

COMPENSATING WINDINGS are used on the stators of some single-phase induction motors, to improve the power factor. By suitable adjustments, unity power factor may be had, or the motor may be made to take leading current.

SERIES MOTORS will operate on either d-c or a-c circuits, but not equally well. To avoid low power factor, excessive

heating and sparking when operated on alternating current, it is necessary to design specially both armature and field of the motor.

Single-phase motors in general are larger, heavier and more expensive than polyphase motors of the same power, voltage, frequency and speed, or than direct-current motors having similar characteristics and rating.

## PROBLEMS ON CHAPTER IX

**Prob. 29–9.** What is the full-load torque of a 15-horsepower, 4-pole, 3-phase, 220-volt, 60-cycle induction motor, operating at 4.8 per cent slip?

**Prob. 30–9.** If the full-load efficiency of the motor of Prob. 29 is 89.6 per cent, what power does it take from the line?

**Prob. 31–9.** What is the line current in Prob. 30 if the power factor is 76 per cent?

**Prob. 32–9.** The driving motor of an electric clock is a small, 2-pole, synchronous motor. Operating on a 60-cycle supply, what gear ratio is required between the motor shaft and the hour hand of the clock?

**Prob. 33–9.** If the motor of Prob. 32 draws 5 watts and has an efficiency of 10 per cent, what torque does it deliver? Express result in inch-ounces.

**Prob. 34–9.** An eight-pole synchronous motor is rated to run at 450 rpm. On a line of what frequency will it run at this speed?

**Prob. 35–9.** At what speed will the synchronous motor of Prob. 34 run if connected to a 25-cycle line?

**Prob. 36–9.** Draw the wiring diagram of a three-phase squirrel-cage motor with resistance starter.

**Prob. 37–9.** Draw the wiring diagram of a three-phase wound-rotor induction motor with external-resistance speed control.

**Prob. 38–9.** A squirrel-cage induction motor having 12 poles is rated as having 5.5 per cent slip on a 60-cycle circuit at full load. What is the speed in revolutions per minute?

**Prob. 39–9.** Construct eight diagrams similar to Fig. 180 (*a* to *h*) for the split-phase motor of Fig. 208, assuming that the current in the split phase is 60° out of phase with the current in main pole windings.

**Prob. 40–9.** Construct a wiring diagram showing the proper connections for reversing the motor of Fig. 208.

**Prob. 41–9.** Construct 8 diagrams similar to 180 (*a* to *h*) for the motor as connected in Prob. 40–9.

**Prob. 42–9.** An auto-transformer has 350 turns. Where would you tap it to obtain 35 volts if there are 115 volts across the terminals of the transformer?

**Prob. 43–9.** An auto-transformer has 900 turns. Between one end and a tap, there are 725 turns and 85 volts. What is the voltage between the outside terminals?

**Prob. 44–9.** It is desired to raise a single-phase line voltage of 110 volts to 125 volts for a single-phase motor. Show how it could be done by tapping an auto-transformer of 600 turns.

**Prob. 45–9.** A transformer coil having 1250 turns in series is connected across a 220-volt line. A load of 25 kw in 110-volt incandescent lamps is connected between one end of the coil and a tap to its middle point. Assuming the losses in the coil to be negligibly small, what current must be drawn from the 220-volt line? What current is delivered at low tension to the lamps? Where does the difference of these currents come from?

**Prob. 46–9.** It is a fact, which may be proved theoretically and experimentally, that the torque which an induction motor exerts at any given value of slip is directly proportional to the square of the voltage applied to the stator; thus, for half voltage we should get one-quarter as much torque for the same slip. What should be the torque when starting with a star-delta switch, expressed as per cent of torque which would be obtained if the motor phases were delta-connected directly to the line at starting?

**Prob. 47–9.** At starting, when the slip is always 100 per cent, the impedance and power factor of each phase of an induction motor remain unchanged for all practical values of starting volts and starting current. The apparent power (volt-amperes) and real power (watts) when starting by star-delta switch, bear what percentage relations to the corresponding values which would be taken if the motor were thrown delta-connected directly upon the line at starting?

**Prob. 48–9.** A certain wound-rotor induction motor when started with a dead short-circuit across its rotor rings, with half rated voltage impressed on its stator, takes two times normal full-

load current at 50 per cent power factor. If enough resistance is inserted in the rotor circuit externally so that the motor takes two times rated current when started with full rated voltage on the stator, what will be the power factor of the starting current (approximately)?

**Prob. 49–9.** If the external resistance added to the rotor circuit of the motor in Prob. 48 were sufficient to reduce the starting current to 100 per cent normal, at normal voltage on stator, what then would be the approximate value of power factor of the motor at starting?

**Prob. 50–9.** What would happen if the brushes were all lifted from the slip rings of a wound-rotor induction motor?

**Prob. 51–9.** A three-phase induction motor with 8 poles, connected to 60-cycle mains, turns 750 rpm. What is the frequency (cycles per second) of the currents induced in the rotor? What frequency in rotor for speed of 450 rpm?

**Prob. 52–9.** At starting, the reactance of the rotor circuit of a certain wound-rotor polyphase induction motor is two times its resistance, and the starting current is two times full-load current. If the resistance of the rotor circuit be doubled, to what percentage of rated-load current will the starting current be lowered? Voltage is same in both cases.

# CHAPTER X

## CONVERTERS AND RECTIFIERS

Because alternating-current power can be more cheaply transmitted over great distances at high voltage and transformed to low voltages for industrial uses, over ninety per cent of all the electric power generated in the United States is alternating current. There are, however, many direct-current power systems, which represent a large investment, and it would not be economically wise to convert them to alternating-current systems. Moreover, it is of some advantage to use direct current, particularly in street railway systems. In certain applications such as electroplating, battery charging, and in radio transmitting and receiving equipment, it is absolutely necessary to have direct-current power available. For this reason it often becomes necessary to convert or rectify alternating-current power into direct-current power. For the conversion of large amounts of power it is customary to use either a synchronous converter (commonly called rotary converter), or a polyphase mercury-arc rectifier. These devices may also be used for the conversion of smaller amounts of power, or one of several types of low-power rectifiers may be used.

**111. The Synchronous Converter. Construction.** Direct-current power may always be obtained from a motor-generator set consisting of two distinct machines mechanically connected, one an alternating-current motor and the other a direct-current generator, and under some conditions this combination is used. A synchronous motor is often used as the driving motor; the direct-current generator is

usually of the compound type. Such a set is shown in Fig. 220.

But instead of using two coupled machines it is possible, and generally preferable, to combine the two into one machine, the converter. A synchronous (or rotary) converter is merely a machine which is a combination of a synchronous motor of the revolving armature type and a direct-current generator. Such a machine is shown in Fig. 221. The al-

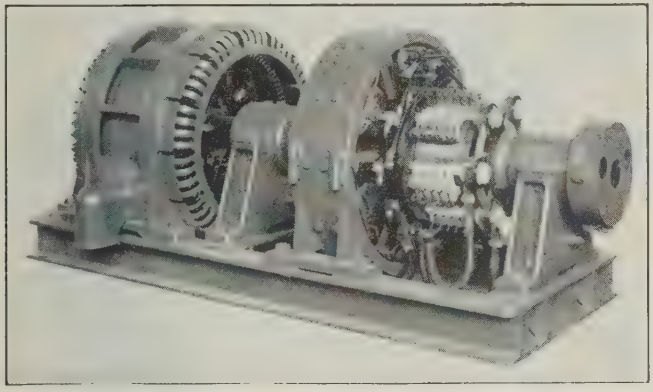


FIG. 220. Motor-generator converter. The motor on the left is a synchronous motor operating on 2300 volts. The direct-current generator delivers 150 kw at 250 volts. *General Electric Co.*

ternating current enters the armature windings through the collector rings shown at the right of the machine and causes the armature to turn in synchronism with the alternations of the current as explained in Chapter IX. We thus have a revolving armature with an alternating current surging back and forth through the windings. We are already familiar with the fact that the armature windings of a direct-current generator always carry such alternating currents, and that a commutator properly connected to the windings is all that is necessary to deliver direct current to a set of brushes.

We have, therefore, only to tap at proper points the windings of the revolving armature of the synchronous motor and connect these taps to the proper segments of a commutator in order to deliver direct current to a set of brushes bearing on the commutator. The commutator is shown at the left of the converter in Fig. 221.

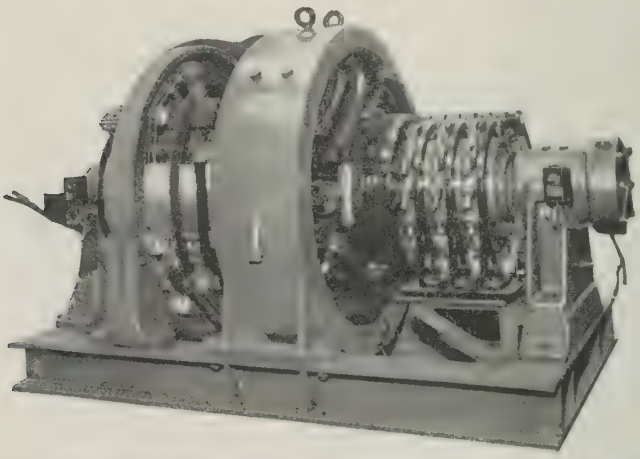


FIG. 221. Synchronous converter. Alternating current is received at the collecting rings on the right and direct current is delivered at the brushes bearing on the commutator at the left.

Thus the same armature fitted with both collecting-rings and a commutator, revolving in a field separately excited from an outside source of direct current, receives alternating current at the rings and delivers direct current at the commutator. Such a machine is called a synchronous converter and may be regarded as a synchronous motor having the revolving armature fitted with a commutator, or as a direct-current generator the armature of which is fitted with collecting rings.

**112. Ratio of the Alternating Voltage to the Direct Voltage of a Synchronous Converter.**

**Single-phase.** Consider the diagram of a simple single-phase synchronous converter shown in Fig. 222. The poles $N$ and $S$ are excited by direct current from an outside source. Alternating-current power is delivered to the collecting-rings $A$ and $B$ from an outside source. Through these rings the lead wires $M$ and $N$ deliver the alternating current to the

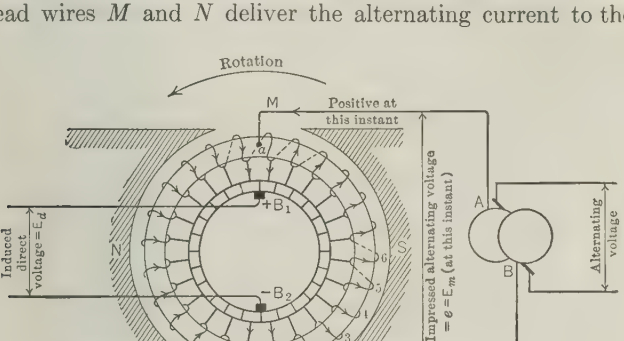



FIG. 222. Diagram of the armature windings and connections of a single-phase synchronous converter. At this instant the maximum value of the alternating voltage is being delivered through the rings to the armature at the tapping points $a$ and $b$, causing it to rotate as marked. The induced voltage marked by arrows on the armature windings is being delivered to the brushes $B_1$ and $B_2$.

armature winding at the two tapping points $a$ and $b$, situated 180 electrical degrees apart from each other. At the instant shown in Fig. 222 the alternating voltage (and current, at unity power factor) would be at a maximum, and, considering the lead $M$ positive at this instant, the armature current would cause the armature to rotate counter clockwise as indicated. There would then be set up in the armature windings an induced voltage as marked in the coils. Note care-

fully that whatever current the alternating line voltage may force through the armature windings at this instant must be forced **against** the induced voltage and must therefore produce a motor effect tending to turn the armature in a counterclockwise direction.

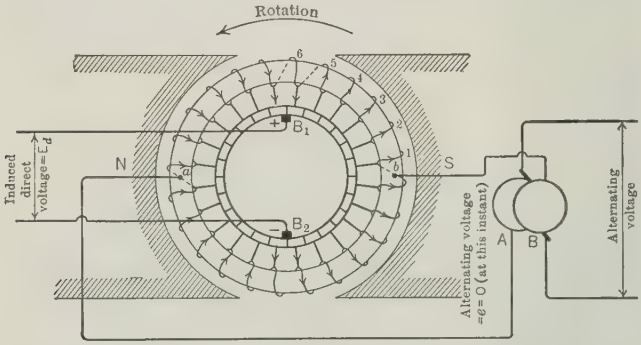



FIG. 223. The armature of Fig. 222 has turned through 90°. The impressed alternating voltage between the tapping points is zero at this instant. The induced voltage between the brushes $B_1$ and $B_2$ is the same as in Fig. 222.

Note also that an alternating current at this instant can flow directly from the wires $M$ and $N$ through the neutral coils to the direct-current brushes without going through the armature. Any appliance attached to the brushes $B_1$ and $B_2$ would at this instant receive all its power directly from the alternating line.

Let us assume, for the sake of simplicity, that the armature resistance and reactance are negligibly small, and that the losses and reactions can be neglected, as is practically true when the converter is running idle. Under these conditions, the voltage induced in the windings is practically equal to the impressed voltage. Now the induced voltage is the voltage which is delivered by the armature to the direct-current brushes, and the impressed voltage is the

maximum instantaneous value of the impressed alternating voltage.

When the armature has turned through 90°, the induced voltage between the taps $a$ and $b$ becomes zero (Fig. 223). But since the machine is in synchronism and in phase with the line voltage, the impressed voltage between the rings $AB$ at this instant has also become zero. There is thus no current in the wires $M$ and $N$. The direct voltage across the brushes, $B_1$ and $B_2$, however, will be the same as before.

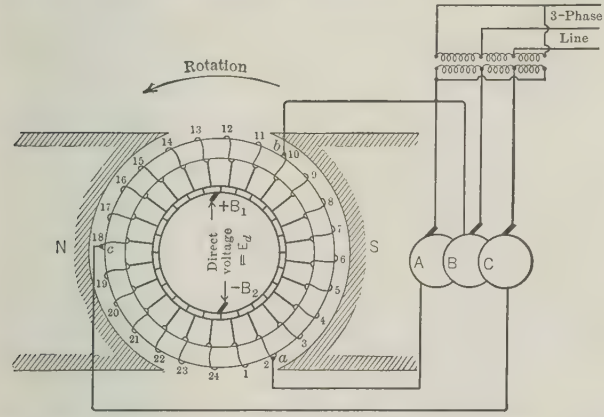



FIG. 224. Diagram of a three-phase three-ring converter. The impressed three-phase alternating voltage is brought to the three equidistant tapping points *abc* on the armature. The induced voltage is delivered as before to the d-c brushes $B_1$ and $B_2$.

We thus have a direct voltage at the brushes which is equal to the maximum value of the alternating voltage at the rings. Of course the alternating voltage applied to the rings is rated in terms of the effective value, which is always 0.707 of the maximum value. Thus the alternating voltage at the rings of a single-phase converter is about 0.707 of the direct voltage at the brushes; the voltage consumed in forcing the current

against the armature resistance causes this ratio to vary slightly from 0.707 at full load.

**Two-phase.** In a two-phase (four-ring) converter, the second phase is tapped at points midway between the single-phase taps, the voltage across one phase being at a maximum when it is zero across the other phase. Thus the voltage across each phase of a two-phase tapping is the same as the voltage across a single-phase tapping. Accordingly, the alternating voltage in each phase of a two-phase converter, also, is 0.707 of the direct voltage.

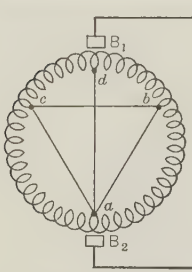



FIG. 225. To produce between brushes $B_1B_2$ a direct voltage represented to scale by the length *ad* requires that there be impressed between three-phase a-c rings a voltage whose maximum instantaneous value is represented to the same scale by *ab* or *bc* or *ac*, or whose voltmeter value is 0.707 times *ab* or *bc* or *ca*.

**Three-phase.** For the voltage relations in a three-phase converter, consider Fig. 224 and 225. The three leads are tapped into the armature windings at three equi-distant points *a*, *b* and *c* (that is, with eight coils between any two taps), and brought out to their respective slip rings *A*, *B* and *C*. The alternating voltage impressed on the rings is thus applied to the armature at these three points and causes the armature to rotate as a synchronous motor.

Fig. 225 represents the voltage relations in the armature windings of the three-phase converter in Fig. 224. The length of the solid lines *ab*, *bc* and *ca* represents the maximum values of alternating voltage applied to the armature windings. The length of *ad* represents the direct voltage between the brushes on the commutator. If these lines are drawn to scale and if the armature windings are laid out on a perfect circle, then the length of either *ab*, *bc* or *ac* will be 0.866 of the length of *ad*.

Thus the maximum value of the alternating voltage is 0.866 of the direct voltage. But we always measure the alternating voltage by its effective value which is 0.707 of the maximum value. Thus the effective value of the alternating voltages is 0.707 of 0.866, or 0.612, of the direct voltage, in a three-phase three-ring converter.

To sum up:

RATIO OF ALTERNATING VOLTAGE TO DIRECT VOLTAGE

| | Ideal. | Actual. |
|---|---|---|
| Single- or two-phase | 0.707 | 0.71 |
| Three-phase * (three-rings) | 0.612 | 0.62 |

**Example 1.** A 220-volt 50-kw single-phase synchronous converter will require what alternating voltage at the rings?

**Solution.** The alternating voltage of a single-phase converter is 0.707 of the direct voltage.

$$\text{Alternating voltage} = 0.71 \times 220$$
$$= 156 \text{ volts.}$$

**Example 2.** What current will flow in the a-c leads to this converter, if the efficiency is 90 per cent, at unity power factor?

**Solution.** If 50 kw (output) is 90 per cent of the power put into the converter, the whole power input is $\frac{50}{0.90} = 55.6$ kw.

$$\text{Current} = \frac{\text{watts}}{\text{volts} \times \text{power factor}}$$
$$= \frac{55{,}600}{156 \times 1.00}$$
$$= 356 \text{ amperes.}$$

**Prob. 1–10.** What a-c voltage must be supplied to a single-phase converter delivering 2500 kw at 550 volts?

**Prob. 2–10.** At 94 per cent efficiency and 0.95 power factor, what current must each a-c lead of the converter in Prob. 1 carry?

* The three-phase transformers may be so connected to a three-phase converter having six rings that the alternating voltage is 0.707 of the direct voltage. This is called a diametrical three-phase connection.

**Prob. 3–10.** What would be the alternating voltage of the converter in Prob. 1 if it were a two-phase machine?

**Prob. 4–10.** What current would each a-c lead of converter in Prob. 3 carry at 94 per cent efficiency and 0.95 power factor?

**Prob. 5–10.** If the converter of Prob. 1 were a three-ring three-phase converter, what would be the voltage between rings?

**Prob. 6–10.** At 0.95 power factor and 94 per cent efficiency, what current would each a-c lead of the converter in Prob. 5 carry?

**113. Control of the Direct Voltage of a Synchronous Converter.** A synchronous motor always operates at a constant speed in synchronism with the alternations of the current. Thus, varying the field strength will not affect the speed. It merely affects the power factor of the alternating current taken by the motor. Similarly, we have seen that the direct voltage is always a certain number of times as large as the alternating voltage. The field strength of the synchronous converter does not affect the direct-current voltage. Too small a field current merely causes the alternating current to lag behind the voltage and too high a field strength causes the alternating current to lead the voltage. Thus we say that when the field is **under-excited** the converter has a **lagging power factor** and when **over-excited** it has a **leading power factor.**

Accordingly, when we wish to change the direct voltage, we naturally resort to the method of changing the alternating voltage, knowing that a corresponding change will take place in the direct voltage. The alternating voltage is sometimes controlled by tapping the secondaries of the transformers so that by means of switches any required changes can be made in the alternating voltage applied to the rings in order to produce the desired change in the direct-voltage at the brushes.

**Example 3.** If it is desired to raise the direct voltage of the 220-volt single-phase converter of Example 1 to 230 volts, what change must be made in the alternating voltage?

**Solution.** The alternating voltage required for a single-phase converter in order to deliver 230 volts direct current equals $0.71 \times 230$, or 163 volts.

The 220-volt converter of Example 1 had an alternating voltage of 156 volts.

The alternating volts would therefore have to be raised from 156 to 163, or 7 volts, in order to raise the direct voltage from 220 volts to 230 volts.

**Prob. 7–10.** What change in the alternating voltage of the converter in Prob. 1 would have to be made in order that the converter may deliver 600 volts direct current?

**Prob. 8–10.** If the alternating voltage for the converter in Prob. 1 were obtained from 2100 turns of a transformer, how many more turns would have to be included in the next tapping interval in order to produce the alternating voltage required for the converter in Prob. 7?

**114. Rectification of Alternating Current.** In many cases where direct-current power is required, some form of rectifier may be used instead of a rotating machine. Because of the modern, high-power rectifiers which are now available, many applications of rectifiers are being made in service which formerly required the use of motor-generators or rotary converters.

A rectifier in general is characterized by the property of **conducting current in one direction only.** If a direct-current voltage is applied with correct polarity to a rectifier element, the resistance of the element will be relatively low. If the polarity of the voltage is reversed, the resistance of the element will be relatively high. Now if an alternating voltage is applied to the rectifier element, it will conduct current more freely in one direction than in the other; hence there will be an **average** flow of current in a single direction. This is equivalent to saying that the rectifier element draws a direct current, even though the impressed voltage is alternating. In actual use, of course, we do not simply connect the rectifier across the alternating-current line. Instead, we connect a rectifier

or a combination of rectifiers in series with a load to which we wish to supply direct current. The rectifier then permits current to flow through itself and the load in one direction and blocks the reverse flow of current in the load.

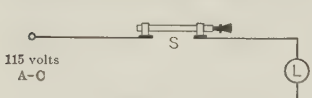



FIG. 226. A knife switch may be used to rectify the supply to the lamp.

An ordinary single-pole knife switch may be considered a simple form of rectifier. In Fig. 226 the switch $S$ is connected in series with the lamp $L$. to an alternating-current line Under these conditions the wave form of the current in the lamp will be that shown in Fig. 227. Let us imagine some mechanical device that is capable of opening and closing the switch $S$ very rapidly, and arrange it to close the switch at point 1 (Fig. 227), open it at point 2, close at 3, open at 4, etc. The resulting current which flows in the lamp is then as shown in Fig. 228. Note that by this process of opening and closing a switch in synchronism with the alternations of the supply voltage, we have stopped the flow of current completely in one direction. The current which flows is not a steady current in this case, but it is a form of direct current known as **unidirectional current.** Any rectifier which produces a current of the form shown in Fig. 228 is called a "half-wave" rectifier, from the fact that only one half of the original alternating-current wave form appears.

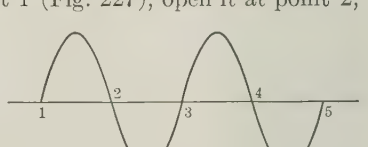



FIG. 227. Wave form of unrectified alternating current.

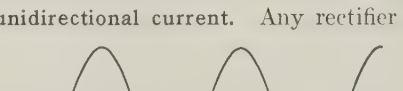



FIG. 228. Wave form from "half-wave" rectifier.

If the single-pole switch of Fig. 226 is replaced by a double-pole reversing switch as shown in Fig. 229, the action of a "full-wave" rectifier may be represented. Referring to Fig. 227, the switch is thrown to the right at point 1, to the left at point 2, right at point 3, etc. The current which then

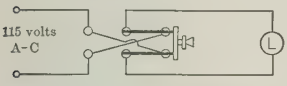



FIG. 229. A double-pole switch represents the action of a full-wave rectifier.

FIG. 230. Wave form from "full-wave" rectifier.

flows through the lamp is shown in Fig. 230. Comparing this current with that of Fig. 228, we see that instead of merely preventing the flow of current in one half of the cycle and leaving gaps between pulses of current, the lower half cycles of the original wave are folded up and fill in the spaces between the upper half cycles. By this means, we are able to double the current capacity of the rectifier for the same alternating-current voltage applied. Furthermore, the "full-wave" current is much smoother than the "half-wave" current and thus approaches more closely the action of a true direct current.

**115. Single-phase Rectifier Connections.** The circuit of Fig. 231 represents schematically the essential connections of a single-phase, half-wave rectifier. The symbol marked "rectifier" indicates that this device allows current to flow in one direction and prevents the flow of current in the opposite direction. Hence the current can flow during only one half-cycle and the wave form must be as shown in Fig. 228.

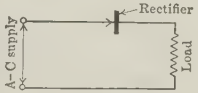



FIG. 231. Schematic representation of a single-phase, half-wave rectifier supplying a resistance load.

Fig. 232 shows a common type of circuit used for obtaining single-phase, full-wave, rectified current. This circuit is

used extensively in radio receivers and in power supplies for amplifiers (see Par. 121). The alternating-current supply is connected to the primary of a transformer $T$. The secondary of this transformer has a tap at the center point of the winding marked $b$.

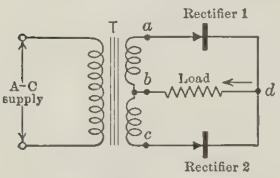



FIG. 232. Schematic representation of a single-phase, full-wave rectifier supplying a load.

Consider the half cycle during which the voltage at $a$ is positive and $c$ is negative. Current then flows away from $a$, through rectifier 1, back through the load to point $b$, and through the transformer winding to point $a$. For the polarities given, no current can flow from $d$ through rectifier 2 because that rectifier blocks the flow of current from $d$ to $c$.

Now if we reverse the polarities, we find that current flows from $c$ through rectifier 2, back through the load to point $b$, and through the transformer winding to point $c$. No current can flow from $d$ to $a$ because of the opposition of rectifier 1.

Comparing the directions of current for the two half-cycles, we find that in both cases the current in the load flows from $d$ to $b$, and is therefore unidirectional. Furthermore, each half cycle contributes to the load current, and the current is therefore full-wave, as shown in Fig. 230.

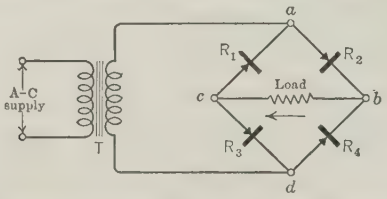



FIG. 233. Bridge connection of a single-phase, full-wave rectifier.

The circuit shown in Fig. 233 represents a method of securing full-wave rectified current with an ordinary trans-

former, using no center tap. This is known as a "bridge" circuit and uses four rectifiers instead of two. However, the transformer is simpler and need be built to deliver only half the voltage required in Fig. 232, for the same value of rectified voltage. In operation, if point $a$ is positive, current flows only through the path $abcd$, since rectifiers $R_2$ and $R_3$ permit current to flow and $R_1$ and $R_4$ block current flow. With reversal of polarity, point $d$ becomes positive, and the direction of current is $dbca$. In either case the direction of current through the load is from $b$ to $c$, and this is a full-wave current.

**Prob. 9–10.** Compare the current ratings of the secondary windings of the transformers in Fig. 232 and 233.

**Prob. 10–10.** Assuming that the loads in Fig. 232 and 233 take the same current and voltage, and that there are no losses in the rectifiers or transformers, compare the volt-ampere rating of the two primary windings of the transformers.

**116. Polyphase Rectifier Connections.** Whenever large amounts of power are to be rectified, as for example in electric railway power supplies, it is more economical to transmit the alternating-current power by the three-phase system. The rectifier is built to operate directly from a three-phase supply or from a six-phase supply.

Fig. 234 shows the essential parts of a three-phase rectifier. A set of transformers is connected with the primaries $A_P$, $B_P$, and $C_P$ in delta, and the corresponding secondaries $A_S$, $B_S$, and $C_S$ in star connection. The secondary terminals $a$, $b$, and $c$ then feed through the rectifiers 1, 2, and 3 to a common junction $d$, and the load is connected between $d$ and the neutral point $n$ of the secondary star. If we trace one of these secondary circuits, that of transformer $A$, for example, when point $a$ is positive current flows from $a$ through rectifier 1, through the load to point $n$ and thus completes the secondary circuit. No other current path is possible for the

current supplied by the transformer *A* because of the blocking action of the other rectifiers.

Note that as far as any single secondary of Fig. 234 is concerned, the circuit is essentially the same as that of Fig. 231. In other words, the three-phase circuit illustrated is really a

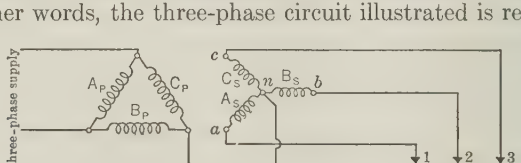



Fig. 234. Schematic representation of a three-phase rectifier.

group of half-wave rectifiers. In Fig. 235, which shows the resultant voltage across the load, it will be noted that phase 1 supplies a half-wave component, followed at 120-degree intervals by half-wave contributions from phases 2 and 3. The dotted lines represent the waves from the individual phases

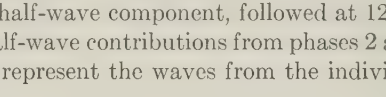



Fig. 235. Form of voltage wave from a three-phase rectifier.

and the solid line represents the resultant voltage. Note that although this voltage is produced by half-wave rectifiers, because of the 120-degree spacing of the individual components, the load voltage never becomes zero as it did in Fig. 228 and 230.

By extending the secondary of each transformer on both sides of the neutral connection, it is possible to produce a six-phase circuit as shown in Fig. 236. This circuit is one commonly used in actual polyphase rectifiers. The operation of this circuit can more easily be grasped by referring to Fig. 237 which shows the secondary connections for transformer

$A$ only. It will be observed that transformer $A$ is supplying full-wave rectified current to the load, exactly as the rec-

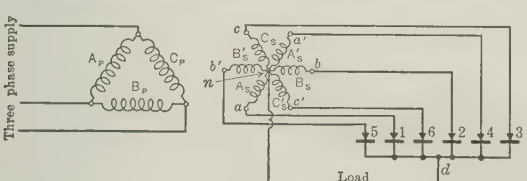



FIG. 236. A six-phase rectifier is composed of a set of three-phase, full-wave rectifiers.

tifier circuit in Fig. 232. Hence, a six-phase rectifier of the type shown in Fig. 237 is really a three-phase, full-wave rectifier.

**Prob. 11–10.** Draw a diagram similar to Fig. 235 showing the wave shape of the voltage produced by the rectifier of Fig. 237.

**Prob. 12–10.** Draw a schematic diagram, similar to Fig. 236, of the connections for a two-phase, full-wave rectifier.

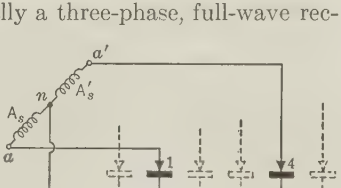



FIG. 237. Part of the circuit of Fig. 236 showing one of the full-wave rectifiers.

**Prob. 13–10.** Draw a diagram, similar to Fig. 235, showing the voltage wave produced by the rectifier of Prob. 12.

**117. Voltage Relations in Rectifier Circuits.** The output voltage of a rectifier in which we are interested is the value indicated on a direct-current voltmeter, that is, the average value. Since the output voltage wave of a rectifier has the same maximum amplitude as the alternating-current input voltage, except for the drop in the rectifier element itself, there is always a definite relation between the average direct-current output voltage at no load, and the alternating-current

input voltage. In the table below, this relation is expressed in two ways: first, the ratio of the average direct-current voltage to the effective value of the alternating-current voltage; second, the ratio of the average direct-current voltage to the maximum alternating-current voltage (1.41 times the effective value).

| Rectifier connection | $\frac{\text{Average d-c volts}}{\text{Effective a-c volts}}$ | $\frac{\text{Average d-c volts}}{\text{Maximum a-c volts}}$ |
|---|---|---|
| Single-phase, half-wave | 0.450 | 0.318 |
| Single-phase, full-wave | 0.900 | 0.636 |
| Three-phase, half-wave | 1.170 | 0.827 |
| Two-phase, full-wave | 1.273 | 0.900 |
| Three-phase, full-wave (six-phase) | 1.350 | 0.955 |

**Example 4.** A three-phase, half-wave rectifier supplies direct current at an output voltage of 3000 volts. What is the effective line voltage of the three-phase supply?

**Solution.** From the table of rectifier voltage ratios we find that

$$\frac{\text{Average d-c volts}}{\text{Effective a-c volts}} = 1.17$$

for a three-phase, half-wave rectifier. Hence

$$\text{Effective a-c volts} = \frac{3000}{1.17} = 2565 \text{ volts.}$$

This, however, is the voltage between the neutral and any line of the alternating-current supply. The line voltage is $1.73 \times 2565$ or 4440 volts.

**Prob. 14–10.** If the same line voltage as in Example 4 were used to supply a three-phase, full-wave rectifier, what would be the direct-current output voltage?

**Prob. 15–10.** What line voltage would be required in Prob. 14 to produce the same direct-current voltage as in Example 4?

**118. Types of Rectifiers in Commercial Use.** A number of different types of rectifiers have been developed to meet the requirements of various applications. These applica-

tions range all the way from the large units which supply power for electric railways, to the very tiny units used in certain kinds of electric instruments.

The **mercury arc rectifier** is the type used where large amounts of rectified power are required, particularly for electric railway supply. It is also built in smaller capacities for general use, such as storage-battery charging.

**Vacuum-tube rectifiers** are used extensively in applications which require a high-voltage, low-current supply. They are used for this purpose in radio transmitters and receivers, X-ray power supplies, high-voltage testing equipment, etc.

**Gaseous conduction rectifiers** are now displacing vacuum-tube rectifiers in certain applications, particularly in radio receivers, because of their higher efficiency. The construction is similar to the vacuum type but they cannot be used at the higher voltages.

**Copper-oxide rectifiers** are used principally in low-voltage applications such as battery charging and telephone power supply. They are also made in very small sizes for use in electric instruments. This permits a direct-current meter to be used for measuring alternating currents and voltages over a wide frequency range.

Miscellaneous rectifiers such as the electrolytic type and the synchronous vibrator type have been quite important in the past but have now been almost entirely displaced by the types listed above.

**119. Mercury Arc Rectifiers.** This is by far the most important rectifier at present for high-power installations. Fig. 238 shows the connection diagram for a small rectifier of this type used in single-phase, full-wave connection. This rectifier is built in a glass tube and represents a type which is still in wide use for low-power work.

The glass tube containing the mercury is exhausted until a very low pressure is obtained. There are two wells, *B* and

$X$, which contain mercury, and two positive graphite electrodes $A$ and $A'$, generally called the anodes.

The anodes $A$ and $A'$ are connected to opposite sides of the line from the transformer. A coil of high reactance but of low resistance ($T_1 - T_2$) is also connected across the transformer. One side of the load is connected to the middle point $C$ of the reactance coil, and the other side to the large mercury well at $B$. The small mercury pool at $X$, which is merely used to start the arc, is connected through a resistance $R$ and switch $S$ to one side of the transformer line. There is such a high resistance offered by the gap between the mercury wells that it would take several thousand volts to start a current through it, so a starting device is necessary. With the switch $S$ closed the tube is tilted until a bridge of mercury is formed across the space between $B$ and $X$. This offers a path from wire $D$ through resistance $R$, from $X$ to $B$, through the load to $C$, through half the reactance coil $T_2$, to the other side of the circuit $E$. An alternating current would therefore flow through this path. If the tube is now tilted back, an arc is formed which vaporizes some of the mercury and so charges it with electricity that the resistance is cut down between the points $A'$ and $B$ and between the points $A$ and $B$. Now mercury vapor possesses the quality in common with almost all metallic vapors of allowing a current to pass easily in one direction and hardly at all in the other direction. Thus the current can now easily pass from either $A$ or $A'$ to $B$, depending upon whether $A$ or $A'$ happens to be positive at this instant. If $A'$ happens to be positive, a current is

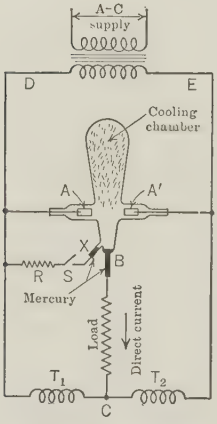



FIG. 238. Connection diagram for a single-phase, full-wave, mercury-arc rectifier.

immediately set up through the vapor between $A'$ and $B$, and flows from $A'$ to $B$, through the load to $C$, through $T_1$ to the other side ($D$) of the transformer $B$. At the next instant $A'$ has become negative and $A$ positive. Practically no current can flow back from $B$ to $A'$, but since $A$ is now positive, a current flows from $A$ to $B$ through the vapor, then through the load to $C$, through $T_2$ to the side $E$ of the transformer. Thus the current through the load is always in one direction.

But the mercury arc has some properties of any other arc, — it requires a voltage to maintain it. Now when $E$ is changing from positive to negative, or vice versa, there is an instant when the voltage is zero, and at this instant the arc tends to go out. The inductive reactance of the coils $T_1$ and $T_2$ is used for the purpose of preserving the arc. For, as we have seen, a current is set up in $T_1$ before the voltage from $A'$ to $B$ dies out. This current, during the decay of the voltage from $A'$ to $B$, tends to keep up its strength because of the inductance of the coil $T_1$ and thus jumps through the vapor from $A$ to $B$ forming a local circuit, — from $A$ to $B$, through the load, through $T_1$ to $A$ again. So,

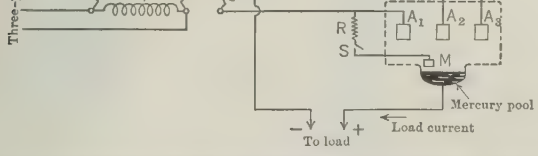



FIG. 239. Schematic connection diagram for a three-phase, half-wave, mercury-arc rectifier. Compare with Fig. 234.

even before $A$ becomes positive on account of the secondary transformer voltage, a current is already flowing from $A$ to $B$, and it is merely increased by the rising positive voltage from $A$ to $B$. Thus the currents overlap one another, as it

were, and maintain a resultant current flowing through the tube continuously.

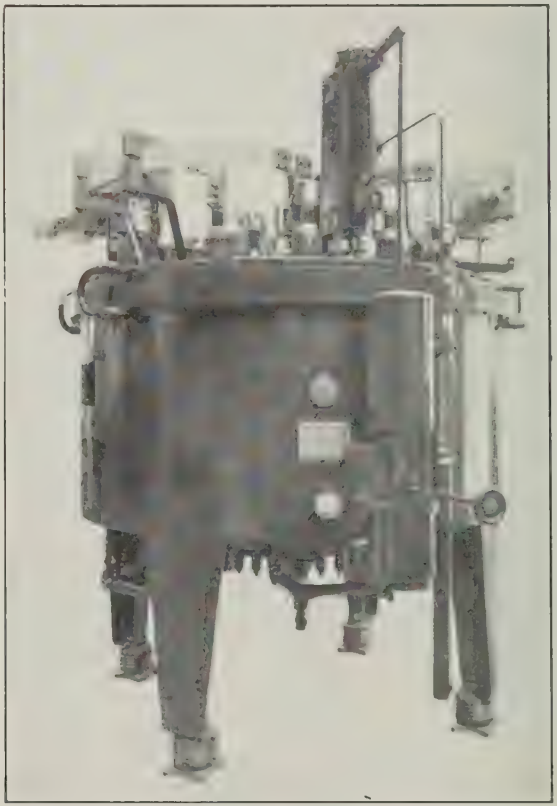


FIG. 240. A 3000-kw, 625-volt, mercury-arc rectifier.
*General Electric Co.*

In polyphase mercury-arc rectifiers, no special provision is necessary for maintaining the arc because the spacing of the polyphase voltage vectors is such that at least one anode

is always positive and the arc is automatically maintained. Fig. 239 shows the connections of a three-phase, half-wave, mercury-arc rectifier. The anodes $A_1$, $A_2$ and $A_3$ are supplied from the star-connected transformer secondaries $A_S$, $B_S$ and $C_S$, respectively. With the switch $S$ closed, the starting anode $M$ is dipped into the mercury pool and then withdrawn. The arc thus established transfers to whichever of the running anodes is at the highest positive potential. During operation the arc transfers from one anode to the next as the rotation of the three-phase voltage vectors causes successive anodes to have the highest positive voltage. Modern rectifiers of this type are built in steel tanks with vacuum pumps provided to maintain the low pressure which is required. Fig. 240 shows a steel-tank rectifier rated to deliver 3000 kw at 625 volts.

**120. Vacuum-tube Rectifiers.** When direct-current power is required at relatively high voltages and low currents, the vacuum-tube rectifier is frequently used. The essential elements of

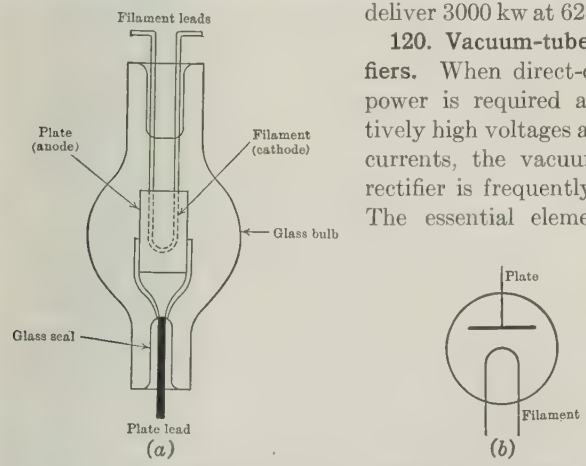



FIG. 241. (*a*) Construction of a high-voltage, vacuum-tube rectifier. (*b*) Symbol used to represent this rectifier in circuit diagrams.

the tube are shown in Fig. 241 (*a*); Fig. 241 (*b*) shows the symbol used to represent this tube in circuit diagrams. A plate made of thin metal bent into an open cyl-

inder is supported within a glass bulb. Inside the plate, but not touching it, there is supported a filament of tungsten which can be heated to incandescence by passing a current through the leads which act as the filament support. This entire assembly is carefully sealed within the tube and the tube is then pumped to a " hard " vacuum, eliminating all gases from the tube.

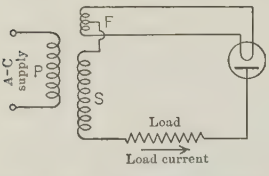



FIG. 242. Half-wave rectifier circuit using a vacuum-tube rectifier.

If the filament of this tube is cold, no appreciable current will be conducted when a voltage is applied between the filament and the plate. However, if the filament is hot, current will flow across the space between the elements when the plate is positive, but will not flow when the plate is negative. This tube therefore can be used as a rectifier.*

Fig. 242 illustrates the use of a vacuum-tube rectifier in a half-wave circuit. The transformer has two secondaries, $S$ and $F$, supplied from the primary $P$. Secondary $F$ is a low-voltage winding used to heat the filament of the tube. A center tap is provided on the filament-heating winding so that the high-voltage secondary $S$ can be connected, through the load, across the plate and filament of the tube. Two tubes of this type are generally used when a full-wave output is required, although some low-voltage, vacuum-tube rectifiers are made with two plates and a single filament in one bulb for use in full-wave circuits. Because of the more efficient

* The action of this device depends upon the fact that a hot, metallic filament emits electrons which can then be attracted toward the plate by a positive voltage. The motion of these electrons constitutes a flow of **negative** current (positive current flows from plate to filament). Since the plate is cold, it does not emit electrons and no current can flow when the polarity is reversed. For a more complete description of the action of these tubes see Chapter XV of " Elements of Electricity," by W. H. Timbie (John Wiley and Sons, Inc.).

operation of gaseous-conduction rectifiers for low-voltage service, the use of vacuum-tube rectifiers is now generally restricted to high-voltage applications, such as in X-ray tube supplies, and in high-voltage testing equipment. For this type of service tubes have been designed to deliver as high as 100,000 volts direct current.

**Prob. 16–10.** Draw a circuit diagram of a full-wave rectifier supplying a resistance load, using two vacuum tubes as rectifiers. A single transformer is to be used, having a secondary with a center tap, and one filament-heating winding with a center tap.

**121. Gaseous-conduction Rectifiers.** The principal drawback of the vacuum-tube rectifier is the large voltage-drop between plate and filament when the tube is operating. This is overcome by putting a small amount of a gas such as argon, or mercury vapor, within the glass tube after the air has been pumped out. Otherwise, the construction of the tube is similar to that of the vacuum-tube rectifier. Current is conducted only when the plate is positive with respect to the filament. Because of the gas or vapor which is present, an arc is established between plate and filament and the voltage drop is quite low, usually less than 20 volts. No reactors are necessary as in Fig. 238 to maintain this arc, however. Because of the heated filament, conduction is started just as in the vacuum-tube rectifier and the arc is established by the current which thus flows.

One of the earliest tubes of this type is that known as the "Tungar." The name comes from the use of a **tungsten** filament and **argon** as the gas. Its principal use is in charging storage batteries. Fig. 243 shows a typical circuit (note that this tube is represented by the same symbol as in Fig. 241*b*), and Fig. 244 is a battery charger using these tubes.

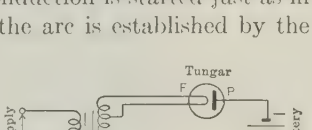



FIG. 243. Half-wave rectifier circuit for battery charging.

The most widely used type of gaseous-conduction rectifier is the one with mercury vapor. Practically all alternating-current radio receivers now contain a rectifying tube of this type to supply the direct-current voltages required.

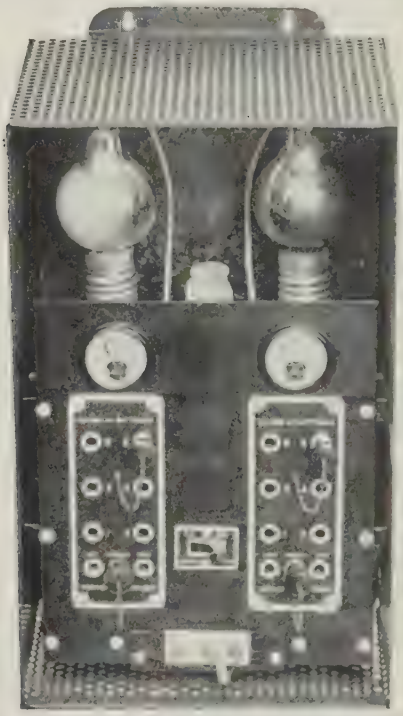


FIG. 244. A "Tungar" battery charger. *General Electric Co.*

They are generally made with two filaments and two plates inside one glass tube, and are used in full-wave circuits. Fig. 245 illustrates the use of a full-wave tube supplying a resistance load; the symbol used merely includes an additional plate so that the tube appears to have two plates, $P_1$

and $P_2$, and a common filament, $F$. If the two halves of the tube are represented by the conventional symbols used in Fig. 232 it will be found that the circuit of Fig. 245 is exactly the same as that of Fig. 232.

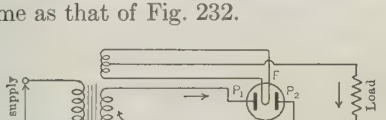



FIG. 245. Mercury-vapor tube with two plates used as a full-wave rectifier.

**122. Copper-oxide Rectifiers.** If a copper disc is heated under proper conditions, a layer of copper oxide can be formed on one side of the disc. Now if two lead plates are pressed against the two faces of the disc, as shown in Fig. 246, it will be found that current flows more readily from the copper oxide to the copper than in the opposite direction. Usually,

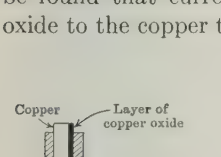



FIG. 246. Basic element of the copper-oxide rectifier.

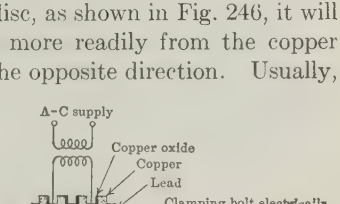



FIG. 247. Cross-section of copper-oxide rectifier assembly. Compare Fig. 233.

a group of these rectifying elements is assembled as shown in Fig. 247, and by comparison with Fig. 233 it will be found that the circuits are identical. Thus, this type of rectifier is generally used in full-wave circuits because of the simple construction required for such service.

The copper-oxide rectifier is used extensively as a means

of obtaining small amounts of direct-current power. Fig. 248 shows a complete rectifier for battery-charging service. It will be noted that metallic fins are provided between the rectifying discs to facilitate cooling and thus increase the rating of the rectifier.

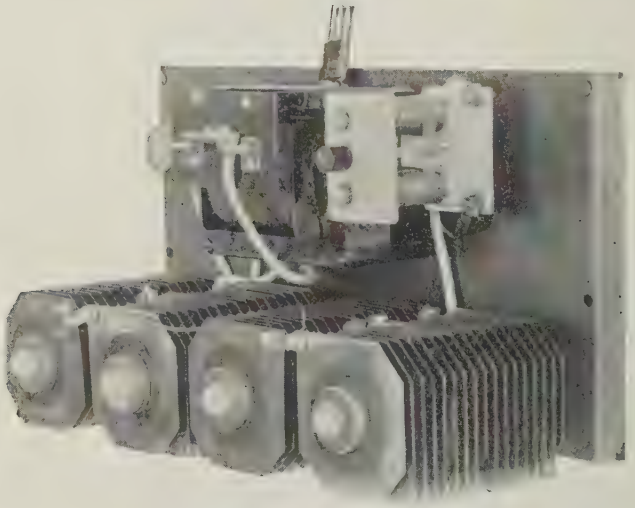


FIG. 248. A copper-oxide rectifier. *General Electric Co.*

**123. Rectifier Load Circuits. "Smoothing" Filters.** The current from a synchronous converter is the same as that delivered by a direct-current generator and can be used wherever direct-current power is required. Many rectifiers, however, deliver current with pulsations of large amplitude and such current cannot be used in all applications. For example, if it were supplied to a circuit containing an inductance with a solid iron core, the pulsations would set up eddy currents and would also cause hysteresis loss. If a polyphase rectifier is used, this pulsation or "ripple" is not so

serious and by connecting an inductive reactance in series with the load the ripple can be considerably decreased.

In battery charging, the wave form of the direct current is of no importance. It should be noted, however, that charging current does not flow during the entire half cycle of voltage. It cannot begin to flow until the instantaneous value of the rectified voltage exceeds the battery electromotive force, and ceases to flow when the rectified voltage is less than the battery voltage. It is the maximum value of the direct-current voltage which determines whether or not a charging current will flow and not the average value. In fact, it is possible to charge a battery with a rectified voltage which has an average value (as read by a direct-current voltmeter) which is less than the battery voltage. The peak values of the rectifier voltage are, of course, greater than the battery voltage.

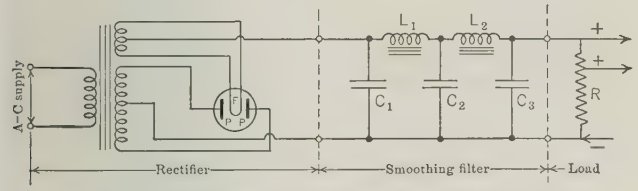



FIG. 249. Full-wave rectifier and smoothing filter.

When a rectifier is used to supply power for an amplifier or a radio receiver, it is necessary to eliminate practically all the ripple in the output wave by using a " smoothing " filter. If this is not done, the ripple is amplified in the receiver and the noise due to it will prevent reception entirely.

A typical rectifier and smoothing filter is shown in Fig. 249. The full-wave rectifier circuit is the same as that shown in Fig. 245. The output of the rectifier is then passed through a series-parallel combination of condensers and inductances before it reaches the load resistance $R$. A condenser will not conduct a steady direct current, but it offers a low impedance

to a varying voltage. Hence the condenser $C_1$ in Fig. 249 conducts the ripple component of the rectifier output readily but the steady component does not flow through the condenser. The inductance $L_1$, on the other hand, passes the steady component of current readily but tends to block the ripple component and causes more of it to flow through $C_1$. The output after passing through $L_1$ will not be steady enough usually, so the condenser $C_2$ and inductance $L_2$ again reduce the ripple and allow only steady current to pass. Finally, condenser $C_3$ is inserted to by-pass the remaining small amount of ripple so that the voltage reaching the resistance $R$ is practically free of ripple. The load resistance $R$ is commonly used as a voltage divider so that direct-current voltages of several values can be supplied to different parts of the receiver.

## SUMMARY OF CHAPTER X

**CONVERTERS** and **RECTIFIERS** are devices for changing alternating-current power into direct-current power. Synchronous (or rotary) converters and motor-generators are used for supplying large amounts of power. The mercury-arc rectifier can also be used for large power requirements, but most rectifiers are used for low-power applications.

**MOTOR-GENERATOR CONVERTER** consists of an a-c motor mechanically coupled to a d-c generator. The following distinguishing features are to be noted:

(a) Motor and generator windings are electrically separate; (b) no definite or necessary voltage ratio between a-c input and d-c output; (c) motor may feed directly from high-voltage a-c mains without transformers, while delivering d-c power at low or moderate voltage; (d) direct-current voltage may be easily controlled independent of all adjustments on the a-c side.

**SYNCHRONOUS CONVERTER** or **ROTARY** is essentially a direct-current generator with collecting-rings added, each ring being connected electrically to certain commutator bars. On account of changed distribution of current in armature winding, synchronous converter can handle more power without overheating than same machine used as straight d-c generator driven mechanically; hence usually commutator and brushes are larger and more prominent than in same size d-c generator.

Other special features to be noted, as follows:

(a) Fixed ratio exists between voltage at a-c rings and voltage at d-c brushes, which necessitates that transformers be used between converters and high-voltage a-c transmission line; (b) range of control over d-c voltage is not wide, and requires either special transformer taps and switches on a-c side, or change of power factor with special reactors; (c) efficiency is higher than that of a motor-generator set; (d) size, cost and space required are less than for a motor-generator converter of equal capacity.

RATIO of effective voltage between a-c rings to constant voltage between d-c brushes is

**0.71** for a single-phase (two-ring) converter.
**0.71** for each phase of a two-phase (four-ring) converter.
**0.62** for each phase of a three-phase (three-ring) converter.

The current ratio depends upon power factor at which converter is operated, and its efficiency. Synchronous converter is really a synchronous motor and a d-c generator combined in one machine, and its power factor may be easily adjusted by changing the field current, as in any synchronous motor. A certain field strength (normal) produces unity power factor; a higher strength (over-excited) makes converter take leading reactive volt-amperes; a lower strength (under-excited) makes converter take lagging reactive volt-amperes.

TO CHANGE VOLTAGE BETWEEN D-C BRUSHES of a synchronous converter, we must change voltage between a-c rings in like ratio. This may be done by:

(a) Having taps for various voltages on the transformers.

(b) Having sufficient reactance between transmission line and rings of converter; then, voltage at rings is raised by over-exciting the converter, causing it to take a leading component of current through the inductive reactance.

RECTIFIERS are devices which conduct current in one direction only. By allowing only half of an a-c wave to pass, a unidirectional current flows and this current has an average d-c value.

A HALF-WAVE rectifier merely blocks the flow of current during one half of the a-c cycle. A FULL-WAVE rectifier supplies unidirectional current during both halves of the a-c cycle.

POLYPHASE RECTIFIERS may be either half-wave or full-wave. A much steadier output current is obtained as the number of phases is increased.

The OUTPUT VOLTAGE of a rectifier can be changed only by changing the a-c supply voltage.

The most common TYPES OF RECTIFIERS are:

(a) Mercury-arc rectifiers, used either in single-phase or polyphase circuits. They are built in large sizes for electric railway supply.

(b) Vacuum-tube rectifiers, used principally in high-voltage, low-current, applications.

(c) Gaseous-conduction rectifiers, now used in many applications where vacuum-tube type was formerly used. Practically all radio receivers now built have this type of rectifier. Also used extensively in storage-battery chargers.

(d) Copper oxide rectifiers, used principally in low-voltage applications. Also used in electric instruments so that d-c meters will read a-c voltages and currents.

MERCURY ARC rectifiers for high-power work are usually polyphase. The anodes and the mercury pool are enclosed in a steel tank which is kept evacuated by an auxiliary pump. Their operation depends upon property of metallic arc, that current may pass into a metal electrode from its vapor, but not from the metal into the vapor.

VACUUM-TUBE RECTIFIER consists of a heated filament and a cold plate spaced apart inside an evacuated glass tube. Current can pass from the plate to the heated filament but not in the opposite direction.

GASEOUS-CONDUCTION RECTIFIER is built almost the same as the vacuum-tube type, but contains a small amount of inert gas or mercury vapor. The effect of the gas is to reduce the internal voltage drop of the tube and thus improve its efficiency.

COPPER-OXIDE RECTIFIER depends for its action on the fact that current passes readily from a layer of copper oxide to the underlying pure copper, but does not pass readily in the opposite direction.

SMOOTHING FILTERS are required between rectifiers and certain types of loads in order to reduce the pulsation or ripple in the rectifier output. In some cases a simple series inductance or shunt capacitance is sufficient. In radio receivers, it is necessary to use combinations of shunt condensers and series inductances in order to obtain a steady output voltage.

## PROBLEMS ON CHAPTER X

**Prob. 17–10.** What must be the alternating voltage of a 50-kw 230-volt single-phase synchronous converter?

**Prob. 18–10.** If the converter of Prob. 17 is operated at 90 per cent efficiency and 95 per cent leading power factor, what current would each a-c lead have to carry?

**Prob. 19–10.** What current would each a-c lead of the converter in Prob. 18 carry if it were a two-phase machine?

**Prob. 20 10.** What current would each a-c lead of machine in Prob. 18 carry, if it were a three-ring three-phase converter, operating at 90 per cent efficiency and 95 per cent power factor?

**Prob. 21–10.** What a-c voltage would the converter of Prob. 17 require if it were a six-ring diametrically-connected converter?

**Prob. 22–10.** What current would each lead of the converter in Prob. 21 carry if it operated at 90 per cent efficiency and 95 per cent power factor?

**Prob. 23–10.** It is desired to raise the direct voltage of a single-phase converter from 220 to 230 volts. How many turns must there be between taps on a transformer to produce the necessary change in the alternating voltage, if the alternating voltage for 220 direct volts is obtained from 1850 turns?

**Prob. 24 10.** Answer the question in Prob. 23 if the converter is a three-phase three-ring machine.

**Prob. 25–10.** If the voltage of a three-phase 2300-volt line were applied directly to the rings of a three-ring three-phase converter, what should be the voltage between the direct-current brushes?

**Prob. 26–10.** Two slip rings are added to an ordinary direct-current generator to make a single-phase synchronous converter. The machine has two poles, two sets of brushes (one positive and one negative), and 36 commutator bars. If we number the bars consecutively starting with any one, and connect one of the rings to Bar No. 1, to which bar should the other ring be connected? If the machine is rated 115-volts d-c, what a-c voltage should we impress between rings?

**Prob. 27–10.** If we desire to make a three-phase three-ring synchronous converter out of the d-c generator of Prob. 26, and we

connect Ring No. 1 to commutator Bar No. 1, to which bars should rings No. 2 and No. 3 be connected, respectively? What a-c voltage should be impressed between rings?

**Prob. 28–10.** Consider that the d-c generator of Prob. 26 had four poles and four brush sets (two positives, bearing on commutator bars 1 and 19, let us say, and two negatives bearing on bars 10 and 28 at the same instant). To what commutator bars should each ring be tapped, to make a two-ring converter?

**Prob. 29–10.** Solve Prob. 28 for a three-phase three-ring converter. Note that each ring must be connected to all bars which at the same instant will be similarly situated under north poles, in order that the currents and heating shall be uniformly distributed in the armature winding.

**Prob. 30–10.** A three-phase 500-kilowatt 650-volt synchronous converter receives power through three single-phase transformers from a three-phase 11,000-volt circuit. The high-tension windings of the transformers are connected in delta and the low-tension windings in wye. What should be the current and voltage rating of the high-tension and of the low-tension windings of each transformer, assuming that the rotary shall be able to operate at 0.90 power factor and 0.94 efficiency at full load?

**Prob. 31–10.** Draw a diagram showing how you would use four single-phase, vacuum-type rectifier tubes to produce a full-wave output, using the bridge type of connection shown in Fig. 233.

**Prob. 32–10.** It will be noted in the table of rectifier voltage ratios on page 346 that a single-phase, full-wave rectifier produces twice the d-c voltage of a single-phase, half-wave rectifier. However, a three-phase, full-wave rectifier does *not* produce twice the voltage of a three-phase, half-wave rectifier. Explain.

**Prob. 33–10.** A full-wave rectifier in an automobile service station delivers an open-circuit d-c voltage of 45 volts. Assuming no voltage drop in the rectifier, how many 6-volt storage batteries connected in series can be charged by this rectifier?

**Prob. 34–10.** Five 6-volt 10-ampere batteries are to be charged simultaneously with two 16-volt 5-ampere batteries and one 8-volt 10-ampere battery. (*a*) How would you arrange them? (*b*) Using a half-wave rectifier and assuming no voltage drop in the rectifier, what open-circuit d-c voltage must the rectifier deliver in order to charge the batteries arranged as in part (*a*)?

# APPENDIX

## TABLE I

### Power Factors and Reactive Factors

| Angle in degrees. | Power factor. | Reactive factor. | Angle in degrees. | Power factor. | Reactive factor. |
|---|---|---|---|---|---|
| 0 | 1.000 | .000 | 46 | .695 | .719 |
| 1 | .999 | .017 | 47 | .682 | .731 |
| 2 | .999 | .035 | 48 | .669 | .743 |
| 3 | .999 | .052 | 49 | .656 | .755 |
| 4 | .998 | .070 | 50 | .643 | .766 |
| 5 | .996 | .087 | 51 | .629 | .777 |
| 6 | .995 | .105 | 52 | .616 | .788 |
| 7 | .993 | .122 | 53 | .602 | .799 |
| 8 | .990 | .139 | 54 | .588 | .809 |
| 9 | .988 | .156 | 55 | .574 | .819 |
| 10 | .985 | .174 | 56 | .559 | .829 |
| 11 | .982 | .191 | 57 | .545 | .839 |
| 12 | .978 | .208 | 58 | .530 | .848 |
| 13 | .974 | .225 | 59 | .515 | .857 |
| 14 | .970 | .242 | 60 | .500 | .866 |
| 15 | .966 | .259 | 61 | .485 | .875 |
| 16 | .961 | .276 | 62 | .469 | .883 |
| 17 | .956 | .292 | 63 | .454 | .891 |
| 18 | .951 | .309 | 64 | .438 | .898 |
| 19 | .946 | .326 | 65 | .423 | .906 |
| 20 | .940 | .342 | 66 | .407 | .914 |
| 21 | .934 | .358 | 67 | .391 | .921 |
| 22 | .927 | .375 | 68 | .375 | .927 |
| 23 | .921 | .391 | 69 | .358 | .934 |
| 24 | .914 | .407 | 70 | .342 | .940 |
| 25 | .906 | .423 | 71 | .326 | .946 |
| 26 | .898 | .438 | 72 | .309 | .951 |
| 27 | .891 | .454 | 73 | .292 | .956 |
| 28 | .883 | .469 | 74 | .276 | .961 |
| 29 | .875 | .485 | 75 | .259 | .966 |
| 30 | .866 | .500 | 76 | .242 | .970 |
| 31 | .857 | .515 | 77 | .225 | .974 |
| 32 | .848 | .530 | 78 | .208 | .978 |
| 33 | .839 | .545 | 79 | .191 | .982 |
| 34 | .829 | .559 | 80 | .174 | .985 |
| 35 | .819 | .574 | 81 | .156 | .988 |
| 36 | .809 | .588 | 82 | .139 | .990 |
| 37 | .799 | .602 | 83 | .122 | .993 |
| 38 | .788 | .616 | 84 | .105 | .995 |
| 39 | .777 | .629 | 85 | .087 | .996 |
| 40 | .766 | .643 | 86 | .070 | .998 |
| 41 | .755 | .656 | 87 | .052 | .999 |
| 42 | .743 | .669 | 88 | .035 | .999 |
| 43 | .731 | .682 | 89 | .017 | .999 |
| 44 | .719 | .695 | 90 | .000 | 1.000 |
| 45 | .707 | .707 | 120 | .500 | .866 |

## Table of Allowable Carrying Capacities of Wires

The following table, showing the allowable carrying capacity of copper wires and cables of 98 per cent conductivity, according to the standard adopted by the National Board of Fire Underwriters, must be followed in placing interior conductors.

For insulated aluminum wire the safe carrying capacity is 84 per cent of that given in the following tables for copper wire with the same kind of insulation.

TABLE II

| B & S. gage number. | Area in circular mils. | Table A. Rubber insulation amperes. | Table B. Other insulation amperes. |
|---|---|---|---|
| 18 | 1,624 | 3 | 6 |
| 16 | 2,583 | 6 | 10 |
| 14 | 4,107 | 15 | 20 |
| 12 | 6,530 | 20 | 30 |
| 10 | 10,380 | 25 | 35 |
| 8 | 16,510 | 35 | 50 |
| 6 | 26,250 | 50 | 70 |
| 5 | 33,100 | 55 | 80 |
| 4 | 41,740 | 70 | 90 |
| 3 | 52,630 | 80 | 100 |
| 2 | 66,370 | 90 | 125 |
| 1 | 83,690 | 100 | 150 |
| 0 | 105,500 | 125 | 200 |
| 00 | 133,100 | 150 | 225 |
| 000 | 167,800 | 175 | 275 |
| | 200,000 | 200 | 300 |
| 0000 | 211,600 | 225 | 325 |
| | 300,000 | 275 | 400 |
| | 400,000 | 325 | 500 |
| | 500,000 | 400 | 600 |
| | 600,000 | 450 | 680 |
| | 700,000 | 500 | 760 |
| | 800,000 | 550 | 840 |
| | 900,000 | 600 | 920 |
| | 1,000,000 | 650 | 1000 |
| | 1,100,000 | 690 | 1080 |
| | 1,200,000 | 730 | 1150 |
| | 1,300,000 | 770 | 1220 |
| | 1,400,000 | 810 | 1290 |
| | 1,500,000 | 850 | 1360 |
| | 1,600,000 | 890 | 1430 |
| | 1,700,000 | 930 | 1490 |
| | 1,800,000 | 970 | 1550 |
| | 1,900,000 | 1010 | 1610 |
| | 2,000,000 | 1050 | 1670 |

## TABLE III

### Resistance of Soft or Annealed Copper Wire

| B. & S. gage, No. | Diameter in mils, $d$ | Area in circular mils, $d^2$ | Ohms per 1000 ft. at 20° C. or 68° F. | Pounds per 1000 ft. | B. & S. gage, No. | Diameter in mils, $d$ | Area in circular mils, $d^2$ | Ohms per 1000 ft. at 20° C. or 68° F. | Pounds per 1000 ft. |
|---|---|---|---|---|---|---|---|---|---|
| 0000 | 460.00 | 211,600 | 0.04893 | 640.5 | 21 | 28.462 | 810.10 | 12.78 | 2.452 |
| 000 | 409.64 | 167,810 | 0.06170 | 508.0 | 22 | 25.347 | 642.40 | 16.12 | 1.945 |
| 00 | 364.80 | 133,080 | 0.07780 | 402.8 | 23 | 22.571 | 509.45 | 20.32 | 1.542 |
| 0 | 324.86 | 105,530 | 0.09811 | 319.5 | 24 | 20.100 | 404.01 | 25.63 | 1.223 |
| 1 | 289.30 | 83,694 | 0.1237 | 253.3 | 25 | 17.900 | 320.40 | 32.31 | 0.9699 |
| 2 | 257.63 | 66,373 | 0.1560 | 200.9 | 26 | 15.940 | 254.10 | 40.75 | 0.7692 |
| 3 | 229.42 | 52,634 | 0.1967 | 159.3 | 27 | 14.195 | 201.50 | 51.38 | 0.6100 |
| 4 | 204.31 | 41,742 | 0.2480 | 126.4 | 28 | 12.641 | 159.79 | 64.79 | 0.4837 |
| 5 | 181.94 | 33,102 | 0.3128 | 100.2 | 29 | 11.257 | 126.72 | 81.70 | 0.3836 |
| 6 | 162.02 | 26,250 | 0.3944 | 79.46 | 30 | 10.025 | 100.50 | 103.0 | 0.3042 |
| 7 | 144.28 | 20,816 | 0.4973 | 63.02 | 31 | 8.928 | 79.70 | 129.9 | 0.2413 |
| 8 | 128.49 | 16,509 | 0.6271 | 49.98 | 32 | 7.950 | 63.21 | 163.8 | 0.1913 |
| 9 | 114.43 | 13,094 | 0.7908 | 39.63 | 33 | 7.080 | 50.13 | 206.6 | 0.1517 |
| 10 | 101.89 | 10,381 | 0.9972 | 31.43 | 34 | 6.305 | 39.75 | 260.5 | 0.1203 |
| 11 | 90.742 | 8,234.0 | 1.257 | 24.93 | 35 | 5.615 | 31.52 | 328.4 | 0.0954 |
| 12 | 80.808 | 6,529.9 | 1.586 | 19.77 | 36 | 5.000 | 25.00 | 414.2 | 0.0757 |
| 13 | 71.961 | 5,178.4 | 1.999 | 15.68 | 37 | 4.453 | 19.82 | 522.2 | 0.0600 |
| 14 | 64.084 | 4,106.8 | 2.521 | 12.43 | 38 | 3.965 | 15.72 | 658.5 | 0.0476 |
| 15 | 57.068 | 3,256.7 | 3.179 | 9.858 | 39 | 3.531 | 12.47 | 830.4 | 0.0377 |
| 16 | 50.820 | 2,582.9 | 4.009 | 7.818 | 40 | 3.145 | 9.89 | 1047 | 0.0299 |
| 17 | 45.257 | 2,048.2 | 5.055 | 6.200 | | | | | |
| 18 | 40.303 | 1,624.3 | 6.374 | 4.917 | | | | | |
| 19 | 35.890 | 1,288.1 | 8.038 | 3.899 | | | | | |
| 20 | 31.961 | 1,021.5 | 10.14 | 3.092 | | | | | |

## TABLE IV

### Table of Reactance of Wire Lines

| Size B. & S. gage. | Reactance in ohms per 1000 ft. of each wire at 60 cycles. Equivalent distance between centers of conductors. (See Par. 81.) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | ½ in. | 1 in. | 2 in. | 2½ in. | 3 in. | 4 in. | 6 in. | 8 in. |
| 14 solid..... | 0.0689 | 0.0848 | 0.1008 | 0.1079 | 0.1101 | 0.119 | 0.126 | 0.135 |
| 12 solid..... | 0.0636 | 0.0795 | 0.0952 | 0.101 | 0.1048 | 0.111 | 0.1207 | 0.127 |
| 10 solid..... | 0.058 | 0.074 | 0.0902 | 0.0953 | 0.0995 | 0.106 | 0.1153 | 0.122 |
| 8 solid..... | 0.0535 | 0.069 | 0.0848 | 0.0899 | 0.0941 | 0.101 | 0.1103 | 0.1165 |
| 6 solid..... | 0.0475 | 0.0635 | 0.0795 | 0.0846 | 0.0888 | 0.0955 | 0.1048 | 0.111 |
| 4 solid..... | 0.0425 | 0.0585 | 0.0741 | 0.0793 | 0.0834 | 0.090 | 0.0993 | 0.1055 |
| 4 stranded. | 0.039 | 0.0555 | 0.0712 | 0.0764 | 0.0805 | 0.087 | 0.0963 | 0.103 |
| 3 stranded. | 0.036 | 0 0525 | 0.0686 | 0.0738 | 0.0779 | 0.084 | 0.0933 | 0.100 |
| 2 stranded. | 0.0335 | 0.0495 | 0.0659 | 0.0711 | 0.0752 | 0.0815 | 0.0908 | 0.097 |
| 1 stranded. | 0.0315 | 0.0475 | 0.0632 | 0.0684 | 0.0725 | 0.079 | 0.0883 | 0.095 |
| 0 stranded. | 0.028 | 0.0445 | 0.0603 | 0.0654 | 0.0696 | 0.076 | 0.0853 | 0.092 |
| 00 stranded. | 0.026 | 0.042 | 0.0577 | 0.0628 | 0.0670 | 0.0735 | 0.0828 | 0.0895 |
| 000 stranded. | 0.023 | 0.039 | 0.0550 | 0.0601 | 0.0643 | 0.071 | 0.0803 | 0.0875 |
| 0000 stranded. | ..... | 0.0365 | 0.0523 | 0.0575 | 0.0616 | 0.068 | 0.0773 | 0.084 |

Note. For the reactance of lines using 25 cycles, multiply the table values by $\frac{25}{60}$. For 40 cycle values, multiply the table values by $\frac{40}{60}$.

## TABLE V

### Values of Maximum Voltage Drop Allowance for Loads which Include Lamps

| | In per cent. | In voltage between wires for | | | |
|---|---|---|---|---|---|
| | | 110 volts. | 115 volts. | 120 volts. | 240 volts. |
| Branches..... | 1.5 | 1.65 | 1.72 | 1.8 | 3.6 |
| Mains..... | 1.0 | 1.10 | 1.15 | 1.2 | 2.4 |
| Feeders..... | 2.5 | 2.75 | 2.88 | 3.0 | 6.0 |
| Total..... | 5.0 | 5.50 | 5.75 | 6.0 | 12.0 |

## TABLE VI

### Full-load Current of Induction Motors*

| Horse-power. | 2-phase† Amperes at | | | | 3-phase Amperes at | | | |
|---|---|---|---|---|---|---|---|---|
| | 110 volts. | 220 volts. | 440 volts. | 550 volts. | 110 volts. | 220 volts. | 440 volts. | 550 volts. |
| 0.5 | 4.3 | 2.2 | 1.1 | 0.9 | 5.0 | 2.5 | 1.3 | 1.0 |
| 1.0 | 5.7 | 2.9 | 1.4 | 1.2 | 6.6 | 3.3 | 1.7 | 1.3 |
| 2.0 | 10.4 | 5.0 | 3.0 | 2.0 | 12.0 | 6.0 | 3.0 | 2.4 |
| 3.0 | .... | 8.0 | 4.0 | 3.0 | .... | 9.0 | 4.5 | 4.0 |
| 5.0 | .... | 13.0 | 7.0 | 6.0 | .... | 15.0 | 7.5 | 6.0 |
| 7.5 | .... | 19.0 | 9.0 | 7.0 | .... | 22.0 | 11.0 | 9.0 |
| 10.0 | .... | 24.0 | 12.0 | 10.0 | .... | 27.0 | 14.0 | 11.0 |
| 15.0 | .... | 33.0 | 16.0 | 13.0 | .... | 38.0 | 19.0 | 15.0 |
| 20.0 | .... | 45.0 | 23.0 | 19.0 | .... | 52.0 | 26.0 | 21.0 |
| 25.0 | .... | 55.0 | 28.0 | 22.0 | .... | 64.0 | 32.0 | 26.0 |
| 30.0 | .... | 67.0 | 34.0 | 27.0 | .... | 77.0 | 39.0 | 31.0 |
| 50.0 | .... | 108.0 | 54.0 | 43.0 | .... | 125.0 | 63.0 | 50.0 |
| 75.0 | .... | 156.0 | 78.0 | 62.0 | .... | 180.0 | 90.0 | 72.0 |
| 100.0 | .... | 212.0 | 106.0 | 85.0 | .... | 246.0 | 123.0 | 98.0 |
| 150.0 | .... | 311.0 | 155.0 | 124.0 | .... | 360.0 | 180.0 | 144.0 |
| 200.0 | .... | 415.0 | 208.0 | 166.0 | .... | 480.0 | 240.0 | 195.0 |

* For full-load current of 208- and 200-volt motors, increase the corresponding 220-volt motor full-load current by 6 and 10 per cent, respectively.

† Values of current are for 2-phase, 4-wire system. For single phase motors, double corresponding current of 2-phase motor. Current in common wire of a 2-phase, 3-wire system is 1.41 times value in table.

## TABLE VII

### Wire Sizes Required for Motor Wiring

| Full-load motor current-amperes. | Minimum wire size-rubber insulation. | Full-load motor current-amperes. | Minimum wire size-rubber insulation. |
|---|---|---|---|
| 1 | 14 | 165 | 0000 |
| 12 | 14 | 180 | 0000 |
| 13 | 12 | 185 | 250,000 C.M. |
| 16 | 12 | 200 | 250,000 C.M. |
| 17 | 10 | 210 | 300,000 |
| 20 | 10 | 220 | 300,000 |
| 22 | 8 | 230 | 350,000 |
| 28 | 8 | 240 | 350,000 |
| 30 | 6 | 250 | 400,000 |
| 40 | 6 | 260 | 400,000 |
| 42 | 5 | 270 | 500,000 |
| 44 | 5 | 320 | 500,000 |
| 46 | 4 | 340 | 600,000 |
| 56 | 4 | 360 | 600,000 |
| 58 | 3 | 380 | 700,000 |
| 64 | 3 | 400 | 700,000 |
| 66 | 2 | 420 | 800,000 |
| 72 | 2 | 440 | 800,000 |
| 74 | 1 | 460 | 900,000 |
| 80 | 1 | 480 | 900,000 |
| 82 | 0 | 500 | 1,000,000 |
| 100 | 0 | 520 | 1,000,000 |
| 105 | 00 | 540 | 1,100,000 |
| 120 | 00 | 560 | 1,200,000 |
| 125 | 000 | 580 | 1,200,000 |
| 140 | 000 | 600 | 1,300,000 |
| 145 | 200,000 C.M. | 625 | 1,400,000 |
| 160 | 200,000 C.M. | | |

## TABLE VIII

### Power Factor of Induction Motors*

(Two- and Three-phase)

| Horse-power. | Power factors. | |
|---|---|---|
| | ¾ load. | Full load. |
| 1 | 0.60 | 0.70 |
| 2 | 0.71 | 0.79 |
| 5 | 0.82 | 0.85 |
| 10 | 0.79 | 0.84 |
| 20 | 0.83 | 0.88 |
| 50 | 0.85 | 0.89 |
| 100 | 0.87 | 0.91 |
| 200 | 0.94 | 0.96 |

* For 60 cycles; 25-cycle motors are practically the same.

## TABLE IX

### Demand Factors for Motor Loads*

| No. of motors. | Character of load. | Demand factor. |
|---|---|---|
| 1 | Individual drives — tools, etc. | 1.25 |
| 2 | Individual drives — tools, etc. | 1.00 |
| 3 | Individual drives — tools, etc. | 0.75 to 0.85 |
| 5 | Individual drives — tools, etc. | 0.60 to 0.70 |
| 10 | Individual drives — tools, etc. | 0.40 to 0.50 |
| 20 | Individual drives — tools, etc. | 0.40 |
| 1 | Group drives | 1.25 |
| 2 or more | Group drives | 0.70 to 0.75 |
| 1 | Fans, compressors, pumps, etc. | 1.25 |
| 2 or more | Fans, compressors, pumps, etc. | 0.85 to 1.00 |

The above values make no allowance for future increase in the load.
* Ratio of probable maximum load to connected load.

## TABLE X

### Equivalent Distance between Conductors in Three-wire Systems

Formula: equivalent distance = $\sqrt[3]{}$ product of the three distances.

| Distance between adjacent wires | ½ in. | 1 in. | 2 in. | 2½ in. | 3 in. | 4 in. | 5 in. | 6 in. | 8 in. |
|---|---|---|---|---|---|---|---|---|---|
| Equivalent distance. . . | 0.63 | 1.26 | 2.52 | 3.15 | 3.78 | 5.04 | 6.3 | 7.56 | 10.08 |

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